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ORIGINAL - SURVEY OR EXPOSITION

Robust portfolio optimization: a categorized bibliographic review

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Abstract
Robust portfolio optimization refers to finding an asset allocation strategy whose behavior under the worst possible realizations of the uncertain inputs, e.g., returns and covariances, is optimized. The robust approach is in contrast to the classical approach, where one estimates the inputs to a portfolio allocation problem and then treats them as certain and accurate. In this paper we provide a categorized bibliography on the application of robust mathematical programming to the portfolio selection problem. With no similar surveys available, one of the aims of this review is to provide quick access for those interested, but maybe not yet in the area, so they know what the area is about, what has been accomplished and where everything can be found. Toward this end, a total of 148 references have been compiled and classified in various ways. Additionally, the number of Scopus© citations by contribution and journal is recorded. Finally, a brief discussion of the review's major findings is provided and some solid leads on future directions are given.

Keywords Robust mathematical programming · Portfolio selection · Bibliographic review

1 Introduction

Optimization affected by parameter uncertainty has long been a focus of the mathematical programming community (Bertsimas et al. 2011). Two of the most popular methodologies for handling parameter uncertainty are stochastic and robust optimization. Stochastic optimization starts by assuming the uncertainty has a probabilistic description. On the other hand, robust optimization, is a more recent approach to optimization under uncertainty, in which

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the uncertainty in the model is not stochastic, but rather deterministic and set-based. Instead of seeking to immunize the solution in some probabilistic sense to stochastic uncertainty, here the decision maker constructs a solution that is feasible for any realization of the uncertainty in each set.

The framework of robust mathematical programming fits perfectly with the modeling of many modern financial optimization problems, as these problems involve future values of security prices, interest rates, exchange rates and so forth, which are not known in advance, but can only be forecasted or estimated. The substantial advances in the areas of robust optimization have proven to be of great importance, especially in the practical applicability and reliability of portfolio optimization procedures.

Unlike the traditional approach, where inputs to the portfolio allocation framework are treated as deterministic, robust portfolio optimization incorporates the notion that inputs have been estimated with errors. In this case, the inputs are not traditional point estimate forecasts, such as for expected returns and asset covariances, but rather uncertainty sets including these point estimates.

In the last 20 years, research activity in robust portfolio optimization has become quite substantial (Ghahtarani and Najafi 2013; Mansini et al. 2014; Gorissen et al. 2015). Kolm et al. (2014) reviewed the 60-year course of portfolio optimization and have confirmed significance of the portfolio robustness trend that has emerged. Results from Google Scholar queries provide some interesting figures about the area's momentum: When searching for modern portfolio theory, we obtained 928,000 results, when searching for portfolio optimization, we obtained 758,000 results, yet when searching for robust portfolio optimization, we obtained 131,000 results. This shows the extent to which robust portfolio optimization has grown to have impact since it has come on the scene.

This paper provides a categorized review on the application of robust mathematical programming to the problem of portfolio selection. A total of 148 references have been compiled and classified. Initially, the contributions are categorized by type of publication and publisher, while the distribution of papers over time is also provided. Moreover, all references are classified by journal, either as coming out of the field of operations research or finance. Additionally, the number of contributions by author is reported. Also, we record the number of Scopus© citations by contribution and by journal. Finally, a brief discussion of the literature's major findings is also provided.

Our fundamental purpose is to provide rapid access to the topic for finance practitioners, graduate students, and others looking for such a vehicle. More specifically, our aim with this paper is to provide quick access for those interested, but not already in the area, so they know: (a) what the area is about, (b) what has been accomplished, (c) where everything can be found, and (d) have some leads on future directions.

The paper proceeds as follows: In Sect. 2 we describe the problem of robust portfolio optimization in a, as much as possible, straightforward and simplified manner. In Sect. 3 we provide details about the underlying bibliographic analysis. In Sect. 4 we report some basic findings with regard to the robust portfolio optimization literature. Finally, concluding remarks are given in Sect. 5.

2 Problem setting

Optimization and more specifically robust optimization has become, as seen by many, an important component in applications. The development of robust methods for optimization is one of the major achievements in the theory of modern financial modeling (Fabozzi et al. 2006). Robust technologies assume that both models and inputs are uncertain. They evaluate the consequences of errors in the models and introduce modifications that mitigate the potentially negative effects of model and estimation errors.

Indeed, inputs to the portfolio allocation process are generally unknown and must be estimated. However, any estimate is subject to error. Therefore, it would be useful if the portfolio optimization problem could handle inputs given in other forms, such as ranges or even as statistical distributions, rather than as typical point estimates (Bertsimas et al. 2011). Additionally, the classical mean–variance framework assumes that all estimates are equally precise and handles all securities equally. But it would be desirable if, when managers calculate optimal portfolios, differences in the precision of the estimates could be explicitly incorporated in the process. Providing this benefit is an underlying target of robust portfolio optimization.

To construct a robust portfolio, a manager needs to realize how uncertainty in return and correlation estimation translates into a distribution of portfolios (Fabozzi et al. 2007a). As an example, the Monte Carlo methodology offers an effective approach; that is, in these methods, one simulates a large number of different portfolios. This approach typically entails a significant computational burden, since every portfolio requires a separate optimization. In contrast, robust mathematical programming is typically in a position to offer computationally a much less expensive approach.

Ben-Tal and Nemirovski (1998, 1999) show how a portfolio manager can solve a robust version of the Markowitz optimization problem efficiently, that is, in about the same time as needed for solving it in a conventional way. The method uses the distribution from the estimation process to find a robust portfolio in the form of a single optimization. It thereby embeds uncertainties about inputs into a deterministic framework. Most classical portfolio optimization formulations have robust counterparts that have roughly the same computational complexity and can be solved in approximately the same amount of time as the classical problem (Goldfarb and Iyengar 2003).

Fabozzi et al. (2006) argue that robust Markowitz portfolios are more stable than other portfolios as inputs fluctuate, and their out-of-sample performance tends to be better than classical mean–variance portfolios. Moreover, the robust optimization approach offers high flexibility and many interesting applications. As an example, robust portfolio optimization can be very effective in large-scale problems involving many complex investment policy constraints on transaction costs, turnover, taxes, etc.

Also, it should be noted that the robust counterparts to the classical mean–variance formalism are typically not regular quadratic programming (QP) problems (Fabozzi et al. 2007b). The resulting type of a robust optimization problem depends on the specific uncertainty set used with many if not most of the uncertainty sets used in practice leading to second-order cone programs (SOCP), which can be solved by contemporary optimization algorithms in approximately the same time as the original problem.

We now present two examples in order to illustrate some of the essentials of robust portfolio optimization. The concepts in these examples are not new and have been around since the beginning of the field, but it is from concepts like these that the field has grown, in many directions, to what it is today.

The *first example* is very small. Consider an investment fund that is entering a difficult period. The board of directors of the fund knows that there are expected return vectors from within a known set of conceivable expected return vectors $U_{\hat{\mu}}$, which when combined with a wrong weighting vector $\boldsymbol{w}$ from the set of all feasible weighting vectors S, that could harm the fund. Also, the board suspects that there could be a number of such combinations.

Putting off usual attempts to optimize portfolio return, suppose the board decides that for the upcoming period top priority is to be placed on avoiding poor outcomes to the greatest guaranteed extent.

Knowing that for each $\boldsymbol{w} \in S$ there is a $\boldsymbol{\mu} \in U_{\hat{\mu}}$ that is worst for that $\boldsymbol{w}$, one way of maximally avoiding poor outcomes is to seek the $\boldsymbol{w}$ whose worst realization of an expected return vector, apart from ties, is better than for any other $\boldsymbol{w} \in S$.

Let the ordinary problem of the investment fund be one whose objective is:

$$\max \boldsymbol{\mu}'\boldsymbol{w} \tag{1}$$

and whose feasible region S contains only the six weighting vectors[1]:

$$S = \{(.1, .1, .8), (.1, .7, .2), (.3, .3, .4), (.5, .2, .3), (.6, .2, .2), (.7, .2, .1)\} \tag{2}$$

The problem is seen as a tiny one for maximizing portfolio return in a future time period. The $\boldsymbol{w} \in S$ that is the board's best option for dealing with its situation is limited of course to one of the six vectors of portfolio weights of length 3 in S.

With the board aware that there is unavoidable uncertainty about the expected return vector in the future period, the board can only suggest a best estimate $\hat{\boldsymbol{\mu}}$ for use as $\boldsymbol{\mu}$ in (1) knowing full well that there can be significant error in the estimate. In keeping the problem simple, albeit abstract, let us assume that the board has knowledge that the correct expected return vector for the future period is one of three possibilities in:

$$U_{\hat{\mu}} = \{(4, 6, -2), (6, -2, 2), (-4, 4, 4)\} \tag{3}$$

but the board does not know which one (where the entries in (3) are profitability indices). Here, $U_{\hat{\mu}}$ is the problem's uncertainty set as it specifies where in the problem the uncertainty that is to be taken into account lies. A way to view the expected return vectors (3) is as being merely three representatives, for purposes of illustration, from what in reality would be a more involved $U_{\hat{\mu}}$.

With the uncertainty causing the board consternation, the board is very concerned about winding up with an unnecessarily bad combination of a $\boldsymbol{w} \in S$ and a $\boldsymbol{\mu} \in U_{\hat{\mu}}$. Note that in this problem there are eighteen different $(\boldsymbol{w}, \boldsymbol{\mu})$ combinations, and who knows, some of them could be very bad. As a consequence, the board would like to immunize the investment fund from the worst combinations of $(\boldsymbol{w}, \boldsymbol{\mu})$ to the greatest guaranteed extent.

In the literature, techniques for doing this fall under the topic of robust optimization. "Robustifying" this problem, that is, re-casting the problem so it deterministically takes into account all of the uncertainties in $U_{\hat{\mu}}$ when optimizing, results in the max–min formulation:

$$\max \left\{ \min_{\mu \in U_{\hat{\mu}}} \left\{ \boldsymbol{\mu}'\boldsymbol{w} | \boldsymbol{w} \in S \right\} \right\} \tag{4}$$

What (4) does is solve for the $\boldsymbol{w} \in S$ that keeps us away from as many of the low objective function value combinations of a $\boldsymbol{w} \in S$ and a $\boldsymbol{\mu} \in U_{\hat{\mu}}$ that can be guaranteed.

[1] Despite the small number of points in S in this example, feasible regions in portfolio selection can be discrete as a result of lot size and other constraints.

In this problem, the solution of (4) is shown numerically by computing the dot product of each $\boldsymbol{w} \in S$ with each $\boldsymbol{\mu} \in U_{\hat{\mu}}$ to form the 6×3 matrix:

$$\begin{bmatrix} -6 & 20 & 32 \\ 42 & -4 & 32 \\ 22 & 20 & 16 \\ 26 & 32 & 0 \\ 32 & 36 & -8 \\ 38 & 40 & -16 \end{bmatrix}$$

In the matrix, for instance, the 36 of element (5, 2) is the dot product of the fifth $\boldsymbol{w} \in S$ with the second $\boldsymbol{\mu} \in U_{\hat{\mu}}$.

From the matrix we see that the solution to (4), which is the robust version of the problem, is the third $\boldsymbol{w} \in S$ as the worst that can happen to the investment fund with this choice of $\boldsymbol{w}$ is an objective function value of 16. In this way, the solution is robust in that it avoids the worst objective function values of -16, -8, -6, -4 and 0 seen elsewhere in the matrix with the other choices of $\boldsymbol{w}$. Note that along with these protections, there is still the possibility of portfolio return being 20 or 22.

The solution of the robust version of this problem was easily obtained because of the small, discrete natures of S and $U_{\hat{\mu}}$. This will not happen in realistic applications where, instead of a few variables, problems are likely to have a few hundred variables, and uncertainty sets can be counted on to be mostly continuous and more complicated. To illustrate, probably the three most popular types of uncertainty sets for expected return vectors are:

1. Box uncertainty

$$U_\delta = \left\{\boldsymbol{\mu} | \left|\mu_i - \hat{\mu}_i\right| \leq \delta_i\right\}$$

where Σ_μ is a special covariance matrix (Fabozzi et al. 2007a).

2. Ellipsoidal uncertainty

$$U_\eta = \left\{\boldsymbol{\mu} | \left(\mu_i - \hat{\mu}_i\right)' \Sigma_\mu^{-1} \left(\mu_i - \hat{\mu}_i\right) \leq \eta^2\right\}$$

3. Ellipsoidal uncertainty

$$U_\gamma = \left\{\boldsymbol{\mu} = \sum_{i=1}^{\kappa} \gamma_i \boldsymbol{y}^i | \gamma_i \geq 0, \sum_{i=1}^{\kappa} \gamma_i = 1\right\}.$$

As for the small example, the three expected return vectors in (3) could be considered as representatives coming from any of the three uncertainty set structures above. For instance, if from a polytopic uncertainty set, the three representatives could easily be the $\boldsymbol{y}^i$, $k = 3$.

Taking on as our *second example* a more standard application, let us consider the problem of conventional mean–variance portfolio optimization:

$$\max \left\{\boldsymbol{\mu}' \boldsymbol{w} - \lambda \boldsymbol{w}' \Sigma \boldsymbol{w} | \boldsymbol{w} \in S\right\} \tag{5}$$

but with box uncertainty set U_δ specified as its best estimate of the future expected return vector. In (5), S is any set defined by linear constraints. Under the same logic as the first problem, the robust version of (5) is obtained by re-casting it as:

$$\max \left\{ \min_{\mu \in U_\delta} \left\{\boldsymbol{\mu}' \boldsymbol{w} - \lambda \boldsymbol{w}' \Sigma \boldsymbol{w} | \boldsymbol{w} \in S\right\} \right\} \tag{6}$$

However, because of the problem's increased number of variables and that U_δ and S are not small discrete sets, this robust formulation is not as easy to solve. Fortunately, as pointed out by Fabozzi et al. (2007a), an optimal solution to (6) is obtained by solving:

$$\max \left\{ \boldsymbol{\mu}'\boldsymbol{w} - \delta'|\boldsymbol{w}| - \lambda \boldsymbol{w}'\Sigma\boldsymbol{w} | \boldsymbol{w} \in S \right\} \tag{7}$$

In this way, after using a trick to get rid of the absolute value signs, (7) results in a quadratic program problem that can be solved on any number of platforms such as MOSEK, GAMS, etc. In other words, the robust version of the problem is tractable. That is, it can solved in acceptable time using known available algorithms. Tractability is a bottom-line issue in robust optimization as there is little purpose to re-cast a problem into a robust version if the robust version is not solvable.

The advantage of the other two types of uncertainty sets is the same. They result in tractable re-cast versions of the problem. However, in the case of ellipsoidal uncertainty sets, the re-cast problems are second-order cone problems that can be solved using the same packages as mentioned above in roughly the same amount of time. Hence, with this being a sampling, we can see why the tools of robust optimization cause the field of robust portfolio optimization to be such an important, attractive, and growing field.

3 Bibliographic analysis

The classification of robust portfolio optimization models is often insightful. Bertsimas et al. (2011) adopt the following categorization of the various models that have been proposed: (a) uncertainty models for return mean and covariance, (b) distributional uncertainty models, (c) robust factor models, and (d) multi-period robust models.

In Fabozzi et al. (2010) there is another classification. They distinguish between: (a) portfolio models with known moments, (b) portfolio models with unknown means (with either box or ellipsoidal uncertainty on the mean), (c) portfolio models with unknown mean and covariance (factor models or models with box uncertainty on the covariance matrix), (d) robust VaR models, (e) robust CVaR models (mixture or discrete distribution models), and (f) models depending on the size and shape of the uncertainty sets (models with specifications, either for factors or for mean and covariance).

Also, an earlier classification by Goldfarb and Iyengar (2003) assumes the following categories: (a) robust mean–variance portfolio selection models (distinct study of the robust minimum variance, maximum return and maximum Sharpe ratio problems), (b) robust VaR portfolio selection models, (c) multivariate regression models with norm selection, and (d) robust portfolio allocation models with uncertain covariance matrices (distinct study of the uncertainty structure for covariance inverse and regular covariance). Similar comprehensive categorizations are also provided by Kim et al. (2014a), Scutella and Recchia (2010) and Tütüncü and Koenig (2004).

This section provides a categorized bibliographic analysis on the application of robust mathematical programming to the portfolio selection problem. In compiling and classifying the 148 references included in this study, the distribution of contributions by type of publication is as follows: operations research journals (91), finance journals (41), books (6), edited volumes (6), and PhD dissertations (4).[2]

[2] As far as the distribution of the contributions by publisher, we report that most appear in publications of Elsevier (52), Springer (43), INFORMS (17), Taylor & Francis (10) and Wiley (6), with the remaining articles spread across 9 other publishers.

Table 1 Distribution of contributions by 3-year periods

3-Year periods	Number of contributions
1995–1997	3
1998–2000	3
2001–2003	7
2004–2006	10
2007–2009	25
2010–2012	22
2013–2015	30
2016–2018	36
2019	12[a]

[a]Through November 30, 2019

Tables 1 and 2 show the distribution of references over time. Specifically, Table 1 shows the references by 3-year periods. Clearly, the publication rate has been on the increase since 1995 with 2007–2009 being the 3-year period in which the number of publications increased the most (25 relative to the 10 of the previous period), while the 2016–2018 period has been the most prolific (36). These findings are supported by the more detailed views of Table 2 with (b) showing years 2014, 2017 and 2018 being of greatest productivity.

Table 3 classifies the contributions by journal in the field of operations research (91 out the 148). The most (22) have been published in the *European Journal of Operational Research* by Elsevier followed by the *Annals of Operations Research* by Springer (12) and *Operations Research* by INFORMS (12). The remaining articles are distributed across 24 other journals.

Similarly, Table 4 classifies the contributions by journal in the field of finance (41 out the 148). The most (9) have been published in the *Journal of Asset Management* by Springer followed by the *Journal of Banking & Finance* by Elsevier (5) and *Quantitative Finance* (5) by Taylor & Francis. The remaining articles are distributed across 11 other journals.

Additionally, Table 5 shows the classification of contributions by various types of publication (16 out the 148). We observe the existence of 6 books, 6 edited volumes articles and 4 PhD dissertations.

Table 6 summarizes the number of contributions by author, with at least 3 connections (papers on which they are authors) being the condition set for an author to be included in the table. The most (18) have been published by Frank J. Fabozzi (EDHEC Business School) and Dessislava Pachamanova (Babson College) (14). The remaining articles are distributed across 21 other authors.

Table 7 presents the 76 most influential papers in the field, of which 56 have been published in operations research journals and 20 in finance journals. At least 10 connected citations was the condition set for a reference to be included in this table. The total number of citations of these 76 papers is 12,637. Furthermore, Table 7 presents the number of Scopus© citations by contribution, as observed through November 30, 2019. The study of Bertsimas and Sim (2004) is the most cited paper with 1842 citations. This is followed by Rockafellar and Uryasev (2002) with 1625, Ben-Tal and Nemirovksi (1998) with 1223, Ben-Tal and Nemirovski (1999) with 1046, Mulvey et al. (1995) with 949, Bertsimas et al. (2011) with 898, and so forth.

Finally, Table 8 shows the number of Scopus© citations by journal through November 30, 2019. In particular, *Operations Research* (INFORMS) has 4095 connected citations, followed by the *Journal of Banking & Finance* (Elsevier) with 1741, *Mathematics of Operations Research* (INFORMS) with 1614, *Operations Research Letters* (Elsevier) with 1092, *SIAM*

Table 2 Panel (a) shows yearly contributions and Panel (b) shows contributions by year sorted from largest to smallest

Year	Number of contributions	Year	Number of contributions
(a)		*(b)*	
1995	1	2014	15
1996	1	2017	14
1997	1	2018	13
1998	1	2011	12
1999	1	2019	12[a]
2000	1	2007	11
2001	0	2016	9
2002	5	2013	8
2003	2	2009	7
2004	4	2010	7
2005	1	2015	7
2006	6	2006	6
2007	11	2008	6
2008	6	2002	5
2009	7	2004	4
2010	7	2012	3
2011	12	2003	2
2012	3	1995	1
2013	8	1996	1
2014	15	1997	1
2015	7	1998	1
2016	9	1999	1
2017	14	2000	1
2018	13	2005	1
2019	12[a]	2001	0

[a]Through November 30, 2019

Review (SIAM) with 898, the *European Journal of Operational Research* (Elsevier) with 752, *Mathematical Programming* (Springer) with 656, the *Review of Financial Studies* (Oxford University Press) with 448, the *Annals of Operations Research* (Springer) with 371, and so forth.

What is striking about all of the above referenced classifications is how different their descriptions can be from one another. This is indicative of the originality of the area, its rapid growth, and ever-growing range of applications to which its tools and techniques have been found to be amenable. In summary, it represents the many different perspectives from which one can look at the potentialities of the field.

4 Major findings

In this section, we briefly report some critical findings with regard to the robust portfolio optimization literature (Pachamanova 2013). First, it has to be mentioned that numerous studies using simulated and real market data provide support for the claim that robust port-

Table 3 Classification of contributions by journal in the field of OR

Journal	Number of contributions	References
European Journal of Operational Research	22	Vassiadou-Zeniou and Zenios (1996), Shen and Zhang (2008), Huang et al. (2010), Gregory et al. (2011), Kawas and Thiele (2011b), Zymler et al. (2011), Dupacova and Kopa (2014), Ehrgott et al. (2014), Fliege and Werner (2014), Kakouris and Rustem (2014), Kim et al. (2014c), Kolm et al. (2014), Mansini et al. (2014), Maillet et al. (2015), Fernandes et al. (2016), Gulpinar and Canakoglu (2017), Ling et al. (2017), Xidonas et al. (2017b), Fakhar et al. (2018), Liu and Chen (2018), Lotfi and Zenios (2018) and Ling et al. (2019)
Annals of Operations Research	12	Tütüncü and Koenig (2004), Fabozzi et al. (2010), Dupacova and Kopa (2012), Kim et al. (2013b), Scutella and Recchia (2013), Marzban et al. (2015), Hasuike and Mehlawat (2018), Kapsos et al. (2018), Kim et al. (2018a, b), Pac and Pinar (2018) and Pandolfo et al. (2019)
Operations Research	12	Mulvey et al. (1995), El Ghaoui et al. (2003), Bertsimas and Sim (2004), Lutgens et al. (2006), Popescu (2007), DeMiguel and Nogales (2009), Natarajan et al. (2009), Zhu and Fukushima (2009), Delage and Ye (2010), Chen et al. (2011), Glasserman and Xu (2013) and Doan et al. (2015)
OR Spectrum	6	Pinar (2007), Kawas and Thiele (2011a), Pae and Sabbaghi (2014), Desmettre et al. (2015), Gulpinar et al. (2016) and Sharma et al. (2017)
Computers & Operations Research	5	Bertsimas and Pachamanova (2008), Moon and Yao (2011), Chen and Kwon (2012), Fonseca and Rustem (2012) and Gulpinar et al. (2014)
Optimization	4	Schöttle and Werner (2009), Pinar (2016), Wang et al. (2017) and Ding et al. (2018)
Operations Research Letters	3	Ben-Tal and Nemirovksi (1999), Pinar and Tütüncü (2005) and Huang et al. (2007)
Mathematics of Operations Research	3	Ben-Tal and Nemirovksi (1998), Goldfarb and Iyengar (2003) and Bo and Capponi (2017)
Mathematical Programming	3	Ben-Tal and Nemirovksi (2002), Lu (2011a) and de Klerk et al. (2019)
Journal of Computational & Applied Mathematics	2	Chen and Tan (2009) and Goel et al. (2019)
Management Science	2	Natarajan et al. (2008) andRujeerapaiboon et al. (2016)
Omega	2	Oguzsoy and Guven (2007) and Gorissen et al. (2015)
Applied Mathematics & Optimization	1	Liu et al. (2019)
Central European Journal of Operations Research	1	Kara et al. (2019)
Computational Management Science	1	Kawas and Thiele (2017)
Expert Systems with Applications	1	Chen and Zhou (2018)
EURO Journal on Computational Optimization	1	Gabrel et al. (2018)

Table 3 continued

Journal	Number of contributions	References
IMA Journal of Management Mathematics	1	Recchia and Scutella (2014)
International Transactions in Operational Research	1	Cacador et al. (2019)
Journal of Combinatorial Optimization	1	Chen and Wei (2019)
Journal of the Operational Research Society	1	Lee et al. (2019)
Journal of Optimization Theory & Applications	1	Kim et al. (2014a)
Metrica	1	Lauprete et al. (2002)
Optimization Methods & Software	1	Lu (2011b)
SIAM Journal on Optimization	1	Calafiore (2007)
SIAM Review	1	Bertsimas et al. (2011)
4OR	1	Scutella and Recchia (2010)

folio optimization outperforms classical mean–variance optimization a high percentage of time (Kim et al. 2018a). The main aspect that differs across robust formulations of the portfolio optimization problem is how the uncertainty sets are modeled (Pachamanova 2006). This suggests that finding a proper balance between robustness and a suitable and practical definition of uncertainty sets may have a nontrivial impact on portfolio performance.

Also, tests using both simulated and market data appear to confirm that robust optimization generally results in more stable portfolio weights. As observed by Fabozzi et al. (2007a), robust optimization tends to avoid corner solutions. This is because at a corner solution an additional security is either added to or dropped from the portfolio. As a result, as robust mean–variance optimization frequently improves worst-case portfolio performance, it also results in smoother and more consistent portfolio returns.

Indeed, one important property of robust efficient portfolios is that they remain relatively unchanged over long periods of time. Re-calculating robust efficient portfolios as new data become available generally generates smaller turnover volumes as compared to their classical counterparts, thus leading to reduced trading costs if the portfolios are to be rebalanced regularly (Bertsimas and Pachamanova 2008). We suggest that as robust optimal portfolios calculated at the beginning of an investment horizon appear to remain optimal or near optimal throughout the horizon, they may be attractive for buy-and-hold investors.

Moreover, the worst-case behavior of portfolios of different assets can be significantly enhanced using robust asset allocation methodologies, often with only minor performance losses on more likely scenarios (Tütüncü and Koenig 2004). Specifically, as the size of uncertainty sets increase, both the benefits of robust portfolios under worst-case scenarios and their under-performance under most likely scenarios appear to increase. This trade-off suggests that a cost–benefit analysis concerning the size of uncertainty sets needs to be performed. The right size will depend on the risk-profile of investors.

Table 4 Classification of contributions by journal in the field of finance

Journal	Number of contributions	References
Journal of Asset Management	9	Ceria and Stubbs (2006), Khodadadi et al. (2006), Scherer (2007), Van Hest and De Waegenaere (2007), Iyengar et al. (2010), Guastaroba et al. (2011), Gulpinar et al. (2011), Deng et al. (2013) and Lu et al. (2019)
Journal of Banking & Finance	5	Rockafellar and Uryasev (2002), Quaranta and Zaffaroni (2008), Gulpinar and Pachamanova (2013), Kim et al. (2014b) and Xing et al. (2014)
Quantitative Finance	5	Zhu et al. (2009), Hellmich and Kassberger (2011), Bergen et al. (2018), Simoes et al. (2018) and Kang et al. (2019)
Finance Research Letters	4	Kim et al. (2013a), Li et al. (2016), Belhajjam et al. (2017) and Han et al. (2017)
Journal of Economic Dynamics & Control	3	Costa and Paiva (2002), Huang et al. (2008) and Plachel (2019)
Journal of Portfolio Management	3	Pachamanova (2006), Fabozzi et al. (2007b) and Pachamanova and Fabozzi (2014)
Economic Modelling	2	Ghahtarani and Najafi (2013) and Xidonas et al. (2017a)
Annals of Finance	2	Ma et al. (2008) and Flor and Larsen (2014)
Journal of Computational Finance	2	Bienstock (2007) and Zhu et al. (2015)
Review of Financial Studies	2	Maenhout (2004) and Garlappi et al. (2007)
Energy Economics	1	Costa et al. (2017)
International Journal of Theoretical & Applied Finance	1	Cong and Oosterlee (2017)
International Review of Financial Analysis	1	Kim et al. (2015)
Mathematical Finance	1	Natarajan et al. (2010)

Table 5 Classification of contributions by various types of publication

Type of publication	Number of contributions	References
Books	6	Kouvelis and Yu (1997), Fabozzi et al. (2007a), Cornuéjols and Tütüncü (2006), Ben-Tal et al. (2009), Kim et al. (2016), Pachamanova and Fabozzi (2016)
Edited volumes	6	Ben-Tal et al. (2002), Pachamanova (2013), Gulpinar and Hu (2016), Iyengar and Ma (2016), Millington and Niranjan (2017), Pachamanova et al. (2017)
PhD dissertations	4	Lobo (2000), Lutgens (2004), Brown (2006), Guastaroba (2010)

Based on the above discussion, a complaint frequently communicated by users of conventional mean–variance methods is that the portfolios generated often contain many small holdings (Fabozzi et al. 2006). While this appears contradictory to the well-known property of diversification, this relates to the mechanics of the underlying mean–variance algorithm, which tries to negotiate the two conflicting objectives of return maximization and risk mini-

Table 6 Number of contributions by author

Author	Number of contributions
Fabozzi, F.J.	18
Pachamanova, D.	14
Kim, J.H.	10
Kim, W.C.	10
Gulpinar, N.	7
Zhu, S.	6
Ben-Tal, A.	5
Huang, D.	5
Nemirovski, A.	5
Tütüncü, R.H.	5
Canakoglu, E.	4
Fukushima, M.	4
Natarajan, K.	4
Pinar, M.C.	4
Thiele, A.	4
Bertsimas, D.	3
Iyengar, G.	3
Kawas, B.	3
Kolm, P.N.	3
Recchia, R.	3
Rustem, B.	3
Scutella, M.G.	3
Zenios, S.A.	3

mization, thus generating two rankings of the assets, i.e., one for high return potentials and one for low risk potentials. A similar phenomenon is observed with the robust portfolio optimization approach. The difference is that the method now ranks the worst-case return potentials of the assets, as well as their worst-case riskiness and then proceeds from the assets with highest worst-case returns to those with lowest worst-case variance, thus resulting in generally smaller portfolios.

However, robust optimization is not a perfect remedy. By using robust portfolio optimization, investors are likely to trade-off the optimality of their portfolio allocation in cases where nature behaves as they predicted for protection against the risk of inaccurate estimation (Fabozzi et al. 2007b). Therefore, managers using the technique should not expect to do better than classical portfolio optimization when estimation errors have little impact or when typical scenarios occur. In contrast, they should expect insurance when their estimates differ from the actual realized values up to the amount they have pre-specified in the modeling process.

5 Concluding remarks

While the literature on robust portfolios from operations research is abundant and insightful, there is a general lack of empirical studies on how the methods work in real world applications.

Table 7 Number of Scopus© citations by contribution

References	Number of citations	References	Number of citations
Bertsimas and Sim (2004)	1842	Glasserman and Xu (2013)	31
Rockafellar and Uryasev (2002)	1625	Pinar (2007)	31
Ben-Tal and Nemirovksi (1998)	1223	Chen and Kwon (2012)	31
Ben-Tal and Nemirovksi (1999)	1046	Zhu et al. (2009)	30
Mulvey et al. (1995)	949	Vassiadou-Zeniou and Zenios (1996)	29
Bertsimas et al. (2011)	898	Lauprete et al. (2002)	29
Ben-Tal and Nemirovksi (2002)	642	Kakouris and Rustem (2014)	28
Delage and Ye (2010)	410	Dupacova and Kopa (2014)	27
Goldfarb and Iyengar (2003)	385	Moon and Yao (2011)	27
El Ghaoui et al. (2003)	310	Pinar and Tütüncü (2005)	27
Garlappi et al. (2007)	234	Flor and Larsen (2014)	27
Maenhout (2004)	214	Dupacova and Kopa (2012)	25
Zhu and Fukushima (2009)	196	Xidonas et al. (2017a, b)	24
Tütüncü and Koenig (2004)	176	Shen and Zhang (2008)	22
Kolm et al. (2014)	126	Lu (2011b)	22
Fabozzi et al. (2010)	119	Huang et al. (2007)	19
Gorissen et al. (2015)	119	Ghahtarani and Najafi (2013)	19
Ehrgott et al. (2014)	115	Maillet et al. (2015)	18
DeMiguel and Nogales (2009)	112	Van Hest and De Waegenaere (2007)	17
Bertsimas and Pachamanova (2008)	97	Gulpinar and Pachamanova (2013)	17
Ceria and Stubbs (2006)	96	Scutella and Recchia (2013)	16
Popescu (2007)	86	Kim et al. (2014c)	15
Natarajan et al. (2009)	85	Kim et al. (2013b)	15
Quaranta and Zaffaroni (2008)	74	Kawas and Thiele (2011a)	15
Fliege and Werner (2014)	71	Xing et al. (2014)	15
Mansini et al. (2014)	71	Lu (2011a)	14
Natarajan et al. (2008)	70	Gülpinar et al. (2014)	13
Calafiore (2007)	61	Fakhar et al. (2018)	12
Chen et al. (2011)	57	Schöttle and Werner (2009)	12
Huang et al. (2010)	56	Hellmich and Kassberger (2011)	12
Costa and Paiva (2002)	53	Kim et al. (2013a)	12
Gregory et al. (2011)	50	Pinar (2016)	11
Zymler et al. (2011)	47	Chen and Tan (2009)	10
Natarajan et al. (2010)	47	Rujeerapaiboon et al. (2016)	10

Table 7 continued

References	Number of citations	References	Number of citations
Fabozzi et al. (2007b)	43	Scutella and Recchia (2010)	10
Huang et al. (2008)	40	Khodadadi et al. (2006)	10
Scherer (2007)	38	Kim et al. (2014b)	10
Kim et al. (2014a)	32	Costa et al. (2017)	10

Table 8 Number of Scopus© citations by journal

Journal	Number of citations
Operations Research	4095
Journal of Banking & Finance	1741
Mathematics of Operations Research	1614
Operations Research Letters	1092
SIAM Review	898
European Journal of Operational Research	752
Mathematical Programming	656
Review of Financial Studies	448
Annals of Operations Research	371
Journal of Asset Management	189
Computers & Operations Research	173
Omega	128
Journal of Economic Dynamics & Control	93
Management Science	80
OR Spectrum	62
SIAM Journal on Optimization	61
Journal of Portfolio Management	56
Mathematical Finance	47
Quantitative Finance	43
Annals of Finance	35
Journal of Optimization Theory & Applications	32
Metrica	29
Finance Research Letters	26
Optimization	24
Optimization Methods & Software	22
Economic Modelling	20
Journal of Computational & Applied Mathematics	12
4OR	10
Energy Economics	10

In addition, it can be suggested that greater interaction between researchers in operations research and finance would be helpful. For example, many of the financial studies focus on data-generating processes and their estimations, and often ignore important ideas and methods from operations research from which they could benefit.

On the other hand, much of the operations research literature takes as given the estimates in applications, without modeling or taking into account the economic forces that drive the data-generating process. Additionally, it rarely studies simultaneously both optimal estimation and associated robust strategies. Also, the operations research literature seldom makes full use of available asset pricing models.

Moreover, in almost all of the studies reviewed, the main conclusion is that robust strategies are preferable to classical ones in terms of the stability of the returned portfolios, and in terms of out-of-sample performance. In a few cases, different robust methods were compared (DeMiguel and Nogales 2009; Ben-Tal et al. 2009). In those cases, the methods compared are of the same kind; methods based on robust estimators in the first case and based on convex risk measures in the second case. On the other hand, it appears that few computational studies have been performed to compare robust portfolio optimization strategies of different kinds, e.g. standard robust models based on uncertainty sets, versus robust VaR or CVaR models.

From one point of view it can be argued that theoretical robust portfolio optimization is mature, but that is only because it is so far in advance of applications. Thus, there are many unanswered questions in the practice of robust portfolio optimization. Indeed, there is a need for more empirical research in order to provide portfolio managers with better guidelines for applying robust optimization in a way that generates superior out-of-sample returns. Practitioners need to understand better the implications of using different types of uncertainty sets and how to calibrate model parameters in an optimization model to deal with the over-conservatism inherent in many robust models. At this point, the growing uneasiness of practitioners with the usability of the robust portfolio optimization tools is something that must be acknowledged and taken into account in future research.

Finally, although our focus in this paper has been on the application of robust optimization to portfolio optimization, robust mathematical programming may be an effective tool in many other financial areas. The robust optimization approach may, for instance, appear as extremely promising, either on the estimation of various econometric models or in the execution of optimal trading programs, etc. In summary, there is no doubt that the methodology's underlying momentum is very strong.

References

Belhajjam, A., Belbachir, M., & El Ouardirhi, S. (2017). Robust multivariate extreme value at risk allocation. *Finance Research Letters, 23,* 1–11.

Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). *Robust optimization. Princeton Series in Applied Mathematics*. Princeton: Princeton University Press.

Ben-Tal, A., Margalit, T., & Nemirovski, A. (2002). Robust modeling of multi-stage portfolio problems. In H. Frenk, K. Roos, T. Terlaky, & S. Zhang (Eds.), *High performance optimization* (pp. 303–328). Dordrecht: Kluwer.

Ben-Tal, A., & Nemirovski, A. (1998). Robust convex optimization. *Mathematics of Operations Research, 23,* 769–805.

Ben-Tal, A., & Nemirovski, A. (1999). Robust solutions of uncertain linear programs. *Operations Research Letters, 25*(1), 1–13.

Ben-Tal, A., & Nemirovski, A. (2002). Robust optimization-methodology and applications. *Mathematical Programming, Series B, 92,* 453–480.

Bergen, V., Escobar, M., Rubtsov, A., & Zagst, R. (2018). Robust multivariate portfolio choice with stochastic covariance in the presence of ambiguity. *Quantitative Finance, 18*(8), 1265–1294.

Bertsimas, D., Brown, D., & Caramanis, C. (2011). Theory and applications of robust optimization. *SIAM Review, 53,* 464–501.

Bertsimas, D., & Pachamanova, D. (2008). Robust multi-period portfolio management in the presence of transaction costs. *Computers & Operations Research, 35,* 3–17.

Bertsimas, D., & Sim, M. (2004). The price of robustness. *Operations Research, 52,* 35–53.

Bienstock, D. (2007). Histogram models for robust portfolio optimization. *Journal of Computational Finance, 11,* 1–64.

Bo, L., & Capponi, A. (2017). Robust optimization of credit portfolios. *Mathematics of Operations Research, 42*(1), 30–56.

Brown, D. (2006). *Risk and robust optimization*. Ph.D. Dissertation, Massachusetts Institute of Technology.

Cacador, S., Dias, J., & Godinho, P. (2019). Portfolio selection under uncertainty: A new methodology for computing relative-robust solutions. *International Transactions in Operational Research*. https://doi.org/10.1111/itor.12674.

Calafiore, G. (2007). Ambiguous risk measures and optimal robust portfolios. *SIAM Journal on Optimization, 18,* 853–877.

Ceria, S., & Stubbs, R. (2006). Incorporating estimation errors into portfolio selection: Robust portfolio construction. *Journal of Asset Management, 7,* 109–127.

Chen, L., He, S., & Zhang, S. (2011). Tight bounds for some risk measures, with applications to robust portfolio selection. *Operations Research, 59,* 847–865.

Chen, C., & Kwon, R. (2012). Robust portfolio selection for index tracking. *Computers & Operations Research, 39,* 829–837.

Chen, W., & Tan, S. (2009). Robust portfolio selection based on asymmetric measures of variability of stock returns. *Journal of Computational & Applied Mathematics, 232,* 295–304.

Chen, C., & Wei, Y. (2019). Robust multiobjective portfolio optimization: A set order relations approach. *Journal of Combinatorial Optimization, 38*(1), 21–49.

Chen, C., & Zhou, Y.-S. (2018). Robust multiobjective portfolio with higher moments. *Expert Systems with Applications, 100,* 165–181.

Cong, F., Oosterlee, C. W. (2017). On robust multi-period pre-commitment and time-consistent mean-variance portfolio optimization. *International Journal of Theoretical & Applied Finance*, *20*(7), art. no. 1750049.

Cornuéjols, G., & Tütüncü, R. (2006). *Optimization methods in finance*. Cambridge: Cambridge University Press.

Costa, O., de Oliveira Ribeiro, C., Rego, E., Stern, J., Parente, V., & Kileber, S. (2017). Robust portfolio optimization for electricity planning: An application based on the Brazilian electricity mix. *Energy Economics, 64,* 158–169.

Costa, O., & Paiva, A. (2002). Robust portfolio selection using linear-matrix inequalities. *Journal of Economic Dynamics & Control, 26,* 889–909.

de Klerk, E., Kuhn, D., & Postek, K. (2019). Distributionally robust optimization with polynomial densities: Theory, models and algorithms. *Mathematical Programming*. https://doi.org/10.1007/s10107-019-01429-5.

Delage, E., & Ye, Y. (2010). Distributionally robust optimization under moment uncertainty with application to data-driven problems. *Operations Research, 58,* 595–612.

DeMiguel, V., & Nogales, F. (2009). Portfolio selection with robust estimation. *Operations Research, 57,* 560–577.

Deng, G., Dulaney, T., McCann, C., & Wang, O. (2013). Robust portfolio optimization with Value-at-Risk-adjusted Sharpe ratios. *Journal of Asset Management, 14*(5), 293–305.

Desmettre, S., Korn, R., Ruckdeschel, P., & Seifried, F. (2015). Robust worst-case optimal investment. *OR Spectrum, 37,* 677–701.

Ding, K.-W., Chen, Z.-Y., & Huang, N.-J. (2018). Robust mean variance optimization problem under Rényi divergence information. *Optimization, 67*(2), 287–307.

Doan, X., Li, X., & Natarajan, K. (2015). Robustness to dependency in portfolio optimization using overlapping marginals. *Operations Research, 63,* 1468–1488.

Dupacova, J., & Kopa, M. (2012). Robustness in stochastic programs with risk constraints. *Annals of Operations Research, 200,* 55–74.

Dupacova, J., & Kopa, M. (2014). Robustness of optimal portfolios under risk and stochastic dominance constraints. *European Journal of Operational Research, 234,* 434–441.

Ehrgott, M., Ide, J., & Schöbel, A. (2014). Min-max robustness for multi-objective optimization problems. *European Journal of Operational Research, 239,* 17–31.

El Ghaoui, L., Oks, M., & Oustry, F. (2003). Worst-case value-at-risk and robust portfolio optimization: A conic programming approach. *Operations Research, 51,* 543–556.

Fabozzi, F., Focardi, S., & Kolm, P. (2006). Trends in quantitative finance. *Research Foundation of CFA Institute, 36*(3), 1–132.

Fabozzi, F., Huang, D., & Zhou, G. (2010). Robust portfolios: Contributions from operations research and finance. *Annals of Operations Research, 176,* 191–220.

Fabozzi, F., Kolm, P., Pachamanova, D., & Focardi, S. (2007a). *Robust portfolio optimization and management*. Hoboken: Wiley.

Fabozzi, F., Kolm, P., Pachamanova, D., & Focardi, S. (2007b). Robust portfolio optimization. *Journal of Portfolio Management, 33,* 40–48.

Fakhar, M., Mahyarinia, M. R., & Zafarani, J. (2018). On non-smooth robust multiobjective optimization under generalized convexity with applications to portfolio optimization. *European Journal of Operational Research, 265*(1), 39–48.

Fernandes, B., Street, A., & Valladao, D. (2016). An adaptive robust portfolio optimization model with loss constraints based on data-driven polyhedral uncertainty sets. *European Journal of Operational Research, 255,* 961–970.

Fliege, J., & Werner, R. (2014). Robust multiobjective optimization & applications in portfolio optimization. *European Journal of Operational Research, 234,* 422–433.

Flor, C., & Larsen, L. (2014). Robust portfolio choice with stochastic interest rates. *Annals of Finance, 10*(2), 243–265.

Fonseca, R., & Rustem, B. (2012). Robust hedging strategies. *Computers & Operations Research, 39,* 2528–2536.

Gabrel, V., Murat, C., & Thiele, A. (2018). Portfolio optimization with pw-robustness. *EURO Journal on Computational Optimization, 6*(3), 267–290.

Garlappi, L., Uppal, R., & Wang, T. (2007). Portfolio selection with parameter and model uncertainty: A multi-prior approach. *Review of Financial Studies, 20,* 41–81.

Ghahtarani, A., & Najafi, A. (2013). Robust goal programming for multi-objective portfolio selection problem. *Economic Modelling, 33,* 588–592.

Glasserman, P., & Xu, X. (2013). Robust portfolio control with stochastic factor dynamics. *Operations Research, 61,* 874–893.

Goel, A., Sharma, A., & Mehra, A. (2019). Robust optimization of mixed CVaR STARR ratio using copulas. *Journal of Computational and Applied Mathematics, 347,* 62–83.

Goldfarb, D., & Iyengar, G. (2003). Robust portfolio selection problems. *Mathematics of Operations Research, 28,* 1–38.

Gorissen, B., Yanikoglu, I., & den Hertog, D. (2015). A practical guide to robust optimization. *Omega, 53,* 124–137.

Gregory, A., Darby-Dowman, K., & Mitra, G. (2011). Robust optimization and portfolio selection: The cost of robustness. *European Journal of Operational Research, 212,* 417–428.

Guastaroba, G. (2010). *Portfolio optimization: Scenario generation, models and algorithms*. Ph.D. Dissertation, University of Bergamo.

Guastaroba, G., Mitra, G., & Speranza, M. G. (2011). Investigating the effectiveness of robust portfolio optimization techniques. *Journal of Asset Management, 12*(4), 260–280.

Gulpinar, N., & Canakoglu, E. (2017). Robust portfolio selection problem under temperature uncertainty. *European Journal of Operational Research, 256,* 500–523.

Gülpinar, N., Canakoglu, E., & Pachamanova, D. (2014). Robust investment decisions under supply disruption in petroleum markets. *Computers & Operations Research, 44,* 75–91.

Gulpinar, N., & Hu, Z. (2016). Robust optimization approaches to single period portfolio allocation problem. *International Series in Operations Research & Management Science, 241,* 265–283.

Gulpinar, N., Katata, K., & Pachamanova, D. (2011). Robust portfolio allocation under discrete asset choice constraints. *Journal of Asset Management, 12,* 67–83.

Gulpinar, N., & Pachamanova, D. (2013). A robust optimization approach to asset-liability management under time-varying investment opportunities. *Journal of Banking & Finance, 37,* 2031–2041.

Gulpinar, N., Pachamanova, D., & Canakoglu, E. (2016). A robust asset–liability management framework for investment products with guarantees. *OR Spectrum, 38,* 1007–1041.

Han, Y., Li, P., & Xia, Y. (2017). Dynamic robust portfolio selection with copulas. *Finance Research Letters, 21,* 190–200.

Hasuike, T., & Mehlawat, M. K. (2018). Investor-friendly and robust portfolio selection model integrating forecasts for financial tendency and risk-averse. *Annals of Operations Research, 269*(1–2), 205–221.

Hellmich, M., & Kassberger, S. (2011). Efficient and robust portfolio optimization in the multivariate generalized hyperbolic framework. *Quantitative Finance, 11,* 1503–1516.

Huang, D., Fabozzi, F., & Fukushima, M. (2007). Robust portfolio selection with uncertain exit time using worst-case VaR strategy. *Operations Research Letters, 35,* 627–635.
Huang, D., Zhu, S., Fabozzi, F., & Fukushima, M. (2008). Portfolio selection with uncertain exit time: A robust CVaR approach. *Journal of Economic Dynamics & Control, 32,* 594–623.
Huang, D., Zhu, S., Fabozzi, F., & Fukushima, M. (2010). Portfolio selection under distributional uncertainty: A relative robust CVaR approach. *European Journal of Operational Research, 203,* 185–194.
Iyengar, G., & Ma, a K C. (2010). A robust optimization approach to pension fund management. *Journal of Asset Management, 11,* 163–177.
Iyengar, G., & Ma, A. (2016). A robust optimization approach to pension fund management. In *Asset management* (pp. 339–363). Cham: Palgrave Macmillan.
Kakouris, I., & Rustem, B. (2014). Robust portfolio optimization with copulas. *European Journal of Operational Research, 235*(1), 28–37.
Kang, Z., Li, X., Li, Z., & Zhu, S. (2019). Data-driven robust mean-CVaR portfolio selection under distribution ambiguity. *Quantitative Finance, 19*(1), 105–121.
Kapsos, M., Christofides, N., & Rustem, B. (2018). Robust risk budgeting. *Annals of Operations Research, 266*(1–2), 199–221.
Kara, G., Ozmen, A., & Weber, G.-W. (2019). Stability advances in robust portfolio optimization under parallelepiped uncertainty. *Central European Journal of Operations Research, 27*(1), 241–261.
Kawas, B., & Thiele, A. (2011a). A log-robust optimization approach to portfolio management. *OR Spectrum, 33*(1), 207–233.
Kawas, B., & Thiele, A. (2011b). Short sales in Log-robust portfolio management. *European Journal of Operational Research, 215*(3), 651–661.
Kawas, B., & Thiele, A. (2017). Log-robust portfolio management with parameter ambiguity. *Computational Management Science, 14*(2), 229–256.
Khodadadi, A., Tütüncü, R. H., & Zangari, P. J. (2006). Optimisation and quantitative investment management. *Journal of Asset Management, 7*(2), 83–92.
Kim, W. C., Kim, J. H., Ahn, S., & Fabozzi, F. J. (2013a). What do robust equity portfolio models really do? *Annals of Operations Research, 205,* 141–168.
Kim, J. H., Kim, W. C., & Fabozzi, F. J. (2013b). Composition of robust equity portfolios. *Finance Research Letters, 10*(2), 72–81.
Kim, J. H., Kim, W. C., & Fabozzi, F. J. (2014a). Recent developments in robust portfolios with a worst-case approach. *Journal of Optimization Theory and Applications, 161*(1), 103–121.
Kim, W. C., Kim, J. H., & Fabozzi, F. J. (2014b). Deciphering robust portfolios. *Journal of Banking & Finance, 45,* 1–8.
Kim, W. C., Kim, J. H., & Fabozzi, F. J. (2016). *Robust equity portfolio management*. Hoboken: Wiley.
Kim, J. H., Kim, W. C., & Fabozzi, F. J. (2018a). Recent advancements in robust optimization for investment management. *Annals of Operations Research, 266,* 183–198.
Kim, W. C., Kim, M. J., Kim, J. H., & Fabozzi, F. J. (2014c). Robust portfolios that do not tilt factor exposure. *European Journal of Operational Research, 234*(2), 411–421.
Kim, J. H., Kim, W. C., Kwon, D.-G., & Fabozzi, F. J. (2018b). Robust equity portfolio performance. *Annals of Operations Research, 266*(1–2), 293–312.
Kim, W. C., Kim, J. H., Mulvey, J. M., & Fabozzi, F. J. (2015). Focusing on the worst state for robust investing. *International Review of Financial Analysis, 39,* 19–31.
Kolm, P. N., Tütüncü, R. H., & Fabozzi, F. J. (2014). 60 years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research, 234*(2), 356–371.
Kouvelis, P., & Yu, G. (1997). *Robust discrete optimization and its applications*. Berlin: Springer.
Lauprete, G. L., Samarov, A. M., & Welsch, R. E. (2002). Robust portfolio optimization. *Metrica, 25*(1–2), 139–149.
Lee, Y., Kim, M. J., Kim, J. H., Jang, J. R., & Chang Kim, W. (2019). Sparse and robust portfolio selection via semi-definite relaxation. *Journal of the Operational Research Society*. https://doi.org/10.1080/01605682.2019.1581408.
Li, P., Han, Y., & Xia, Y. (2016). Portfolio optimization using asymmetry robust mean absolute deviation model. *Finance Research Letters, 18,* 353–362.
Ling, A., Sun, J., & Wang, M. (2019). Robust multi-period portfolio selection based on downside risk with asymmetrically distributed uncertainty set. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2019.01.012.
Ling, A., Sun, J., Xiu, N., & Yang, X. (2017). Robust two-stage stochastic linear optimization with risk aversion. *European Journal of Operational Research, 256*(1), 215–229.
Liu, J., & Chen, Z. (2018). Time consistent multi-period robust risk measures and portfolio selection models with regime-switching. *European Journal of Operational Research, 268*(1), 373–385.

Liu, J., Chen, Z., Lisser, A., & Xu, Z. (2019). Closed-form optimal portfolios of distributionally robust mean-CVaR Problems with unknown mean and variance. *Applied Mathematics & Optimization, 79*(3), 671–693.
Lobo, M. (2000). *Robust and convex optimization with applications in finance*. Ph.D. Dissertation, Stanford University.
Lotfi, S., & Zenios, S. A. (2018). Robust VaR and CVaR optimization under joint ambiguity in distributions, means, and covariances. *European Journal of Operational Research, 269*(2), 556–576.
Lu, Z. (2011a). A computational study on robust portfolio selection based on a joint ellipsoidal uncertainty set. *Mathematical Programming, Series A, 126,* 193–201.
Lu, Z. (2011b). Robust portfolio selection based on a joint ellipsoidal uncertainty set. *Optimization Methods & Software, 26,* 89–104.
Lu, I.-C., Tee, K.-H., & Li, B. (2019). Asset allocation with multiple analysts' views: A robust approach. *Journal of Asset Management, 20*(3), 215–228.
Lutgens, F. (2004). *Robust portfolio optimization*. Ph.D. dissertation, University of Maastricht.
Lutgens, F., Sturm, S., & Kolen, A. (2006). Robust one-period option hedging. *Operations Research, 54*(6), 1051–1062.
Ma, X., Zhao, Q., & Qu, J. (2008). Robust portfolio optimization with a generalized expected utility model under ambiguity. *Annals of Finance, 4*(4), 431–444.
Maenhout, P. J. (2004). Robust portfolio rules and asset pricing. *Review of Financial Studies, 17*(4), 951–983.
Maillet, B., Tokpavi, S., & Vaucher, B. (2015). Global minimum variance portfolio optimization under some model risk: A robust regression-based approach. *European Journal of Operational Research, 244*(1), 289–299.
Mansini, R., Ogryczak, W., & Speranza, M. G. (2014). Twenty years of linear programming-based portfolio optimization. *European Journal of Operational Research, 234*(2), 518–535.
Marzban, S., Mahootchi, M., & Khamseh, A. (2015). Developing a multi-period robust optimization model considering American style options. *Annals of Operations Research, 233,* 305–320.
Millington, T., & Niranjan, M. (2017). Robust portfolio risk minimization using the graphical lasso. *Lecture Notes in Computer Science, 10635,* 863–872.
Moon, T., & Yao, T. (2011). A robust mean absolute deviation model for portfolio optimization. *Computers & Operations Research, 38*(9), 1251–1258.
Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations Research, 43*(2), 264–281.
Natarajan, K., Pachamanova, D., & Sim, M. (2008). Incorporating asymmetric distribution information in robust value-at-risk optimization. *Management Science, 54*(3), 573–585.
Natarajan, K., Pachamanova, D., & Sim, M. (2009). Constructing risk measures from uncertainty sets. *Operations Research, 57*(5), 1129–1141.
Natarajan, K., Sim, M., & Uichanco, J. (2010). Tractable robust expected utility and risk models for portfolio optimization. *Mathematical Finance, 20*(4), 695–731.
Oguzsoy, C. B., & Guven, S. (2007). Robust portfolio planning in the presence of market anomalies. *Omega, 35*(1), 1–6.
Pac, A. B., & Pinar, M. C. (2018). On robust portfolio and naive diversification: Mixing ambiguous and unambiguous assets. *Annals of Operations Research, 266*(1–2), 223–253.
Pachamanova, D. (2006). Handling parameter uncertainty in portfolio risk minimization: The robust optimization approach. *Journal of Portfolio Management, 32*(4), 70–78.
Pachamanova, D. (2013). Robust portfolio selection. Wiley Encyclopedia of Operations Research & Management Science. In: J. Cochran, L. Cox, P. Keskinocak, J. Kharoufeh & J. C. Smith (Eds.). Wiley.
Pachamanova, D., & Fabozzi, F. J. (2014). Recent trends in equity portfolio construction analytics. *Journal of Portfolio Management, 40*(3), 137–151.
Pachamanova, D., & Fabozzi, F. J. (2016). *Portfolio construction and analytics*. Hoboken: Wiley.
Pachamanova, D., Gulpinar, N., & Canakoglu, E. (2017). Robust approaches to pension fund asset liability management under uncertainty. *International Series in Operations Research & Management Science, 245,* 89–119.
Pae, Y., & Sabbaghi, N. (2014). Log-robust portfolio management after transaction costs. *OR Spectrum, 36*(1), 95–112.
Pandolfo, G., Iorio, C., Siciliano, R., & D'Ambrosio, A. (2019). Robust mean-variance portfolio through the weighted L^p depth function. *Annals of Operations Research*. https://doi.org/10.1007/s10479-019-03474-x.
Pinar, M. C. (2007). Robust scenario optimization based on downside-risk measure for multi-period portfolio selection. *OR Spectrum, 29*(2), 295–309.
Pinar, M. C. (2016). On robust mean-variance portfolios. *Optimization, 65*(5), 1039–1048.

Pinar, M. C., & Tütüncü, R. H. (2005). Robust profit opportunities in risky financial portfolios. *Operations Research Letters, 33*(4), 331–340.
Plachel, L. (2019). A unified model for regularized and robust portfolio optimization. *Journal of Economic Dynamics and Control*, *109*, art. no. 103779.
Popescu, I. (2007). Robust mean-covariance solutions for stochastic optimization. *Operations Research, 55*(1), 98–112.
Quaranta, A. G., & Zaffaroni, A. (2008). Robust optimization of conditional value at risk and portfolio selection. *Journal of Banking & Finance, 32*(10), 2046–2056.
Recchia, R., & Scutella, M. G. (2014). Robust asset allocation strategies: Relaxed versus classical robustness. *IMA Journal of Management Mathematics, 25*(1), 21–56.
Rockafellar, R. T., & Uryasev, S. (2002). Conditional value at risk for general loss distributions. *Journal of Banking & Finance, 26*(7), 1443–1471.
Rujeerapaiboon, N., Kuhn, D., & Wiesemann, W. (2016). Robust growth-optimal portfolios. *Management Science, 62,* 2090–2109.
Scherer, B. (2007). Can robust portfolio optimization help to build better portfolios? *Journal of Asset Management, 7*(6), 374–387.
Schöttle, K., & Werner, R. (2009). Robustness properties of mean-variance portfolios. *Optimization, 58*(6), 641–663.
Scutella, M. G., & Recchia, R. (2010). Robust portfolio asset allocation and risk measures. *4OR, 8*(2), 113–139.
Scutella, M. G., & Recchia, R. (2013). Robust portfolio asset allocation and risk measures. *Annals of Operations Research, 204*(1), 145–169.
Sharma, A., Utz, S., & Mehra, A. (2017). Omega-CVaR portfolio optimization and its worst-case analysis. *OR Spectrum, 39*(2), 505–539.
Shen, R., & Zhang, S. (2008). Robust portfolio selection based on a multi-stage scenario tree. *European Journal of Operational Research, 191,* 864–887.
Simoes, G., McDonald, M., Williams, S., Fenn, D., & Hauser, R. (2018). Relative robust portfolio optimization with benchmark regret. *Quantitative Finance, 18*(12), 1991–2003.
Tütüncü, R. H., & Koenig, M. (2004). Robust asset allocation. *Annals of Operations Research, 132*(1–4), 157–187.
Van Hest, T., & De Waegenaere, A. (2007). Optimal robust and consistent active implementation of a pension fund's benchmark investment strategy. *Journal of Asset Management, 8*(3), 176–187.
Vassiadou-Zeniou, C., & Zenios, S. A. (1996). Robust optimization models for managing callable bond portfolios. *European Journal of Operational Research, 91*(2), 264–273.
Wang, M.-H., Yue, J., & Huang, N.-J. (2017). Robust mean variance portfolio selection model in the jump-diffusion financial market with an intractable claim. *Optimization, 66*(7), 1219–1234.
Xidonas, P., Hassapis, C., Soulis, J., & Samitas, A. (2017a). Robust minimum variance portfolio optimization modelling under scenario uncertainty. *Economic Modelling, 64,* 60–71.
Xidonas, P., Mavrotas, G., Hassapis, C., & Zopounidis, C. (2017b). Robust multiobjective portfolio optimization: A minimax regret approach. *European Journal of Operational Research, 262,* 299–305.
Xing, X., Hu, J., & Yang, Y. (2014). Robust minimum variance portfolio with L-infinity constraints. *Journal of Banking & Finance, 46,* 107–117.
Zhu, S., & Fukushima, M. (2009). Worst-case conditional Value-at-Risk with application to robust portfolio management. *Operations Research, 57,* 1155–1168.
Zhu, S., Ji, X., & Li, D. (2015). A robust set-valued scenario approach for handling modeling risk in portfolio optimization. *Journal of Computational Finance, 19*(1), 11–40.
Zhu, S., Li, D., & Wang, S. (2009). Robust portfolio selection under downside risk measures. *Quantitative Finance, 9*(7), 869–885.
Zymler, S., Rustem, B., & Kuhn, D. (2011). Robust portfolio optimization with derivative insurance guarantees. *European Journal of Operational Research, 210*(2), 410–424.