Earnings Inequality and the Minimum Wage:
Evidence from Brazil[*]

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Abstract

We show that a minimum wage can have large effects throughout the earnings distribution, using a combination of theory and empirical evidence. To this end, we develop an equilibrium search model featuring empirically relevant worker and firm heterogeneity. We use the estimated model to evaluate a 119 percent increase in the real minimum wage in Brazil from 1996 to 2012. Direct and indirect effects of the policy account for a substantial decline in earnings inequality, with modest negative employment consequences. Using administrative linked employer-employee data and two household surveys, we find reduced-form evidence supporting the model predictions.

Keywords: Worker and Firm Heterogeneity, Equilibrium Search Model, Monopsony, Spillover Effects
JEL classification: E24, E25, E61, E64, J31, J38

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1   Introduction

To what extent do minimum wage policies shape earnings inequality? In light of historically high levels of income inequality in many countries, understanding the effects of such policies on labor market outcomes is increasingly important. Several OECD nations have recently implemented higher wage floors in an attempt to reverse a decades-old trend of rising inequality and falling real wages. In the US, for example, an active debate remains over the connection between the decline in the real minimum wage and a well-documented rise in income inequality since the 1970s. Maybe less known, several countries in Latin America—among them Brazil—have seen falling income inequality since the 1990s and a concurrent rise of legal minimum wages. Yet a quantitative assessment of the effects of the minimum wage has been challenging due to sparse policy variation, data limitations, and methodological disagreements.

In this paper, we attempt to unify theory and measurement to analyze the distributional consequences of the minimum wage. Our main contribution is to quantify the effects on earnings inequality of a large, 119 percent increase in the real minimum wage in Brazil from 1996 to 2012. The apparent size of the minimum wage change and the availability of rich administrative linked employer-employee data together with detailed household surveys make Brazil a natural testing ground for studying the effects of this policy. To this end, we develop and estimate an equilibrium search model of the Brazilian labor market and confront it with reduced-form evidence on the impact of the observed minimum wage increase. We find that the policy induces a notable inequality decline, reducing the variance of earnings by 11 log points and leading to "spillover effects" up to the 80th percentile of the earnings distribution.

Our analysis proceeds in three steps. In the first step, we build an equilibrium model of the labor market in the spirit of Burdett and Mortensen (1998) featuring worker and firm heterogeneity and endogenous job creation. Workers who differ in their ability, off- and on-the-job search efficiency, and value of leisure engage in undirected job search in markets segmented by worker type. Firms that differ in productivity choose what wages and how many vacancies to post in each market. Because of frictions in the labor market, individual firms face an upward-sloping labor supply curve, which shifts with the wages offered by other firms. Consequently, firms' equilibrium choices are interdependent, such that the minimum wage induces all firms to adjust in markets where it is binding, while participants in markets where it does not bind remain unaffected. As a result, a minimum wage leads to spillovers higher up the wage distribution within,

but not across, markets. This suggests that in a monopsonistic environment the segmentation of the market into worker and firm heterogeneity matters for the strengh of spillover effects.

In the second step, we bring the theory to the data in order to quantify the effects of a large increase in the minimum wage in Brazil on inequality and aggregate economic outcomes. To this end, we semiparametrically estimate the model via indirect inference using detailed administrative matched employer-employee data on tens of millions of Brazilian workers. We exploit the fact that the model's equilibrium wage equation includes as a special case the two-way fixed effects specification due to Abowd, Kramarz, and Margolis (1999, henceforth AKM), which we use as an auxiliary framework to inform the relative importance of firm and worker heterogeneity in the model vis-à-vis the data. We use the estimated model to assess the impact of a 44 log points increase in the real, productivity-adjusted minimum wage.

The minimum wage hike reduces the variance of earnings by 11 log points, or 55 percent of the empirical decline in Brazil over this period. Two thirds of the total impact is due to equilibrium effects. While wage growth is most pronounced at the bottom of the distribution, spillovers reach up to the 80th percentile of the distribution. The policy leads to a fall in frictional wage dispersion by reducing the firm productivity pay premium, and it diminishes permanent pay differences between individuals by lowering the worker skill premium, consistent with facts characterizing the decline in wage inequality in Brazil over this period (Alvarez et al., 2018). At the same time, we find a muted negative employment and output response due to two opposing forces. On the one hand, the minimum wage squeezes firms' profit margins, leading all firms—particularly those at low productivity levels—to post fewer vacancies. On the other hand, the reduced congestion in the labor market increases the chance that a given vacancy contacts a worker, which leads to a relatively smaller decline in employment than in aggregate vacancies and partly counters firms' incentives to reduce vacancy creation. Additionally, labor shifts to more productive firms as the profit margin of low productivity firms falls disproportionately, resulting in an increase in labor productivity. Overall, these forces partly offset each other, leading to modest disemployment and efficiency gains. While we find a large inequality response and a small unemployment response, our sensitivity analysis highlights that different parameter estimates would have lead to the opposite conclusion.

In the third step, we confront the model predictions with evidence on the impact of the minimum wage in Brazil using administrative data and household surveys. Exploiting variation in the effective bindingness of the federal minimum wage across Brazilian regions over time, we find

that a higher minimum wage has nonlinear effects on wage inequality. Our regression estimates imply spillovers associated with compression up to the 80th wage percentile of the earnings distribution, consistent with the results from the estimated model. This is despite the fact that only around two percent of workers earn the minimum wage. We also show that, in line with our model predictions, the minimum wage is only weakly negatively correlated with employment, formality, and firm dynamics.

Our investigation was motivated by the rich, and at times conflicting, set of results on the impact of a minimum wage documented in the literature. An advantage of our theory-guided empirical approach is that it allows us to speak to the sensitivity of our results with respect to various labor market configurations. One of our contributions is to propose the relative composition of worker and firm heterogeneity in the labor market as a key determinant of minimum wage effects. Our estimated model predicts a large decline in inequality and a small decline in employment due to the minimum wage increase in Brazil. These conclusions are mostly due to the fact that Brazilian labor markets are far from the perfectly competitive benchmark. For example, the Brazilian economy initially shows a low labor share, low job-to-job transition rates, and large magnitudes of between-firm wage dispersion relative to other developed countries. We find that productivity dispersion among active firms is among the most important determinants of our estimated effects. A less dispersed firm productivity distribution would have led us to find significantly smaller effects on inequality effect and larger effects on disemployment than what we estimated. This sensitivity analysis potentially sheds new light on a dispersed set of previous results in the literature.

Related literature. Our findings contribute to three strands of the literature. The first provides a structural assessment of minimum wage effects in frictional labor markets. Eckstein and Wolpin (1990) estimate a generalization of the Albrecht and Axell (1984) model with a minimum wage and no within-firm wage differences. Koning et al. (1995) and van den Berg and Ridder (1998) assess minimum wage effects on unemployment in a wage posting model with on-the-job search and homogeneous firms competing in segmented labor markets. Burdett and Mortensen (1998) and Bontemps et al. (1999) formalize the idea that minimum wage spillovers may affect higher wages in an equilibrium search model. They do so in a framework that features either firm productivity heterogeneity or worker productivity heterogeneity, but not both. In contrast, our model features two-sided heterogeneity and highlights that the relative importance of worker versus

firm heterogeneity limits the reach of spillover effects of the minimum wage. Flinn (2006) stresses the importance of endogenous contact rates for optimal minimum wage levels in a search and bargaining framework with match-specific productivities. Flinn et al. (2017) study the minimum wage in a framework with on-the-job search where firms endogenously choose whether or not to renegotiate wages as in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006). Doppelt (2018) characterizes welfare implications of the minimum wage in a search model with both extensive and intensive margins of labor supply. The recent advent of administrative linked employer-employee data allows for such models to be estimated in a way that disentangles worker and firm heterogeneity, which we argue is crucial for correctly inferring the effects of the minimum wage. We complement these previous works by highlighting the determinants of minimum wage effects in an equilibrium search model with two-sided heterogeneity.

The second literature is concerned with reduced-form estimates of the impact of a minimum wage. A long list of papers has focused on employment effects, with summaries contained in Card and Krueger (1995) and Neumark and Wascher (2008). Most findings point to small negative effects on the number of jobs, though less is known for a minimum wage change as large as that in Brazil. Fewer studies examine the effects on wage inequality, although notable exceptions include Gramlich (1976), Grossman (1983), DiNardo et al. (1996), Machin et al. (2003), and Brochu et al. (2018). We further refine their insights by distinguishing between the effects of the minimum wage on individual versus employer wage components. A seminal paper by Lee (1999) uses survey data exploiting variation in the effective bindingness of the minimum wage across US states to estimate spillovers reaching high up in the distribution. In contrast, Autor et al. (2016) argue that spillovers cannot be distinguished from measurement error due to data limitations in the Current Population Survey. Rinz and Voorheis (2018) use US Social Security extracts to study minimum wage effects on annual earnings but lack information on hours worked or hourly wages. Overcoming some of these hurdles, we use sizable policy variation combined with detailed administrative data to document widespread spillovers and little displacement due to the minimum wage in Brazil— striking findings that we reconcile through the lens of our structural model.

Complementing previous empirical work with an equilibrium model also helps to answer the call by Neumark (2017) to better understand conditions under which the effects of a minimum wage can be more or less favorable. By studying Brazil, we provide an "existence proof" of a case in which the minimum wage led to a remarkable decline in wage inequality with relatively little associated disemployment. By matching the predictions of our structural model to the microdata

surrounding a rise in the minimum wage so large that in many contexts it would be considered out-of-sample, our results lend reassurance to future research evaluating the effects of unprecedented minimum wage increases such as those recently considered in the US.[1]

Finally, the third literature studies changes in between-firm pay differences driving inequality trends, as is the case with a large inequality decline in Brazil documented by Alvarez et al. (2018).[2] While the econometric framework due to AKM has been widely used in applied empirical research, structural interpretations have been perceived as problematic (Gautier and Teulings, 2006; Eeckhout and Kircher, 2011; Lopes de Melo, 2018). Consequently, the fundamental causes behind observed changes in the wage anatomy remain largely unexplored. A small number of papers have provided microfoundations for the AKM specification in cross-sectional contexts, including Barlevy (2008), Bagger et al. (2014), and Burdett et al. (2011, 2016). Relative to these works, we provide a tractable equilibrium model that nests the AKM wage equation as a special case. The AKM wage decomposition—although generally misspecified—is useful in estimating key parameters of our structural model. To the best of our knowledge, we are the first to evaluate a change in the minimum wage as a driver of observed dynamics of AKM pay components over time. Bridging the aforementioned literatures, we conclude that the joint distribution of AKM fixed effects provides valuable information that helps discipline the model predictions about the effects of a minimum wage and, conversely, that the minimum wage shapes the dispersion of AKM fixed effects in a way that helps explain Brazil's remarkable inequality decline over this period.

Outline. The paper proceeds as follows. Section 2 presents motivating facts related to inequality and the minimum wage in Brazil. Section 3 develops our equilibrium search model and characterizes the effects of the minimum wage in this environment. Section 4 estimates the model, which we use in Section 5 to quantify the equilibrium effects of the minimum wage. Section 6 provides empirical evidence in support of the model predictions. Finally, Section 7 concludes.

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[1]Other mechanisms that could give rise to spillover effects include skill assignments with comparative advantage (Teulings, 1995, 2000, 2003), hierarchical matching (Lopes de Melo, 2012), fairness considerations (Card et al., 2012), educational investment (Bárány, 2016), endogenous union formation (Taschereau-Dumouchel, 2017), and hedonic compensation (Phelan, 2018).

[2]See Davis and Haltiwanger (1991), Dunne et al. (2004), Song et al. (2018), Barth et al. (2016), and Abowd et al. (2018) for the US; Cardoso (1999) for Portugal; Iranzo et al. (2008) for Italy; Nordström Skans et al. (2009), Akerman et al. (2013), and Lindqvist et al. (2015) for Sweden; Faggio et al. (2010) for the UK; Eriksson et al. (2013) for the Czech Republic; Card et al. (2013) and Kantenga and Law (2016) for Germany; and Helpman et al. (2017) for Brazil.

2 Motivating Facts

To examine a decline in earnings inequality in relation to a concurrent rise in the minimum wage in Brazil from 1996 to 2012, we combine comprehensive administrative linked employer-employee data (RAIS) with two household surveys (PNAD and PME). The three datasets are described in Appendix A.1. Detailed summary statistics are presented in Appendix A.2. The main conclusion we draw from examining the raw data is that there was a significant decrease in the dispersion of monthly earnings—henceforth "wages"[3]—with little sign of declining employment among formal sector workers in Brazil between 1996 and 2012.

2.1 Dissecting Brazil's inequality decline

Our investigation is motivated by a remarkable decline of 20 log points in the standard deviation of wages in Brazil between 1996 and 2012. To understand this inequality decline, we follow Alvarez et al. (2018) in implementing a statistical decomposition of the wage distribution in Brazil's formal sector using the RAIS linked employer-employee data. Noting that seemingly identical workers experience large pay differences across firms, we decompose wage differences into worker and firm heterogeneity. Specifically, we estimate a two-way fixed effects framework due to AKM that decomposes log wages w_{ijt} of individual i working at firm j in year t within five-year periods as

$$w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}, \tag{1}$$

where α_i denotes an individual fixed effect, α_j denotes a firm fixed effect, γ_t is a year dummy, and ε_{ijt} is a residual satisfying the strict exogeneity condition $\mathbb{E}\left[\varepsilon_{ijt} \mid i, j, t\right] = 0$.[4]

Table 1 presents a variance decomposition based on equation (1) over repeated time windows. During 1996–2000, half of the total variance of wages of 68 log points is accounted for by worker pay heterogeneity and a quarter by firm pay heterogeneity. Between 1996–2000 and 2008–2012, the total variance dropped by 20 log points, primarily due to a decline in between-firm pay dispersion, constituting 44 percent of the decline in the variance of log wages over this period.

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[3]We also observe contractual hours, although we find little cross-sectional variation and changes over time along this margin, plausibly due to Brazil's rigid labor laws restricting part-time work arrangements. See Section 6.2 for a more formal regression analysis and Appendix D.4 for further details.

[4]Equation (1) is identified off workers switching employers across years for the largest set of individuals at firms connected through worker flows. There has been a fruitful debate around the merits and potential biases of this framework, including recent work by Andrews et al. (2008); Eeckhout and Kircher (2011); Bonhomme et al. (2017); Lopes de Melo (2018); Card et al. (2018); and Borovičková and Shimer (2018). Alvarez et al. (2018) present a battery of specification tests and robustness checks, and conclude that the model describes well the Brazilian data during this period.

Table 1. AKM variance decomposition, 1996–2000 and 2008–2012

	1996–2000	2008–2012	Change
Total variance of log wages, $Var(w_{ijt})$	0.68	0.48	-0.20
Variance of worker fixed effects, $Var(\hat{\alpha}_i)$	0.33	0.28	-0.06
Variance of firm fixed effects, $Var(\hat{\alpha}_j)$	0.16	0.07	-0.09
2×Covariance b/w workers and firms, $2 \times Cov(\hat{\alpha}_i, \hat{\alpha}_j)$	0.13	0.10	-0.03
Residual variance, $Var(\hat{\varepsilon}_{ijt})$	0.05	0.03	-0.01
Number of worker-year observations	65,645,691	103,188,795	
Share in largest connected set	99.86%	99.96%	
R^2	0.93	0.93	

Notes: Predicted variances (shares) due to components in log wage decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$ for the population of male workers of age 18–49. Omitted are variance terms involving year dummies γ_t, which account for a negligible share of the total variance. Source: RAIS.

2.2 A firm ladder

Given the importance of between-firm pay differences in accounting for observed wage dispersion, what economic factors sustain pay differences for identical workers across employers? To motivate our structural approach in the next section, Table 2 documents that job-to-job mobility is an important source of wage growth for Brazilian workers, and that growth in firm AKM fixed effects accounts for a significant share of the overall growth in wages experienced by job-to-job movers. Overall, adult male workers in Brazil experienced 0.8 log points monthly wage growth during 1996–2000. However, a small but important number of job-to-job switchers of around 1.4 percent of the workforce each month make significant gains of around 6.5 log points in wages and 5.7 log points (or 88 percent of the total gain) in AKM firm fixed effects from transitioning between employers. Thus, job-to-job transitions from low- to high-paying firms—although less frequent than in the US (Engbom, 2017)—present a powerful engine for individual wage growth.[5]

Table 2. Changes in wage and AKM firm fixed effect conditional on job transition status

	Overall	Job-to-job transitions
Monthly rate	–	0.014
Mean change in log wage, $\mathbb{E}\left[\Delta w_{ijt}\right]$	0.008	0.065
Mean change in AKM firm fixed effect, $\mathbb{E}\left[\Delta\hat{\alpha}_j\right]$	0.001	0.057

Notes: Monthly job-to-job transition rate is defined as the mean indicator for switching employers between two consecutive months. Mean change in log wage and mean change in AKM firm fixed effect are based on the decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$ for the population of male workers of age 18–49 during the period 1996-2000. Source: RAIS.

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[5]Corresponding to our findings in the previous section, we find that the mean change in AKM firm fixed effects from job-to-job transitions declined somewhat from 5.7 log points to 4.1 log points over the 1996–2012 period.

2.3 The minimum wage in Brazil

Motivated by Brazil's inequality decline and the link between job-to-job mobility and individual pay dynamics in the data, we now turn to a salient change in the labor market over this period: the rise in the minimum wage.[6] Brazil's statutory minimum wage is set at the federal level and stated in terms of a monthly—not hourly—earnings floor. There are no provisions for legal subminimum or differentiated minimum wages across demographics or economic subdivisions (Lemos et al., 2004). The minimum wage is set for full-time workers with 44-hour contracts and adjusted proportionately for part-time workers.[7]

Brazil's real minimum wage deteriorated under high inflation leading up to 1996 when a switch in government ignited a gradual ascent of the wage floor by 119 percent in real terms, reaching 622 BRL or 410 PPP-adjusted USD per month by 2012. Accounting for aggregate productivity growth, this corresponds to a 44 log points rise in the minimum wage. To put these numbers into context, the minimum wage as a fraction of the median wage increased from around 34 percent in 1996 to 60 percent in 2012. The negative comovement of the minimum wage and the variance of log earnings over the preceding 25-year period is shown in Figure 1.[8]

Figure 1. Evolution of earnings inequality and the real minimum wage in Brazil, 1988–2012
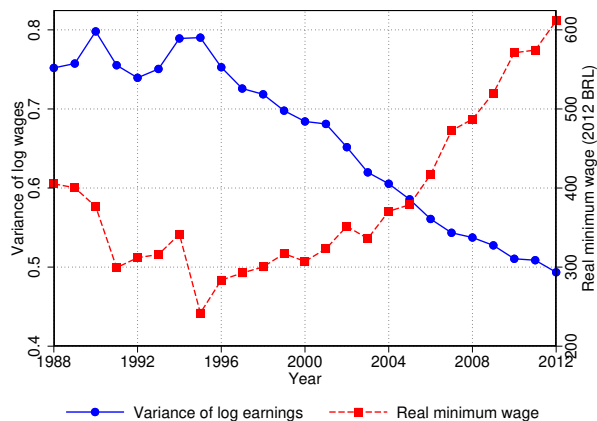
Notes: Statistics are for males of age 18–49. Real minimum wage is the annual average of the monthly time series. Source: RAIS, IPEA.

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[6]While Brazil enacted other social policies during the mid-2000s, such as the *Bolsa Família* transfer program for needy families launched in 2003, the minimum wage predates many of these policies. By exploiting cross-sectional variation in the data in Section 6.1, we identify effects of the minimum wage net of aggregate trends, so the fact that inequality declined in Brazil over this period is neither necessary nor sufficient for our conclusions.

[7]Using information on hours in the RAIS data, we find a small initial share of such workers and no significant changes related to the minimum wage over time–see also Section 6.2 and Appendix D.4. Special labor contracts allow for parts of the minimum wage to be paid in-kind in the form of accommodation and food, although in the PNAD data only 0.8 percent of workers report receiving nonmonetary remuneration in 1996, and 0.3 percent of workers in 2012.

[8]Matching this time series pattern, in Section 6.1 we show that our empirical estimates and counterfactual model simulations imply an asymmetric and convex response of wage dispersion to changes in the minimum wage.

Much of the previous empirical literature has interpreted the mass of workers employed at the minimum wage as a measure of the bindingness of the wage floor (Flinn, 2006, 2010). Although there is substantial heterogeneity in the bindingness of the minimum wage across subpopulations in our administrative data from Brazil, we robustly find a small spike at the wage floor for male workers of age 18–49. Panel (a) of Figure 2 shows across states an average of approximately two percent of workers earn exactly the minimum wage in 1996 and 2012 against the relative bindingness of the minimum wage measured by the Kaitz index, defined as the log minimum-to-median wage.[9] We find a weak positive correlation between the Kaitz index and the share of workers earning the minimum wage, but the fraction remains at relatively small levels even in states where the minimum wage is most binding.

We can broaden our definition of "mass point" to three measures whose evolution from 1996 to 2012 is depicted in panel (b) of Figure 2. The share of workers earning exactly the minimum wage, shown by the blue line, remains flat at two percent. A little more than 1.5 percent of workers in 1996 and around three percent of workers in 2012 report earning less than the minimum wage, shown by the red line. These observations are likely due to a mix of legal exceptions, misreporting, and illegal employment. Our most generous definition includes workers within a 5 percent band around the minimum wage, shown in green. This most generous measure rises from 3.5 to 7 percent over this period, far from the roughly 30 percent of workers between the old and the new minimum wage.

Figure 2. Data: Worker shares in relation to the minimum wage, 1996–2012

(a) Heterogeneity across states and over time

(b) Share at, below, or around the minimum wage
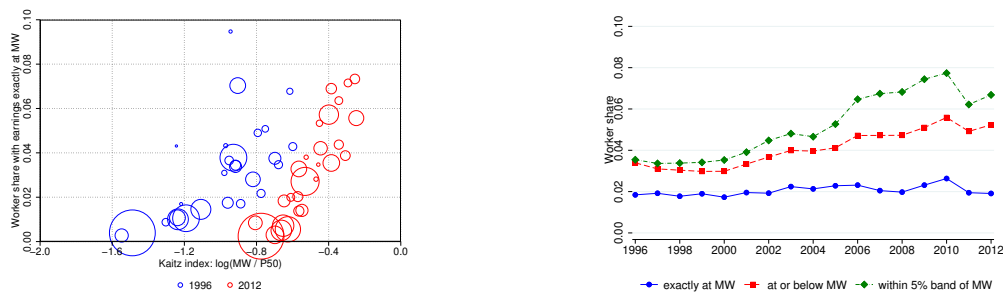
Notes: Panel (a) shows share of male workers of age 18–49 earning exactly the minimum wage against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, across states in 1996 and 2012. Area of circles is proportional to population size. In panel (b), the blue line shows share of workers earning exactly the minimum wage, the red line shows share at or below the minimum wage, and the green line plots share within 5 percent of the minimum wage. Source: RAIS.

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[9]For comparison, 3.3 percent of hourly paid workers in the US earned the prevailing federal minimum wage or less in 2015 (U.S. Bureau of Labor Statistics, 2017).

To what extent can the rise in the minimum wage account for Brazil's concurrent inequality decline from 1996 to 2012? Evaluating the effects of the minimum wage in Brazil over this period, as well as designing such policies in other contexts, requires a model that is consistent with the cross-section of worker and firm pay heterogeneity, the prevalence of a firm ladder explaining wage gains from job-to-job transitions, and the absence of a large mass point at the wage floor.

3 Equilibrium Model

This section develops a version of the Burdett and Mortensen (1998) equilibrium model with worker and firm heterogeneity that we use to assess the effects of a minimum wage increase. In line with salient empirical facts from Section 2, this framework can generate endogenous wage dispersion for identical workers across employers. The Burdett-Mortensen model is widely used to study wage determination, and our exposition closely follows that of Bontemps et al. (1999, 2000), Mortensen (2003), and Jolivet et al. (2006). Our contribution is to allow for both worker and firm heterogeneity in a tractable manner to use the estimated framework for quantitative analysis.

3.1 Environment

We study a stationary economy cast in continuous time that consists of a unit mass of infinitely-lived workers and a mass M_0 of firms who meet in a frictional labor market.

Workers. Workers differ in ability θ distributed over $\left[\theta, \overline{\theta}\right]$ with pdf h_θ. We think of θ as capturing an amalgamate of educational qualifications and skill competencies that are valued in the labor market. As in Lise and Robin (2017) and Bagger and Lentz (2018), we abstract from human capital accumulation. Workers can be employed or nonemployed, the latter of which we map to the pool of unemployed plus informally employed in the data. Workers value a stream of consumption equal to their wage when employed or b_θ when nonemployed, discounted at rate ρ. In both states, workers search for jobs within markets segmented by ability type, as in van den Berg and Ridder (1998), which can be thought of as separate Burdett-Mortensen economies indexed by θ.

The assumption that labor markets are perfectly segmented by worker ability type deserves some discussion. Modeling workers by unobservable ability type, rather than based purely on observables as in van den Berg and Ridder (1998), allows us to match the empirical regularity that substantial earnings dispersion remains within narrowly defined demographic groups (Alvarez

et al., 2018). Segmenting markets across θ-types is a simple, albeit stylized, way to achieve two objectives: First, it allows us to handle two-sided heterogeneity in a tractable manner. Second, it helps us to connect our structural model to a rich empirical literature on between- versus within-firm pay inequality in the spirit of AKM. [10]Although it would be very interesting to develop a model in which workers of different types search in the same market, this is beyond the scope of this paper. It is worth nothing that by segmenting markets, we effectively shut down spillovers across markets, thereby limiting the extent of spillover effects of a minimum wage.

Let λ_θ^u denote the job offer arrival rate for the nonemployed and $\lambda_\theta^e = s_\theta \lambda_\theta^u$, for some parameter s_θ, the arrival rate for the employed. A job offer entails a wage draw $w \sim F_\theta(\cdot)$ over support $[\underline{w}_\theta, \overline{w}_\theta]$. Although workers take arrival rates and the wage offer distribution as given, both are determined endogenously through firms' equilibrium vacancy and wage posting decisions, possibly subject to a minimum wage. Matches dissolve exogenously at rate δ_θ, leading a share $u_\theta = \delta_\theta / (\delta_\theta + \lambda_\theta^u)$ of workers to be frictionally nonemployed. As employed workers find higher-paying jobs through on-the-job search, the realized wage distribution G_θ first-order stochastically dominates the wage offer distribution F_θ. Indeed, $G_\theta(w) = F_\theta(w) / (1 + \kappa_\theta (1 - F_\theta(w)))$, where $\kappa_\theta \equiv \lambda_\theta^e / \delta_\theta$ governs the effective speed of climbing up the job ladder.

The values of nonemployed workers, W_θ, and of workers employed at wage w, $S_\theta(w)$, satisfy[11]

$$\rho W_\theta = b_\theta + \lambda_\theta^u \int_{\underline{w}_\theta}^{\overline{w}_\theta} \max\{S_\theta(w) - W_\theta, 0\} \, dF_\theta(w)$$

$$\rho S_\theta(w) = w + \lambda_\theta^e \int_w^{\overline{w}_\theta} [S_\theta(w') - S_\theta(w)] \, dF_\theta(w') + \delta_\theta [W_\theta - S_\theta(w)].$$

The optimal strategy of a nonemployed worker involves a reservation threshold ϕ_θ equal to the flow value of nonemployment plus the forgone option value of remaining nonemployed:

$$\phi_\theta = b_\theta + (\lambda_\theta^u - \lambda_\theta^e) \int_{\phi_\theta}^{\overline{w}_\theta} \frac{1 - F_\theta(w)}{\rho + \delta_\theta + \lambda_\theta^e (1 - F_\theta(w))} \, dw.$$

In contrast to Albrecht and Axell (1984), our model features heterogeneity in the reservation

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[10]Modeling labor markets as segmented by worker type, rather than pooled across types as in Heise and Porzio (18), allows our model to tractably generate both within-firm and between-firm inequality with a positive covariance between worker and firm types as documented empirically in Section 2.1.

[11]Following most of the literature, we abstract from an hours margin here. Motivated by the empirical evidence in Section D.4 that hours are highly concentrated at full-time work in the cross-section of the data, and that hours show no significant relation with the level of the minimum wage over time, we think this is an innocuous assumption for our setting. Our framework could be readily extended to include an intensive margin of labor supply, requiring a very lower elasticity of labor supply to match the invariance in the data.

threshold across, instead of within, θ-markets.[12] We say the minimum wage is *binding* in market θ whenever $w^{min} > \phi_\theta$ so that workers' reservation wage is $R_\theta = \max\{\phi_\theta, w^{min}\}$.

Firms. Firms are characterized by a productivity level p distributed continuously over $[p_0, \overline{p}]$ with cdf Γ_0. They operate a linear production technology combining l_θ workers of each ability type θ to produce flow output

$$y(p, \{l_\theta\}_{\theta \in \Theta}) = p \int_{\theta \in \Theta} \theta l_\theta d\theta.$$

In contrast to the assumption in van den Berg and Ridder (1998) and Bontemps et al. (1999, 2000) but in line with Flinn (2006), we model contact rates as endogenous. Specifically, we assume that firms attract type-θ workers by posting v_θ job openings subject to a convex cost $c_\theta(v_\theta)$: $c'_\theta, c''_\theta > 0$. The firm commits to a wage w_θ for its vacancies in market θ. That firms make equilibrium wage posting decisions, rather than bargain with individual workers over the match surplus with some exogenous bargaining parameter, provides a better description of the Brazilian labor market given its low skilled labor force and institutional setting.[13] We later show that our assumption is consistent with the empirical distribution of wages, in particular the absence of a pronounced mass point at the minimum wage.

A firm's wage rank $1 - F_\theta(w_\theta)$ together with its recruiting intensity v_θ jointly determine a firm's employment level $l_\theta(w_\theta, v_\theta)$. As production and the recruitment process are independent across markets, a productivity p firm's problem coincides with separate profit maximization in each market:

$$\forall \theta : \max_{w_\theta \geq w^{min}, v_\theta} \{(p\theta - w_\theta) l_\theta(w_\theta, v_\theta) - c_\theta(v_\theta)\}.$$

A firm makes positive profits in market θ only if it posts a wage between workers' reservation wage R_θ and its productivity p. Hence, there is an active mass of firms $M_\theta = M_0(1 - \Gamma_0(\underline{p}_\theta))$ distributed $\Gamma_\theta(p) = \Gamma_0(p \mid p > \underline{p}_\theta)$ with lower bound $\underline{p}_\theta = R_\theta/\theta$. Given optimal wage and vacancy posting policies $(w_\theta(p), v_\theta(p))$ in market θ, the wage offer distribution for a given aggregate vacancy mass $V_\theta = M_\theta \int_{\underline{p}_\theta}^{\overline{p}} v_\theta(p') d\Gamma_\theta(p')$ is simply $F_\theta(w_\theta(p)) = M_\theta \int_{\underline{p}_\theta}^{p} v_\theta(p') d\Gamma_\theta(p') / V_\theta$.

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[12]To capture the rich empirical heterogeneity in this and other moments, our implementation allows for flexible heterogeneity both within and across groups of observable worker characteristics.

[13]In the literature, variants of the Burdett and Mortensen (1998) model have been popular in studying wage determination, while bargaining frameworks have been more widely used in the study of labor flows. Furthermore, in the words of Flinn (2006): "the validity of [fixing the value of the bargaining parameter] is questionable, particularly when considering large policy (i.e., minimum wage) changes."

Matching. Denote by $S_\theta = h_\theta \left(u_\theta + s_\theta \left(1 - u_\theta \right) \right)$ the efficiency-weighted number of searching workers in market θ. The total number of meetings between workers and firms in market θ, m_θ, can be represented by a Cobb-Douglas function $m_\theta = \chi_\theta S_\theta^{1-\alpha} V_\theta^\alpha$, where χ_θ is a matching efficiency parameter and α governs the elasticity of matches with respect to vacancies.

3.2 Effects of the minimum wage

We define, characterize, and outline a solution algorithm for a *search equilibrium with a minimum wage* in Appendix B.1–C.1. We illuminate here the model's mechanism giving rise to wage dispersion for identical workers across firms and the effects of the minimum wage in this environment.

We focus first on a single θ-market. Job-to-job mobility renders firms' wage and vacancy policies interdependent. In choosing a wage, firms take as given the distribution of competing wage offers F_θ and weigh two opposing forces. On the one hand, a lower wage increases per-worker profits. On the other hand, a higher wage rank raises steady-state employment through increased poaching and decreased voluntary quits. As has been well known since Burdett and Mortensen (1998), this trade-off leads more productive firms to post higher wages, leading to equilibrium wage dispersion for identical workers.[14] Perturbations to this environment lead to *spillovers* between all employers in a θ-market, even if only a subset of firms is directly affected.

Concretely, let us consider the effects of a minimum wage raise between steady states. A set of firms will adjust their wage offers to comply with the new wage floor. As firm optimization induces the equilibrium wage offer distribution to be continuous and wages to be strictly increasing in productivity, other firms adjust wages in equilibrium to retain their pay rank. Such competitive pressure leads the minimum wage to spill over to higher-paying firms. Finally, fewer vacancy postings due to lower profit margins and firm exit will result in higher frictional unemployment.

We now turn to the economy with a continuum of segmented markets. In markets where the minimum is binding, the strength of the above mechanism depends on the labor market configuration. Minimum wage bindingness is determined by workers' reservation wages. Markets where the minimum wage is nonbinding remain unaffected. In this sense, labor market segmentation puts a cap on the spillover effects that are omnipresent in the original Burdett and Mortensen (1998) framework. Evidently then, the mix of worker versus firm heterogeneity will be a crucial input to our estimated model when considering equilibrium effects of the minimum wage.

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[14]In the empirically relevant case with firm heterogeneity, there exists a unique pure strategy equilibrium in firms' wage posting game. In contrast, the model with homogeneous firms has a unique mixed strategy equilibrium with an upward-sloping wage density, different from the heterogeneous firms equilibrium considered here.

Our framework relates closely to the empirical literature on pay decompositions into worker and firm heterogeneity due to AKM. To see this, with exogenous contact rates, $(\lambda_\theta^u, \lambda_\theta^e, \delta_\theta)$ constant across θ-markets, $b_\theta \propto \theta$, and a non-binding minimum wage, equilibrium wages in our model coincide with the log additive specification (1) that is popular in empirical studies of the wage distribution and its changes over time (Card et al., 2013; Alvarez et al., 2018; Song et al., 2018):

$$\log w_\theta(p) = \alpha_\theta + \alpha(p),\tag{2}$$

where the "worker effect" $\alpha_\theta = \log \theta$ is an increasing function of ability, while the "firm effect" or piece rate $\alpha(p) = p - \int_{\underline{p}}^p [(1 - \Gamma_0(\underline{p}) + \kappa(1 - \Gamma_0(p)))/(1 - \Gamma_0(\underline{p}) + \kappa(1 - \Gamma_0(x)))]^2 dx$ is independent of worker ability and strictly increasing in firm productivity. Under more general parameterizations, the wage equation (2) is perturbed and the AKM specification in equation (1) no longer has a structural interpretation. Nevertheless, we argue that it provides valuable information about the underlying unobserved worker and firm heterogeneity.

How does the minimum wage affect wage inequality in this equilibrium framework? It is instructive to characterize the spillover effects of the minimum wage for a special case of the model, which we later numerically show continues to approximately hold under more general numerical parameterizations.

Proposition. *Assume exogenous contact rates, constant $(\lambda_\theta^u, \lambda_\theta^e, \delta_\theta) \in \mathbb{R}_{++}^3$ for all θ, and $b_\theta \propto \theta$. Then for markets where the minimum wage binds, $\{\theta | w^{min} \geq \phi_\theta\}$, a marginal increase in the minimum wage*

1. *increases wages at all firms: $\partial w_\theta(p; w^{min}) / \partial w^{min} > 0 \,\forall p$;*

2. *decreases the productivity pay premium across firms: $\partial \left[\partial w_\theta(p; w^{min}) / \delta p \right] / \partial w^{min} < 0$; and*

3. *decreases the returns to worker ability: $\partial \left[\partial w_\theta(p; w^{min}) / \delta \theta \right] / \partial w^{min} < 0$.*

Proof. See Appendix B.3. □

We interpret the proposition as follows. Part 1 states that wages increase for all workers with a reservation wage below the minimum wage. In the presence of search frictions, rent sharing is an equilibrium outcome, and the minimum wage acts as a transfer from firms to workers in the markets it affects. Part 2 characterizes the nature of spillovers between firms within a market. Wage increases at the initially lowest-paying firms are one-for-one with the minimum wage but gradually decline for higher-paying firms, leading to a flattening of the firm productivity-pay

gradient. Finally, part 3 shows that among all affected markets, lower ability workers gain more from the minimum wage, leading to a flattening of the worker ability-pay gradient.

Our model nests two important benchmarks: that of perfectly competitive labor markets with workers paid their marginal product ($\lambda_\theta^e / \delta_\theta \to +\infty$), and the monopsony outcome where all observed wage heterogeneity reflects differences in workers' outside option ($\lambda_\theta^e / \delta_\theta = 0$). In both cases, though for different reasons, there is no "frictional wage dispersion" across firms so that the minimum wage induces no spillovers. For the intermediate range, the paramterization of the model determines the strength of equilibrium spillovers. Hence, the model's predictions for minimum wage effects depend crucially on estimates of the heterogeneous labor market parameters.

4 Estimation

This section estimates the model based on Brazil's RAIS matched employer-employee data from the "pre-period" 1996–2000. We want to use the estimated model to consider the following policy experiment: given the structural parameters estimated on a "pre-period," what are the equilibrium effects the subsequent rise in the minimum wage seen in Brazil?

4.1 Methodology

The general model we presented in Section 3 is rich enough to capture many facets of heterogeneity among workers, firm attributes, and features of the Brazilian labor market. Consequently, we want to determine a large set of parameter values when bringing the model to the data.

Pre-set parameters. We start by setting a few parameters based on standard values in the literature. Without data on vacancies, matching efficiency is not separately identified from the cost intercept of posting vacancies, so we normalize the fomer to $\chi_\theta \equiv 1$. Following Petrongolo and Pissarides (2001), we set the elasticity of the aggregate matching function with respect to vacancies to $\alpha = 0.3$. We adopt a monthly frequency and set the discount rate to the annual equivalent of a five percent real interest rate.

Indirect inference. Next, we estimate the remaining parameters via indirect inference (Gourieroux et al., 1993) by targeting a set of auxiliary moments that summarize the data in key dimensions, thus informing the underlying structural parameters. Our estimation strategy balances two

considerations: On the one hand, it is important to capture accurately the distribution of worker and firm heterogeneity underlying the observed wage distribution. On the other hand, estimating a model with many parameters requires efficiency. We trade off these competing objectives by adopting a semiparametric approach: We do not impose a parametric assumption on the distribution of worker ability (the most important determinant of wage differences), but we take a parametric approach to estimating log firm productivity by assuming that it follows a Gamma distribution with shape parameter γ_1 and scale parameter γ_2. We demonstrate below that this semiparametric approach provides sufficient flexibility to capture well the distribution of wages in the data, as well as its decomposition into firm and worker AKM components.

This leaves us with the following parameters to estimate: market-specific separation rates, $\{\delta_\theta\}_\theta$; relative search efficiencies, $\{s_\theta\}_\theta$; flow values of leisure, $\{b_\theta\}_\theta$; scalar cost intercepts, $\{c_\theta\}_\theta$, of the assumed isoelastic vacancy cost function, $c_\theta(v) = c_\theta v^{1+\eta} / (1 + \eta)$; curvature of vacancy creation, η; mass of potential firms, M_0; parameters of the Gamma distribution describing firm productivity, γ_1 and γ_2; and the distribution of worker ability as summarized by its median, θ^{50}, and pth percentiles relative to the median, $\tilde{\theta}^p$, for $p = 5, 10, 25, 75, 90, 95$, between which we interpolate. We normalize the initial minimum wage to be the numeraire.

We target the following 55 moments in our estimation, with details on their computation relegated to Appendix C.3: the log median-to-minimum wage; the employment-weighted mean log firm size; the employment-weighted standard deviation of log firm size; the log 5–50, log 10–50, log 25–50, log 75–50, log 90–50 and log 95–50 percentile ratios of AKM worker and firm fixed effects; and a set of auxiliary labor market moments by worker type. The set of auxiliary labor market moments includes the employment-to-nonemployment (EN) hazard rate, the nonemployment-to-employment (NE) hazard rate, the effective speed at which workers climb the firm ladder, and workers' reservation wage, all by decile of AKM worker fixed effects.

To reduce the dimensionality of our indirect inference step, we proceed in two steps. In the first step, we estimate outside of our main estimation loop the 4×10 labor market parameters by worker ability decile. To this end, we target the EN hazard, NE hazard, the effective speed at which workers climb the firm ladder, and workers' reservation wage by decile of worker AKM fixed effects. Mapping the rank in AKM worker fixed effects into the rank of underlying ability rank works to the extent that more able workers do not have sufficiently worse labor market mobility patterns.[15] We then compute by empirical AKM worker fixed effect decile the following auxiliary

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[15]We test for the robustness of this identification by simulating the model for different slopes of the labor market

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labor market moments. An estimate of the EN transition rate directly informs the exogenous job destruction rate, δ_θ. A reduced-form moment capturing the relative arrival rate of job offers while employed, $\hat{s}_\theta$, directly corresponds to the relative on-the-job search efficiency parameter s_θ. The vacancy costs intercept, c_θ, is picked so that the model matches the empirical NE transition hazard. Finally, it suffices to solve for the reservation wage of workers, ϕ_θ, instead of the underlying flow value of leisure, b_θ. We solve the model and later back out the underlying flow value of leisure b_θ that rationalizes our estimate of ϕ_θ given the other parameter values.

This leaves us with 11 parameters to estimate in our main estimation via indirect inference: $c_1, M_0, \gamma_1, \gamma_2, \theta^{50}, \left\{\tilde{\theta}^p\right\}_{p=5,10,25,75,90,95}$. Although the remaining structural parameters are jointly identified, we outline here a brief heuristic identification argument for what moments inform each of the moments. The median-to-minimum wage informs the median worker ability level, θ^{50}, while the log percentile ratios of the AKM worker fixed effects distribution particularly inform the percentile ratios of the underlying worker ability distribution, $\tilde{\theta}^p$. The shape and scale of the distribution of AKM firm fixed effects particularly inform the parameters governing the firm productivity distribution, γ_1 and γ_2. Average firm size informs the measure of firms relative to the mass of workers, M_0. Finally, the standard deviation of firm sizes informs the curvature of vacancy creation, η, because a lower value of this elasticity makes it relatively less costly for productive firms to grow large relative to unproductive, small firms. We pick as our estimate the parameter vector that minimizes the sum of equally weighted squared percentage differences between the remaining 15 moments in the model and the data.[16]

4.2 Parameter estimates and model fit

Following Appendix C.3, we derive a set of model-consistent auxiliary moments summarizing workers' rate of leaving formal employment (δ_θ), time out of formal employment (informative of λ^u_θ), job transition rate while employed (informative of λ^e_θ), and lowest starting wage (informative of ϕ_θ). Figure 3 plots the derived auxiliary moments by worker ability decile.

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parameters across ability groups, in particular s_θ, and find that the joint information contained in the distribution of measured AKM worker fixed effects and observed mobility patterns is enough to recover the correct ranking of underlying worker ability types.

[16]In practice, we solve the model approximately 5 million times under different parameter configuations from a space of potential values. To make this feasible, for a given parameter configuration we solve for the continuous-time wage and vacancy policies obtained from the equilibrium system of differential equations described in Section C.1 and run an AKM decomposition on the solution. This avoids simulating a large number of individual worker histories, which speeds up estimation by an order of magnitude. After the model estimation, we simulate a large number of worker histories at monthly frequency under the estimated parameter values and add orthogonal normal disturbances to simulated log wages before estimating AKM on the resulting dataset.

Figure 3. Targeted labor market moments by worker ability decile, 1996–2000

| (a) EN hazard rate | (b) NE hazard rate | (c) Rel. search efficiency | (d) Lowest acc. wage |
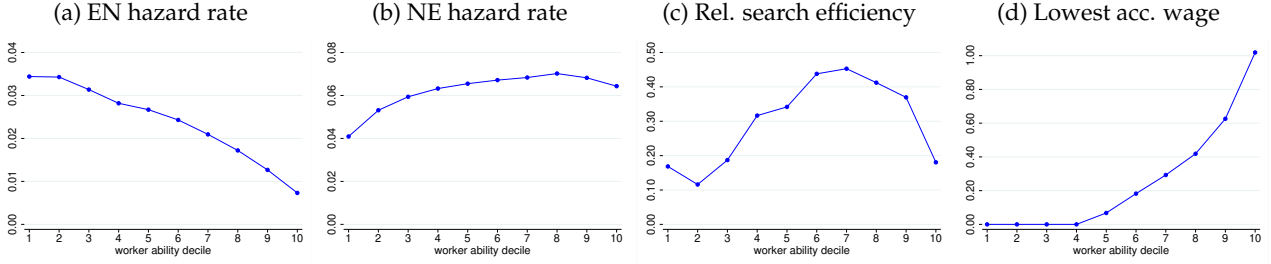
Notes: Each worker ability decile contains around 9 million observations. Standard errors are all $< 10^{-3}$ and omitted. Exit hazard rate in panel (a) and entry hazard rate in panel (b) are monthly rates. Relative on-the-job search intensity in panel (c) is a scalar. Estimated lowest accepted wage in panel (d) is in log multiples of the current minimum wage. Source: RAIS.

Figure 4 plots the resulting estimates of the separation rate δ_θ, the scalar cost of posting vacancies c_θ, the relative search intensity of employed workers s_θ and the flow value of nonemployment b_θ by worker ability. We find substantial heterogeneity in these parameters across worker ability groups, as a result of significant differences in observed labor market mobility patterns by worker types in the data. The separation rate of the most able workers is only a quarter of that for the least able, and they search with twice the efficiency on the job. As we will see, this heterogeneity in labor market mobility implies sorting of higher-paid workers to higher-paying firms. The scalar cost of posting vacancies is non-monotone in worker ability due to two offsetting forces. On the one hand, it is more attractive for firms to post vacancies in higher ability markets since such workers produce more, which tends to drive up the estimated c_θ. On the other hand, observed NE flows are higher for more able workers, implying a lower scalar cost c_θ.

The estimated flow value of nonemployment is mildly negative and declines in worker ability. It is well-known that this class of models imply a low flow value of leisure, and our framework is not too different in this regard (Hornstein et al., 2011). As there are many possible explanations for this observations—including a large curvature of utility, financial constraints, or a high real interest rate—and we think of this issue as orthogonal to our objective, we do not pursue it further in this paper. Note that we cannot estimate the flow value in markets where the minimum wage binds, since we do not observe workers reservation wage in these cases. This is not a problem, fortunately, since the policy experiment we consider increases the minimum wage, we do not require an estimate of the flow value of leisure for those workers who are in markets where the minimum wage initially binds.[17]

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[17]In some of the later counterfactual experiments, we simulate a lower minimum wage than what Brazil had initially. When conducting these experiments, we linearly extrapolate the observed reservation wage by worker type in Figure 3 for workers below the initial minimum wage in Brazil when estimating the flow value of leisure for these workers.

Figure 4. Estimated values of δ_θ, c_θ, s_θ and b_θ by worker ability quantiles
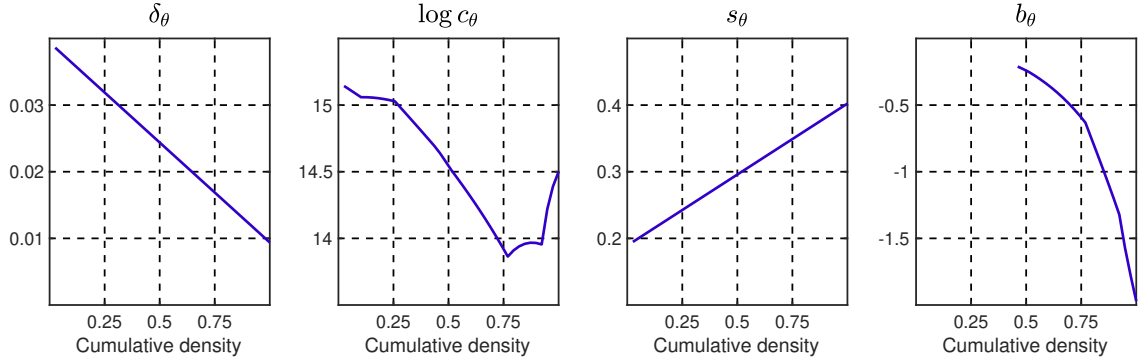
Note: Estimated continuous-time separation rate δ_θ, flow scalar cost of posting vacancies c_θ, $c_\theta(v) = c_\theta v^{1+\eta}/(1+\eta)$, relative search intensity of employed workers s_θ, and flow value of leisure b_θ, by cdf of worker ability. Source: simulations.

Table 3 presents the parameters of the model estimated in the indirect inference step. The implied variance of firm productivity is seven log points versus nine for worker ability. To preview our later findings, the estimates imply that the least able workers remain productive at the least productive firms in response to the empirically relevant 44 log point increase in the minimum wage. Hence, the employment reponse to the minimum wage hike that we later find is driven by adjustment on the intensive margin—changes in vacancy creation conditional on remaining active—not the extensive margin. Finally, we highlight that the estimated cost of hiring is not particularly convex, implying a relatively elastic employment response to a minimum wage hike.

Table 3. Parameter estimates

Description	Parameter	Value
Panel A. Worker ability, flexible interpolation points		
Median	θ^{50}	2.29
5th percentile, relative to median	$\tilde{\theta}^5$	0.80
10th percentile, relative to median	$\tilde{\theta}^{10}$	0.82
25th percentile, relative to median	$\tilde{\theta}^{25}$	0.99
75th percentile, relative to median	$\tilde{\theta}^{75}$	1.07
90th percentile, relative to median	$\tilde{\theta}^{90}$	1.64
95th percentile, relative to median	$\tilde{\theta}^{95}$	2.66
Panel B. Firm productivity, $\log(p) \sim \Gamma(\gamma_1, \gamma_2)$		
Shape	γ_1	2.19
Scale	γ_2	0.15
Panel C. Labor market parameters		
Mass of firms	M_0	0.07
Curvature of vacancy cost	η	0.45

Note: Estimated parameter values of worker ability distribution, firm productivity distribution, and job creation. Source: simulations.

Table 4 compares targeted moments used in the indirect inference procedure in the model and in the data. The model replicates well the empirical distribution of AKM worker and firm fixed effects, suggesting that our parametric assumption on the firm productivity distribution is sufficiently flexible to allow us to capture the distribution of AKM firm fixed effects in the data.

4.3 Validation

To validate our proposed theory of wage setting, we first note that our estimated value of the rate at which workers move up the firm ladder, κ_θ, indicates that Brazilian workers indeed do climb a firm ladder through gradual on-the-job mobility. In fact, given that we estimate this parameter by comparing the distribution of overall firm AKM fixed effects to that among workers who reenter the labor market and that we target this as an auxiliary moment, by construction the estimated model matches the extent to which such mobility moves workers up the firm ladder.

Table 4. Data vs. model: Targeted moments, 1996–2000

Moment	Data	Model
Panel A. Worker fixed effects		
P5–50	-0.578	-0.579
P10–50	-0.498	-0.468
P25–50	-0.253	-0.271
P75–50	0.365	0.389
P90–50	0.976	0.874
P95–50	1.203	1.207
Panel B. Firm fixed effects		
P5–50	-0.570	-0.637
P10–50	-0.503	-0.503
P25–50	-0.305	-0.263
P75–50	0.309	0.274
P90–50	0.503	0.509
P95–50	0.570	0.655
Panel C. Other moments		
Log minimum-to-median wage	-1.121	-1.157
Mean log firm size	4.439	4.579
Standard deviation of log firm size	2.340	2.458

Note: Percentile ratios summarizing targeted worker and firm AKM firm fixed effects in the model and the data. Model moments are based on a simulated, discrete-time monthly approximation of model with 1m individuals over five years, including orthogonal normal noise added to annual log earnings in order to match the variance of the estimated AKM residual. Both model and data include individual-years with earnings greater than the minimum wage and at firms with at least 10 employees. Source: RAIS and simulations.

To further validate the theory, we next illustrate that the estimated model not only captures

the separately targeted AKM worker and firm effects distributions, but also replicates many features of the empirical wage distribution. Specifically, because higher ability workers climb the job ladder faster, the estimated model replicates the positive assortative matching between workers and firms evident in the data, matching closely the empirical correlation between AKM worker and firm fixed effects of 0.28 with associated covariance of 0.07. In spite of matching well large shares of the AKM fixed effects distributions, a small number of extreme outliers—confined to the top and bottom few percent of the AKM fixed effects distributions—leads us to underestimate the overall variance of wages by five log points.[18] Figure 5 plots the resulting distribution of wages, illustrating the model fit with respect to the empirical wage distribution.

Figure 5. Data vs. model: Histogram of wages, 1996–2000

(a) Data   (b) Model
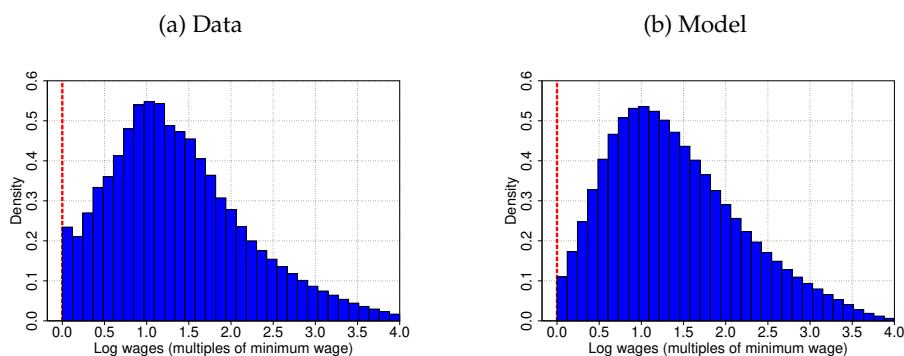
Note: Model moments are based on a simulated, discrete-time monthly approximation of model with 1m individuals over five years, including orthogonal normal noise added to annual log earnings in order to match the variance of the estimated AKM residual. Both model and data include individual-years with earnings greater than the minimum wage and at firms with at least 10 employees. Source: RAIS and simulations.

A feature of the Burdett and Mortensen (1998) model that carries over to our framework is the continuity of the wage distribution. This prediction is the direct result of strategic wage posting by firms, which in equilibrium rules out mass points anywhere, in particular at the minimum wage. Several authors have noted that this feature of the model is at odds with the data in certain contexts, such as Manning (2003) for the US, suggesting that the benchmark wage posting model is not a good description of those labor markets. In contrast, in the case of Brazil a relatively small share of workers is bunched at the minimum wage. While the model somewhat understates the density of workers at wages close to the minimum wage, we expect that this will if anything lead us to find smaller effects of the minimum wage.

Another way to assess the model's ability to match the overall distribution of wages given our

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[18]While we believe that it is more useful to capture the great majority of the distributions of AKM fixed effects as summarized by the targeted percentile ratios, we have verified that our results remain robust to inflating the dispersion of worker ability and firm productivity to match the variances of the AKM fixed effects.

targeted distribution of worker and firm AKM fixed effects is to evaluate the behavior of the AKM residual. Comparing panels (a) and (b) of Figure 6 shows that the model reproduces the pattern of residual variation across worker and firm effects groups found in the data. The minimum wage induces low-low matches to be associated with a positive residual, indicative of the nonlinear nature by which the wage floor affects pay schedules across worker and firm types. While the model slightly overpredicts this pattern in the data, in both cases the systematic deviation from the AKM two-way fixed effect decomposition is small.

Figure 6. Data vs. model: AKM wage residuals, 1996–2000

(a) Data
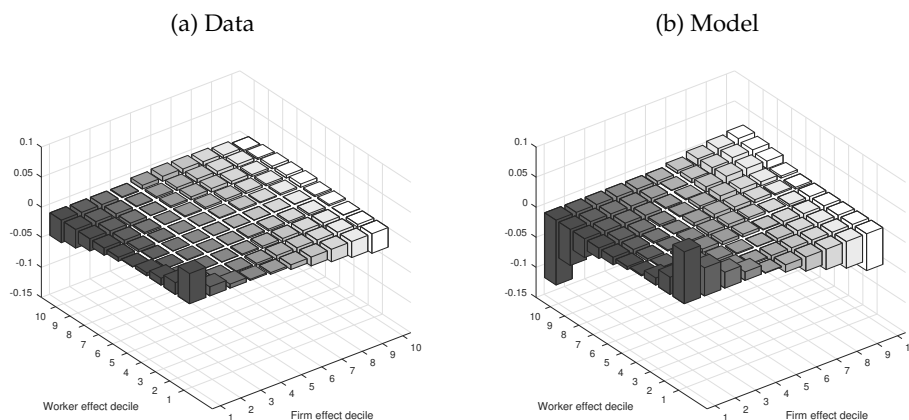
(b) Model

Notes: Figure shows estimated AKM residual $\hat{\varepsilon}_{ijt} = w_{ijt} - \hat{\alpha}_i - \hat{\alpha}_j - \hat{\gamma}_t$ by worker and firm effect deciles. Model moments are based on a simulated, discrete-time monthly approximation of model with 1m individuals over five years, including orthogonal normal noise added to annual log earnings in order to match the variance of the estimated AKM residual. Both model and data include individual-years with earnings greater than the minimum wage and at firms with at least 10 employees. Source: RAIS and simulations.

Finally, in order to assess whether the theory matches the nature of wage dynamics in the data, we confront our model with an AKM diagnostic tool proposed by the empirical literature (Card et al., 2013). Specifically, Figure 7 plots average wages for moving workers starting in the first and fourth firm effect quartiles depending on their destination quartile of firm effects. Wage gains and losses of switchers across the firm effects distribution are largely symmetric, in the sense that the wage losses experienced by those moving down the firm ladder mirror the wage gains of those moving up the firm ladder. The empirical literature has interpreted this as suggesting a lack of assortative matching, an interpretation that our model supports. We conclude from these exercises that the model matches well key stylized facts on wage dynamics in the data.

Figure 7. Data vs. model: Event study graph of wage gains from switching firms, 1996–2000

(a) Data　　　　　　　　　　　　(b) Model
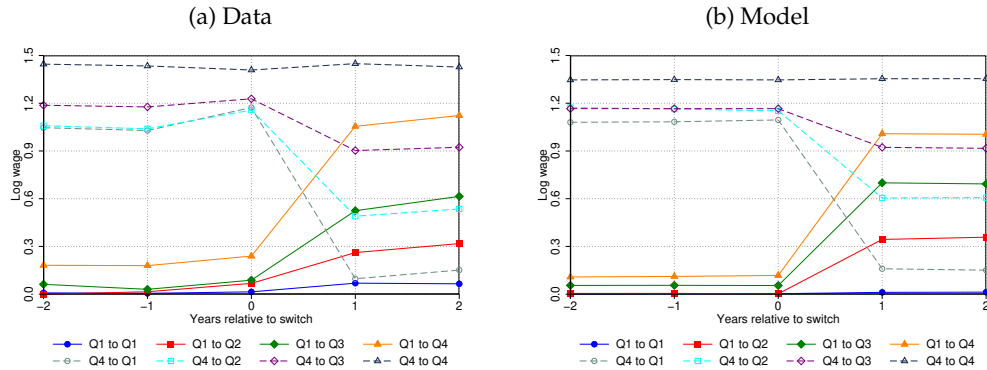
Notes: Figure plots changes in mean log wage upon switching employers between year 0 and year 1. Different lines show transitions from first and fourth quartiles of AKM firm fixed effects distribution for the period 1996–2000. Model moments are based on a simulated, discrete-time monthly approximation of model with 1m individuals over five years, including orthogonal normal noise added to annual log earnings in order to match the variance of the estimated AKM residual. Both model and data include individual-years with earnings greater than the minimum wage and at firms with at least 10 employees. Source: RAIS and simulations.

Figure 8 illustrates why, although misspecified, the AKM regression equation (1) provides valuable information that informs the underlying structural parameters guiding worker and firm heterogeneity. Panel (a) shows the close to linear structure of log wages in worker and firm types, with wage profiles shifting approximately in parallel with the notable exception of the bottom firm productivity decile. Estimated AKM worker effects in panel (b) are strictly increasing in worker ability θ while estimated AKM firm effects in panel (c) are strictly increasing in firm productivity.

Figure 8. Model: Wages and AKM components across worker and firm types, 1996–2000

(a) Wage structure　　　(b) AKM worker effects vs. ability　　　(c) AKM firm effects vs. prod.
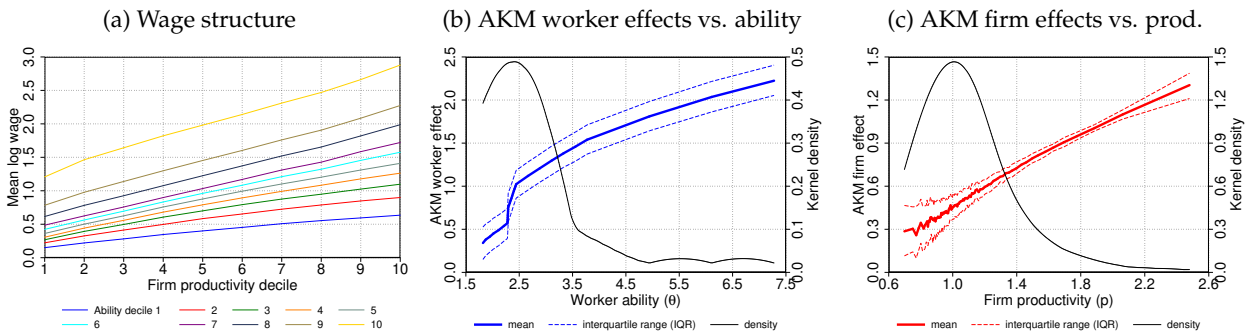
Notes: Worker ability (firm productivity) approximated using an Epanechnikov kernel density with bandwidth 0.53 (0.17). Model moments are based on a simulated, discrete-time monthly approximation of model with 1m individuals over five years, including orthogonal normal noise added to annual log earnings in order to match the variance of the estimated AKM residual. Both model and data include individual-years with earnings greater than the minimum wage and at firms with at least 10 employees. Source: simulations.

5 Effects of a Higher Minimum Wage

We use the estimated model to simulate the following policy experiment: what are the steady-state effects of a 44 log points increase in the real productivity-adjusted minimum wage between

1996–2000 and 2008–2012 on the wage distribution and aggregate economics outcomes, holding fixed all structural parameters?

5.1 Effect on the distribution of wages

The 44 log point rise in the minimum wage generates a significant decline in earnings inequality, as summarized in Table 5. The variance of log earnings declines by 11 log points, compared to an overall decline of 20 log points in Brazil over this period. Hence, the minimum wage accounts for 55 percent of the overall compression in inequality experienced by Brazil between 1996 and 2012. Both the variance of worker and firm AKM fixed effects decline markedly, with the former falling by 0.04 and the latter by 0.03. For comparison, the corresponding declines in the data are 0.06 and 0.09, respectively. The minimum wage accounts for most of the empirical decline in the covariance, driven primarily by the fall in the variance of the worker and firm AKM fixed effects. Finally, the model predicts no decline in the residual, whereas this falls by 0.02 in the data.[19]

As in the data, the compression in inequality in response to the minimum wage hike is concentrated at the bottom of the earnings distribution. The log 50-10 percentile ratio of earnings falls by 13 log points, while the log 90-50 ratio declines by seven log points. Hence in a relative sense, the minimum wage accounts for a similar fraction of the declines in earnings inequality throughout the earnings distribution in Brazil. It is worth highlighting that the share of workers employed at the minimum wage is well below 10 percent throughout this period, suggesting far-reaching spillover effects of the minimum wage. In line with part 1 of our model proposition, the decline in inequality is driven by substantially higher wages at the bottom of the ability distribution: Wages for the lowest-skill group are on average 30 log points higher after the minimum wage hike. On the other hand, the minimum wage affects wages up to the 80th percentile of worker ability, highlighting its far-reaching effects.

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[19]To address this, one could easily extend our estimation procedure to include relative noise, rather than in levels.

Table 5. Data vs. model: Inequality, 1996–2000 and 2008–2012

	1996–2000		2008–2012		Change	
	Data	Model	Data	Model	Data	Model
Panel A: AKM variance decomposition						
Total variance	0.68	0.61	0.48	0.51	-0.20	-0.11
Worker fixed effects	0.33	0.27	0.28	0.23	-0.06	-0.04
Firm fixed effects	0.16	0.15	0.07	0.12	-0.09	-0.03
2 x Covariance	0.13	0.12	0.10	0.09	-0.04	-0.04
Residual	0.05	0.07	0.03	0.07	-0.02	-0.00
Panel B: Wage percentiles						
P5–50	-1.03	-0.97	-0.63	-0.80	0.40	0.18
P10–50	-0.85	-0.81	-0.55	-0.68	0.30	0.13
P25–50	-0.46	-0.48	-0.33	-0.41	0.13	0.07
P75–50	0.58	0.60	0.48	0.55	-0.09	-0.05
P90–50	1.22	1.23	1.07	1.17	-0.15	-0.07
P95–50	1.62	1.61	1.47	1.55	-0.14	-0.07

Notes: Predicted worker and firm AKM fixed effects based on $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Omitted are variance terms involving year dummies γ_t, which account for a negligible share of the total variance. Source: RAIS and simulations.

To understand the importance of minimum wage spillovers higher up in the income dsitribution, let us define the "direct effect" as moving workers up to the new wage floor and the "indirect effect" as the additional adjustment due to workers' and firms' equilibrium responses. Panels (a)–(c) of Figure 9 show this two-step decomposition for the model-simulated data (which matches well the empirical earnings distribution). The first step of the decomposition, going from panel (a) to panel (b) of the figure, illustrates the counterfactual wage distribution that would result if all workers previously employed below the new minimum wage were moved up to the new wage floor. Clearly, the result is a large spike in the wage distribution exactly at the minimum wage, which we show in Section 2.3 of the paper is in stark contrast to the empirical wage distribution at any point in time during this period. The second step of the decomposition, going from panel (b) to panel (c) of the figure, allows for the equilibrium adjustment of firm wage offers and worker mobility in the new steady state of the model economy.

Table 6 implements this decomposition of the total decline in the variance of wages in the model and in the data, both showing that only 25–30 percent of the total decline in the variance of wages is explained by the direct effect of the minimum wage. The results from our estimated model suggest that indirect effects or spillovers lead to sizable propagation of the minimum wage.

Figure 9. Model: Direct and indirect effects on wages, 1996–2000 and 2008–2012

| (a) Initial | (b) Only direct effect | (c) Direct + indirect effects |
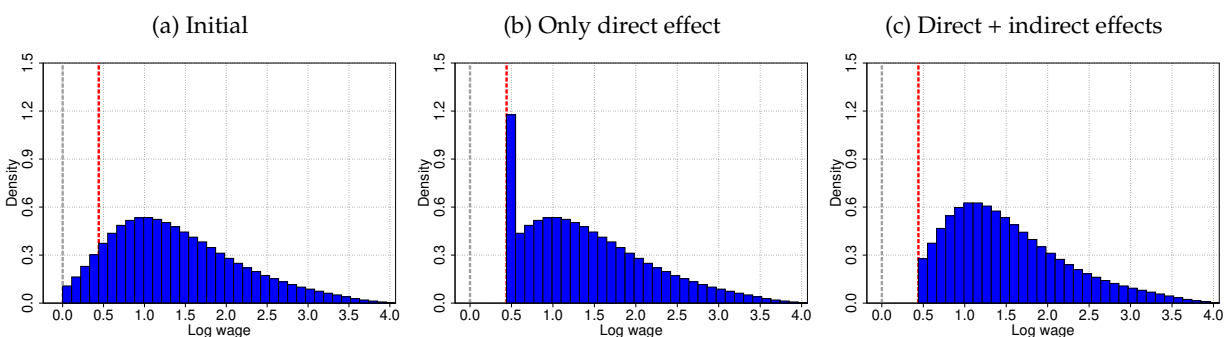
Notes: Counterfactual decomposition of the overall compression due to the minimum wage in the estimated model into a "direct effect" and "indirect effect." The grey dashed vertical line shows the initial level of the minimum wage, normalized to 0. The red dashed vertical line shows the final level of the minimum wage. Panel (a) shows the initial wage distribution in the estimated model. Panel (b) shows the counterfactual "direct effect" that results when moving all workers between the old and the new minimum to the new wage floor. Panel (c) shows the final equilibrium wage distribution. Source: simulations.

Table 6. Data vs. model: Decomposition into direct and indirect effects

	Data		Model	
	$Var\,(w)$	Δ (% total)	$Var\,(w)$	Δ (% total)
Initial period (1996–2000)	0.68	-	0.61	-
+ "Direct effect "	0.63	-0.05 (25%)	0.58	-0.03 (30%)
+ "Indirect effect" = final period (2008–2012)	0.48	-0.15 (75%)	0.51	-0.07 (70%)

Notes: Table shows decomposition result into "direct" and "indirect" effects of the minimum wage in the model and in the data. Direct effect results when moving all workers between the old and the new minimum to the new wage floor implements the decomposition. Indirect effect is due to remaining shifts in the wage distribution. Source: RAIS and simulations.

5.2 Effect on aggregate economic outcomes

Table 7 summarizes the impact of the minimum wage on macroeconomic outcomes. As in Flinn (2006), firms respond by creating fewer jobs, leading to a 1.9 log points increase in the aggregate nonemployment rate. This increase, however, is heterogeneous across the ability distribution, with the least able workers experiencing a much larger increase. Gross output increases slightly because the negative employment effect is offset by an increase in labor productivity, as employment reallocates towards more productive firms as in Eckstein and Wolpin (1990). While not targeted, the model approximately matches the initial 55 percent labor share, or portion of value added going to worker pay in the data. In spite of the sizable minimum wage hike, our model predicts that the aggregate labor share moves by a relatively modest 1.6 log points, due to the reallocation of workers toward higher-productivity firms with lower labor shares. For comparison, Brazil's labor share rose by approximately 0.9 log points in the data over this period.

Table 7. Model: Effects on macroeconomic outcomes, 1996–2000 and 2008–2012

	1996–2000	2008–2012	Change
Nonemployment	0.278	0.298	0.019
Output	1.973	1.979	0.006
Labor productivity	2.299	2.332	0.033
Net output	1.968	1.975	0.007
Labor share	0.504	0.520	0.016

Notes: Output is the log of total output; labor productivity is the log of total output minus the log of total employment; net output is the log of total output minus vacancy costs; labor share is total wages divided by total output. Source: simulations.

It is worth highlighting the different forces leading to the relatively modest employment response to the minimum wage. To this end, it is useful to decompose the elasticity of nonemployment to the minimum wage in market θ as

$$
\frac{\partial \log u_\theta}{\partial \log MW} = \underbrace{\frac{\partial \log u_\theta}{\partial \log \lambda_\theta^u}}_{\text{"job finding channel"}} \times \underbrace{\frac{\partial \log \lambda_\theta^u}{\partial \log V_\theta}}_{\text{"vacancy conversion channel"}} \times \underbrace{\frac{\partial \log V_\theta}{\partial \log MW}}_{\text{"profit squeeze channel"}}
\tag{3}
$$

Equation (3) decomposes the total unemployment response by skill group into three terms. The first term (or "job finding channel") reflects the marginal effect of a change in the job finding rate on the nonemployment rate. This term equals $-\lambda_\theta^u / \left(\delta_\theta + \lambda_\theta^u\right)$ so that for a given separation rate, the steady-state unemployment rate is inversely related to the job finding rate from unemployment. The second term (or "vacancy conversion channel") equals $\alpha / \left(1 - \alpha(1 - s_\theta)\delta_\theta\lambda_\theta^u / \left((\delta_\theta + s_\theta\lambda_\theta^u)(\delta_\theta + \lambda_\theta^u)\right)\right)$ and captures the marginal effect of a change in aggregate vacancies on the job finding rate. This term is positive and captures two distinct congestion externalities in the labor market: that a vacancy crowds out other vacancies, and that searching workers crowd out other searching workers.[20] The third term (or "profit squeeze channel") stands for the change in aggregate recruiting intensity in response to a marginal change in the minimum wage. This term is negative, because a higher minimum wage cuts into firms' profit margins and leads them to post fewer vacancies as an equilibrium response.

This decomposition sheds light on why we find a relatively small response of the nonemployment rate to the large increase in the minimum wage. It also helps understand what moments of the data may lead to a larger effect of the minimum wage on nonemployment. First, we estimate

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[20]For instance, if the employed do not search, $s_\theta = 0$, the second term reduces to $\alpha / (1 - \alpha\lambda_\theta^u / (\delta_\theta + \lambda_\theta^u))$, while if they continue to search with the same efficiency as the nonemployed, $s_\theta = 1$, the second term is α. This refects the fact that if nonemployed workers stop searching as they become employed, a fall in vacancy creation and the assocated increase in the nonemployment rate crowds out other nonemployed job seekers' job prospects.

a relatively low nonemployment-to-employment transition rate, λ_θ^u, in Brazil (relative to for instance the US), with an associated value of 0.67 on the "job finding channel". Second, we estimate that workers continue to search with a nontrivial intensity as employed, which together with a benchmark estimate in the literature of the elasticity of the matching function results in a "vacancy conversion channel" of around 0.38. Third, we find a large, almost one-for-one, reduction of aggregate vacancies to the minimum wage, implying a "profit squeeze channel" of approximately 1.00. Together, these estimates imply that nonemployment increases by a factor equal to 0.25, substantially mitigating the aggregate vacancy response by a factor of around one quarter.

5.3 Minimum wage channels

The effect of the minimum wage on wages and nonemployment can be decomposed into an extensive margin adjustment of firms' hiring policy—exit—and an intensive margin adjustment of hiring—changes in the vacancy policy conditional on continuing to hire of firms—as well as in the case of wages, changes in the pay policy of firms. Table 8 decomposes the effect of the minimum wage on mean wages, the variance of wages and nonemployment into these separate components. Specifically, the first column shows the effect of exit only, holding the vacancy policy and the wage policy fixed. As we noted earlier, our estimates imply that all workers remain productive at all firms also after the 44 log point increase in the minimum wage. The second column shows the additional effect of changes in the intensive margin of hiring—the vacancy policy conditional on hiring. This accounts for over a third of the increase in wages, but less than a quarter of the fall in inequality. That is, the reallocation of workers to more productive, higher paying firms primarily boosts average earnings, but has less of an effect on the dispersion of earnings. The final column shows the additional effect of adjusting the wage policy, illustrating that this is the most important source of changes in inequality.[21]

Table 8. Variance decomposition: Allocation versus wage policy

Moment	Exit	Vacancy policy	Wage policy	Total effect
Mean wage	0.000	0.048	0.085	0.133
Variance of wage	0.000	-0.034	-0.108	-0.142
Nonemployment	0.000	0.019	0.000	0.019

Notes: Effects on average log wages, the variance of log wages and the nonemployment rate (percentage points) of only changing the exit policy of firms, holding the vacancy policy and wage policy fixed (column 1), the additional effect of changing also the vacancy policy (column 2) and the additional effect of changing also the wage policy. Source: model.

[21] Although this decomposition is not invariant to the order in which we add the different effects, we reach effectively the same conclusion regardless of the order.

Figure 10. Model: Gradient effects of the minimum wage, 1996–2000 and 2008–2012

(a) Lower productivity pay premium   (b) Lower returns to worker ability
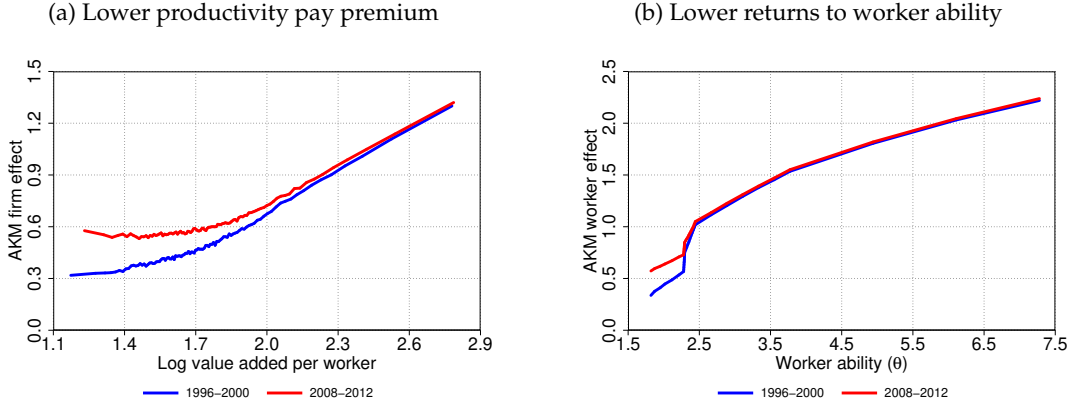
Notes: Predicted worker and firm AKM fixed effects are based on $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Panel (a) plots mean AKM firm fixed effect against log value added per worker, computed from the model simulated data as $VAPW_{jt} = \sum_{i \in \{i|J(i,t)=j\}} \theta_i p_j \, / \sum_i \mathbf{1}\,[i \in \{i|J\,(i,t) = j\}]$. Panel (b) plots the mean AKM worker fixed effect against worker ability θ. Source: simulations.

The result that changes in the allocation of workers to firms account for a small share of the total minimum wage impact matches empirical findings in Alvarez et al. (2018) that compositional changes account for little of the decline in inequality in Brazil over this period. Instead, as predicted by parts 2 and 3 of our model proposition, the minimum wage reduces the returns to pay-relevant worker and firm characteristics. Figure 10 illustrates that this mechanism generates a flattening of AKM firm effects across value added per worker in panel (a) and of AKM worker effects across ability in panel (b), thereby providing an explanation for the empirical facts describing Brazil's decline in inequality over this period (Alvarez et al., 2018). Therefore, it is through a change in the productivity-pay premium and in skill prices, rather than changes in the underlying distribution, that the minimum wage reduces wage inequality.

5.4   Out-of-sample predictions

While a complete welfare evaluation of the minimum wage is beyond the scope of this paper, a natural question to ask in our framework is what the distributional and aggregate consequences of setting a minimum wage outside of the range seen in Brazil are? Figure 11 illustrates the impact of the minimum wage on inequality, nonemployment and net output as the minimum wage varies over a wider domain, up to 120 log points (70 percent) less than and up to 160 log points (500 percent) higher than its level in 1996–2000. Evidently, Brazil started from a favorable position in terms of the relative benefits and costs of the subsequent minimum wage hike, in the sense that for a lower initial minimum wage the subsequent increase would have had little effect on either

inequality or nonemployment whereas for a higher initial minimum wage the effect on inequality would have been smaller but the effect on nonemployment larger. Throughout the domain, the effect on net output is small and nonmonotone due to the offsetting productivity gains and fall in resources spent on recruiting counteracting the disemployment effect of the minimum wage.

Figure 11. Model: Out-of-sample predictions for minimum wage effects
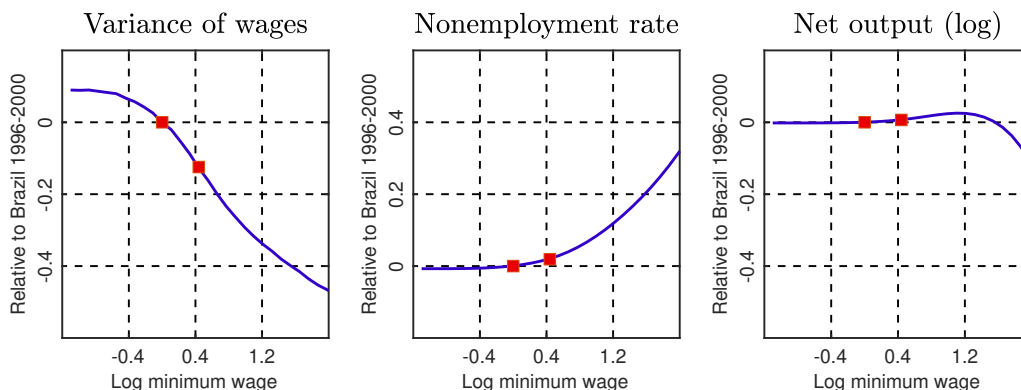
Notes: Out-of-sample predictions for minimum wage effects on the variance of wages (left), nonemployment rate (center), and net output in logs (right). All panels are normalized to the initial level in Brazil in 1996–2000, represented by the left most red dot. The red dot on the right marks the final level of the minimum wage in 2008–2012. Source: model.

A related question is what moments of the data inform the magnitude of the effects of a minimum wage increase on inequality and nonemployment. To illustrate this, Figure 12 varies our targeted moments between 50 percent and 150 percent of those observed in Brazil in 1996–2000. Specifically, the two left panels vary all the percentile ratios summarizing the worker AKM fixed effect distribution by the same factor (between 0.5 and 1.5) relative to their empirical values in Brazil in 1996–2000, reestimates all parameters instead targeting these inflated or deflated moments, and graphs in the top panel the estimated impact of the 44 log point increase in the minimum wage on the variance of log wages and in the bottom panel the impact on the nonemployment rate. The middle panels do the same for all percentile ratios summarizing the firm AKM fixed effect distribution, while the right panels do it varying the median to minimum ratio. Evidently, the impact of the minimum wage remains roughly the same regardless of the dispersion in worker AKM fixed effects. In contrast, the higher is the dispersion in firm AKM fixed effects, the more favorable does a minimum wage hike appear in the sense that the impact on inequality is larger while the effect on nonemployment is smaller. Finally, the higher is the initial median wage to the minimum wage—the less binding is the initial level of the minimum wage—the lower are the effects of the increase in the minimum wage on both inequality and nonemployment.

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Figure 12. Model: Inequality and nonemployment effects across moment ranges
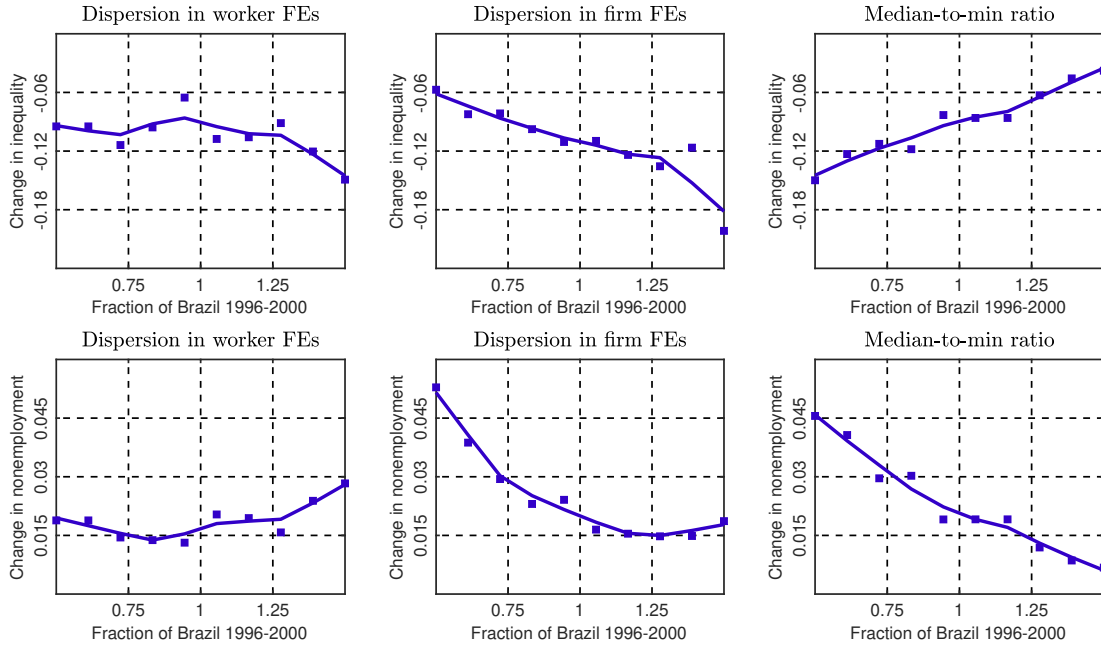
Notes: Effect of 44 log point increase in minimum wage on inequality (top panels) and nonemployment (bottom panels) varying either all the targeted percentiles characterizing the worker AKM fixed effect distribution by the same factor between 0.5 and 1.5 of the corresponding empirical moments in Brazil from 1996–2000 (left panels), all the targeted percentiles characterizing the firm AKM fixed effect distribution (middle panels), or the log median-to-minimum wage ratio (right panels), reestimating the model to fit these deflated/inflated moments holding all other targeted moments fixed at their targeted levels, and tracing out the impact of a 44 log point increase in the minimum wage in the reestimated model. Source: model.

5.5 Robustness

To illustrate the sensitivity of our results to variation in the underlying parameter values of the model, Figures 13–14 plot the predicted impact of the 44 log point rise in the minimum wage on inequality and nonemployment, respectively, varying one parameter at a time. The estimated impact of the minimum wage on both inequality and nonemployment is primarily sensitive to variation in two structural parameters: the level of worker ability summarized by its median value, θ^{50}, and the scale of firm productivity, γ_2. In particular, a higher median worker ability implies a smaller effect of the minimum wage on both inequality and nonemployment, as workers are less bound by the minimum wage in the first place. A higher scale of firm productivity is associated with a larger effect of the minimum wage on inequality but a smaller effect on nonemployment, as firms differ more in underlying productivity. In this sense, the greater is dispersion in firm productivity, the more it tilts the balance in favor of a minimum wage. Appendix C discusses further identification of the key parameters of the model in light of these findings.

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Figure 13. Model: Minimum wage effect on inequality, varying one parameter at a time
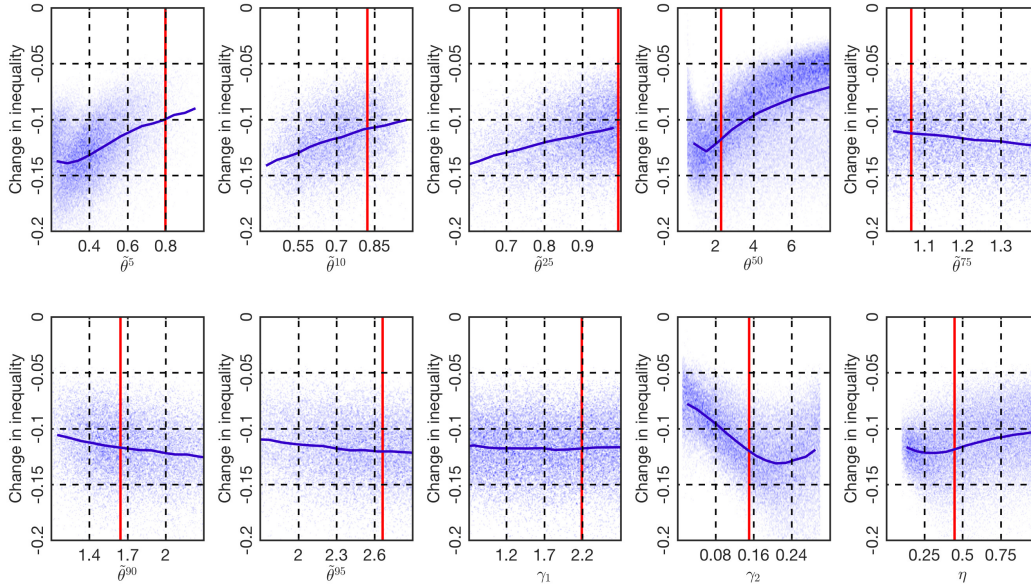
Notes: Predicted impact of 44 log point increase in the minimum wage on variance of log earnings varying one parameter at a time. Each blue dot represents one model simulation, the bold blue line represents the movering average over model simulations, and the red vertical line represents the estimated parameter value. Source: model.

## Figure 14. Model: Minimum wage effect on nonemployment, varying one parameter at a time
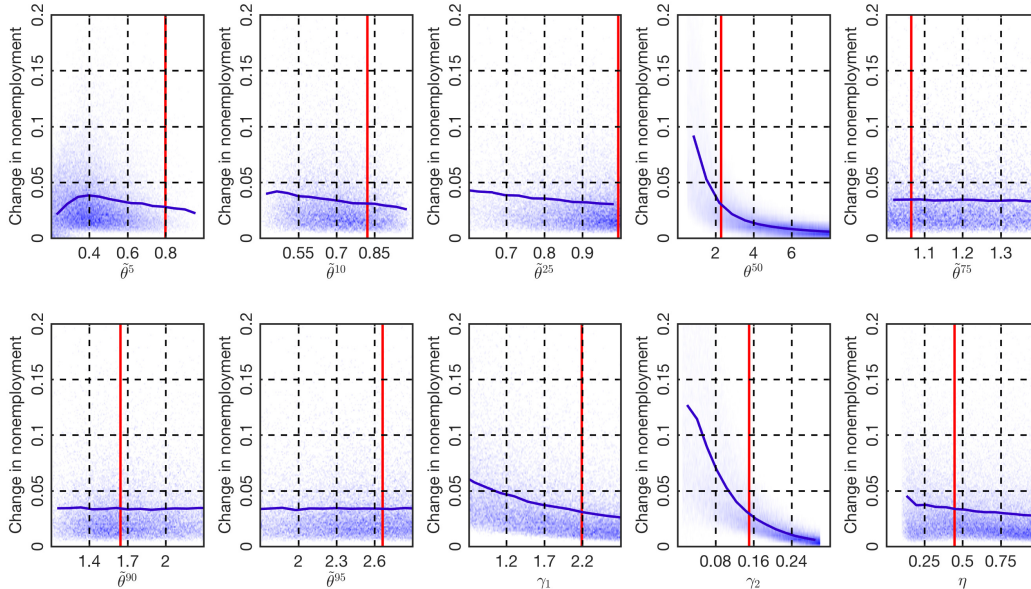
Notes: Predicted impact of 44 log point increase in the minimum wage on aggregate nonemployment rate varying one parameter at a time. Each blue dot represents one model simulation, the bold blue line represents the movering average over model simulations, and the red vertical line represents the estimated parameter value. Source: model.

6 Reduced-Form Evidence Using Cross-State Variation

We confront our model with novel empirical facts on the impact of the minimum wage in Brazil. Our starting observation is that wage inequality, while declining overall in Brazil between 1996 and 2012, fell disproportionately in initally low-income regions for which the minimum wage was more binding. Figure 15 groups into "low income" and "high income" the three lowest and three highest among Brazil's 27 states ranked by mean log wage in 1996, and plots normalized wage inequality measures between 1996 and 2012. Panel (a) shows that the variance of log wages drops by more than half in initially low-income states, but by less than one-fifth in initially high-income states. Panel (b) shows that lower-tail inequality drops especially in initially low-income states, with the P50–P10 and P50–P25 for this group declining by 50 and 40 percent, respectively, but by markedly less for initially high-income states. In contrast, upper-tail inequality, measured by the P75–P50 or the P90–P50, declines only in initially low-income states, as shown in panel (c).

Figure 15. Data: Evolution of wage inequality measures across state groups, 1996–2012

(a) Variance of log wages   (b) Lower-tail inequality   (c) Upper-tail inequality
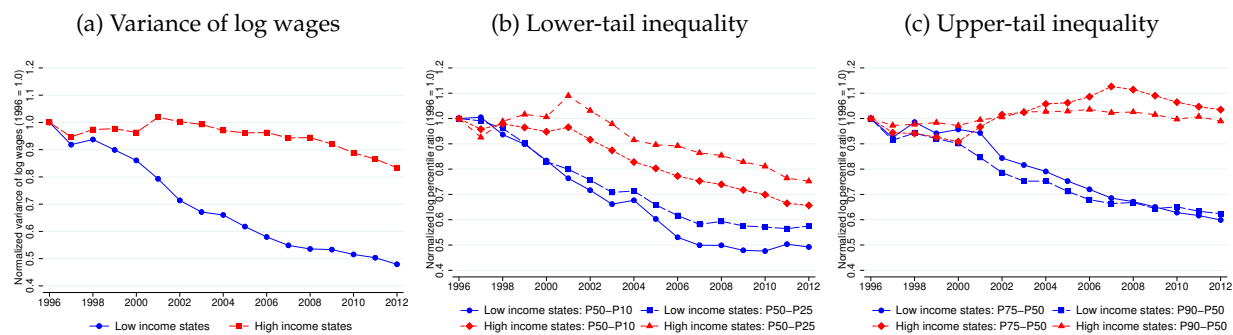
Notes: All panels group into "low income" and "high income" the three lowest and three highest states ranked by mean log wage in 1996, and plot between 1996 and 2012 different wage inequality measures normalized to 1.0 in 1996. Panel (a) shows the variance of log wages, panel (b) shows lower-tail wage percentile ratios, and panel (c) shows upper-tail wage percentile ratios. Source: RAIS.

6.1 Spillover effects identified off regional variation

These patterns lead us to ask: to what extent can the rise in the minimum wage rationalize the observed heterogeneity in wage inequality in the cross section and over time? As the policy is set at the federal level, it is hard to find exogenous variation in its treatment intensity. To identify minimum wage effects in this environment, we follow Lee (1999) and Autor et al. (2016) in exploiting heterogeneous exposure across subpopulations that differ in initial bindingness with respect to the federal minimum wage. We define the "effective minimum wage" or Kaitz index for subpopulation s at time t, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, as the log difference between the

prevailing minimum wage, w_t^{min}, and the median wage of subpopulation s, w_{st}^{median}.[22]

Figure 16 plots the relation between different wage percentile ratios and the Kaitz index for the data compared to the model.[23] Panel (a) plots empirical lower-tail inequality, measured by the P50–P10, against the Kaitz index across Brazilian states over time. The negative 45 degree line marks states where the minimum wage is binding for the lower 10 percent of workers. Panel (b) repeats the same exercise on our simulated data. Both plots show a negative relationship between the P50–P10 and the Kaitz index that grows more pronounced for more binding states in the cross section and over time. For comparison, panels (c)–(d) show a weaker relationship between top inequality, measured by the P90–P50, and the Kaitz index.[24]

Following Lee (1999) and Autor et al. (2016), we regress an outcome variable $y_{st}(p)$ for wage percentile p of state s in year t on the effective bindingness of the minimum wage and controls:

$$y_{st}(p) = \sum_{n=1}^{N} \beta_n(p) \, kaitz_{st}^n + \gamma_{st}(p) + \varepsilon_{st}(p) \tag{4}$$

where N is the polynomial order of the Kaitz index, $\gamma_{st}(p)$ denotes either year dummies or linear state-time trends plus year dummies, and $\varepsilon_{st}(p)$ is an error term that we assume satisfies the strict exogeneity assumption $\mathbb{E}[\varepsilon_{st}(p)|kaitz_{st}, \ldots, kaitz_{st}^n, \gamma_{st}(p)] = 0$. After estimating equation (4) separately by wage percentile p, we compute an estimate of the marginal effect of the minimum wage, $\rho(p) \equiv \sum_{n=1}^{N} n\beta_n(p) \, kaitz_{st}^{n-1}$, evaluated at the worker-weighted median state-year. Allowing for polynomials of order $N \geq 2$ is important to capture the nonlinear effects of the minimum wage as it becomes more binding.

We first consider as an outcome variable in equation (4) the log ratio between wage percentile p and the median wage, $y_{st}(p) = \log w_{st}(p) - \log w_{st}^{median}$, for various values of p.[25] To the extent that the minimum wage leads to higher wage growth at lower percentiles, we expect the estimated marginal effect $\rho(p)$ to be weakly decreasing across wage percentiles p. We interpret positive

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[22]Figure 23 in Appendix D.1 shows that variation in the Kaitz index across Brazilian states is large initially and declines as the minimum wage increases, while roughly preserving the ranking of states over time.

[23]We produce data from our model by simulating 27 separate economies with all parameters held at their estimated values, varying only the level of the minimum wage to match the empirical Kaitz index distribution across Brazil's states. This is consistent with the original idea of Lee (1999) of a minimum wage differentially masking the same underlying latent wage distribution across regions with different average income levels. Our simplifying assumption that each state is a separate labor market is stark but motivated by the fact that only 3–5 percent of all workers switch jobs between states in a given year.

[24]Figure 24–25 in Appendix D.2 show that our conclusions are robust to considering a broader set of earnings percentile ratios and to running the analysis at a more granular level for Brazil's 559 microregions.

[25]The identifying assumption then becomes that conditional on controls $\gamma_{st}(p)$, the "centrality measure," or Kaitz index, is not systematically correlated with "underlying" wage dispersion across states and over time. Appendix D.3 provides evidence in support of this identifying assumption holding in Brazil.

Figure 16. Data vs. model: Wage percentile ratios across Brazilian states over time, 1996–2012

(a) Data: P50–P10

(b) Model: P50–P10

(c) Data: P90–P50

(d) Model: P90–P50
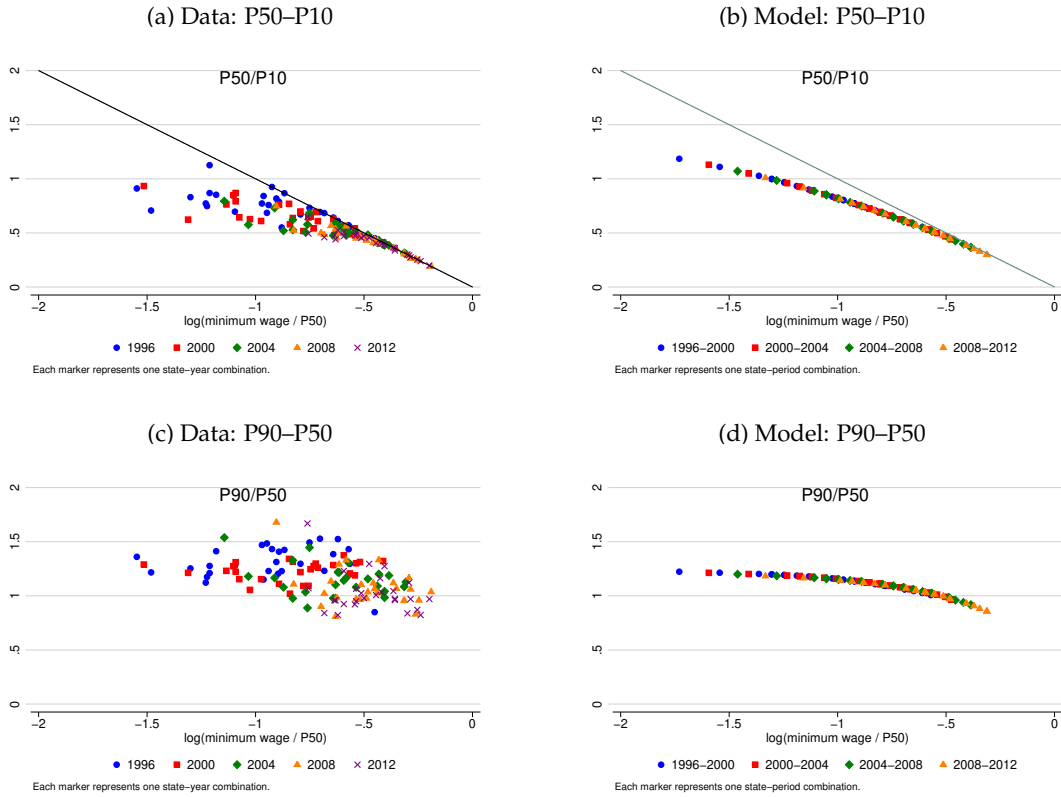
Notes: Each marker represents one state-year combination for each of Brazil's 27 states in the data, and 27 separate model simulations with estimated mean wages for the model to match the empirical Kaitz index distribution. Source: RAIS and simulations.

(negative) point estimates of $\rho(p)$ for $p < 50$ ($p > 50$) as an increase in the minimum wage leading to compression in the lower (upper) tail of the wage distribution. We interpret as *spillovers* the downward-sloping range of $\rho(p)$ estimates.

Table 9 shows results from estimating equation (4) with polynomial order $N = 2$ in the RAIS microdata and in our model-simulated data.[26] Specification (1) is a variant of that in Lee (1999) with only year effects estimated across Brazil's 27 states. We find that the minimum wage has significant marginal effects up to the 80th percentile of the wage distribution, with spillovers (i.e., negative gradient of marginal effect estimates) reaching up to just below the 80th percentile. For example, a ten percent increrase in the minimum wage increases the tenth wage percentile by five percent relative to the median. Specification (2) is the preferred OLS specification from Autor et al. (2016) and adds a set of linear state-time trends, leading to qualitatively similar conclusions although slightly higher point estimates. As additional robustness checks, we run specification (1) across Brazil's 556 microregions and across 54 2-digit industries in specifications (3) and (4),

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[26]We tried polynomials of order $N > 2$ without obtaining significantly different results to those presented below.

respectively. The estimated spillover effects under these two specifications reach between the 70th and the 90th wage percentile, though standard errors make them hard to distinguish from our modal estimate of spillovers up to the 80th percentile.

Applying the same regression model as in specification 4 to our model-generated state-level data, the estimated marginal effects for lower-tail wage percentile ratios are strikingly congruent between the model and the data, suggesting that we can interpret the empirical estimates as due to minimum wage spillovers. The model also predicts upper-tail elasticities in line with the data up to the 90th percentile. Above that, the model shows a more negative point estimate relative to the data, although little evidence of spillovers (i.e., a small gradient of marginal effect estimates). We interpret this as relative wages in the top decile comoving with the minimum wage bindingness for reasons outside of our model.

In some specifications, we find estimated marginal effects that are upward sloping in the uppermost range—usually above the 90th percentile—of the wage distribution in line with the findings by Autor et al. (2016). This could be a sign of transitory shocks affecting the upper tail and the Kaitz index simultaneously, possibly due to concurrent changes in Brazilian labor markets. Nevertheless, by cutting the data along different dimensions and at levels of aggregation, as well as using alternative specifications, we consistently find significant minimum wage effects up to the 80th percentile of the wage distribution.

Table 9. Data vs. model: Marginal effect on various wage percentile ratios relative to median

Marginal effect, $\rho(p)$	Data				Model
	(1)	(2)	(3)	(4)	(5)
$p = 10$	0.467*** (0.014)	0.748*** (0.032)	0.466*** (0.024)	0.558*** (0.035)	0.467*** (0.001)
$p = 20$	0.283*** (0.011)	0.492*** (0.028)	0.313*** (0.022)	0.353*** (0.026)	0.263*** (0.001)
$p = 30$	0.149*** (0.008)	0.392*** (0.023)	0.182*** (0.016)	0.222*** (0.020)	0.147*** (0.001)
$p = 40$	0.057*** (0.005)	0.185*** (0.020)	0.070*** (0.007)	0.109*** (0.011)	0.063*** (0.001)
$p = 60$	-0.063*** (0.006)	-0.111*** (0.023)	-0.041*** (0.008)	-0.110*** (0.014)	-0.046*** (0.001)
$p = 70$	-0.117*** (0.013)	-0.143*** (0.038)	-0.064*** (0.017)	-0.225*** (0.028)	-0.079*** (0.002)
$p = 80$	-0.092*** (0.022)	0.023 (0.047)	-0.047* (0.026)	-0.356*** (0.046)	-0.102*** (0.003)
$p = 90$	0.006 (0.027)	-0.013 (0.051)	0.054 (0.035)	-0.483*** (0.065)	-0.116*** (0.003)

Notes: Table shows predicted marginal effects evaluated at the worker-weighted mean across years 1992–2012. * = significant at the 10% level, ** = 5%, *** = 1%. Underlying regressions are variants of equation (4) with polynomial degree $N = 2$ and with each observation equally weighted, where dependent variable is the log wage percentile ratio Pp/P50. Specification (1) includes year effects and is run across 27 states. Specification (2) includes additional linear state-time trends. Specification (3) is the same as specification (1) run across 556 microregions with standard errors clustered at the microregion level. Specification (4) is the same as specification (1) run across 54 2-digit industries with standard errors clustered at the industry level. Model specification (5) is the same as specification (1) run on computer-simulated data across 27 states differing in their distance to the minimum wage. Source: RAIS and simulations.

Our reduced-form estimates of spillover effects of the minimum wage broadly match those

obtained from our estimated structural model. It is worth highlighting that these two sets of estimates were informed by very different moments in the data. On the one hand, our reduced-form effects were estimated off systematic comovement of the wage distribution with the Kaitz index. On the other hand, our model effects were disciplined by the microstructure of worker and firm heterogeneity that we estimated from an AKM decomposition. It is reassuring that both methods yield similar results, lending confidence to our conclusions.

Our finding of spillover effects up to the 80th percentile may appear surprising. For comparison, recent results in the literature by Autor et al. (2016) show spillovers up to the 20th percentile of the wage distribution in the US. Nevertheless, we are confident in our results based on three grounds. First, the large administrative dataset we base our work on plausibly admits less measurement error than the CPS household survey, alleviating some of the concern of bias in the estimates of $\beta_n(p)$ in equation (4) above. Second, as highlighted in our structural model, the large dispersion in firm productivities and degree of search frictions, with associated large employer pay components in the AKM wage decomposition, explain why the effects of the minimum wage are large in our setting.

Third, we now provide a model-motivated empirical test of the reach of spillovers that can be easily implemented in longitudinal worker data. Our proposed test ranks workers by their current wage and computes for each wage rank the share of individuals who previously, say over the past five years, earned the minimum wage. Through the lens of our model, we expect spillovers to reach up to the highest wage rank at which a positive mass of workers have been previously employed at the minimum wage. Figure 17 shows the results of implementing our test on the Brazilian data for 1996 and 2012. Confirming our previous results, we find a positive mass of workers previously earning the minimum wage up to around the 80th percentile of the wage distribution in 1996 and up to the 90th percentile in 2012.[27]

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[27] Another validation of our findings comes from inspection of Figure 18 in Appendix **??**, which shows a pronounced elevation of average wages and a number of pay-relevant worker covariates above the 80th wage percentile. Reassuringly, the minimum wage appears to have had little effect on the highest skill groups including college graduates whose population shares increase steeply after the 80th wage percentile, as shown in Figure 19 in Appendix **??**.

Figure 17. Data: Worker share who recently earned the minimum wage, 1996 and 2012
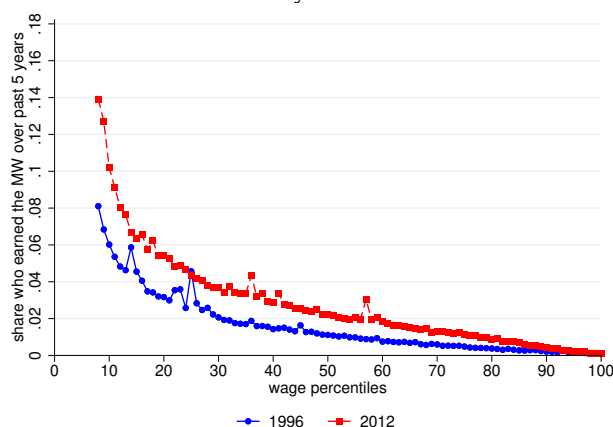
Notes: Figure shows share of workers who have held a job paying the minimum wage over the past five years across percentiles of the current wage distribution. Source: RAIS.

6.2 Employment effects identified off regional variation

So far, our analysis has been silent on the issue of informality in Brazil. The distinction between informality and unemployment is important to the extent that each may represent a separate margin of adjustment. We now extend our regression framework to investigate the effects of the minimum wage on both formal and informal employment in Brazil between 1996 and 2012. To this end, we combine administrative data with two household surveys to estimate variants of the specification in equation (4) with the dependent variable, y_{st}, at the state-year or metropolitan area-year level.

Panel A of Table 10 shows that the minimum wage has precisely estimated zero effects on the population size, labor force participation rate, employment rate, and formal employment share. These estimates accord well with our model prediction of muted employment effects due to the minimum wage.[28] Results from the PME data in panel B show statistically significant but moderate negative effects on transition rates from nonformal to formal as well as from formal to nonformal employment. The mild slowdown of recruitment of workers from outside the formal sector matches our model prediction of fewer aggregate vacancies.[29] Finally, panel C shows that the estimated effects on mean hours worked, as well as firm entry and exit rates, defined as the employment share at new firms relative to the previous year and the employment share at firms that exit in the following year, respectively. Our estimates indicate that the effects of the minimum wage on these outcome variables are statistically indistinguishable from zero.[30]

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[28]In the PNAD data, the formal employment share rose by 16 percentage points over this period, largely accounted for by changes in educational composition of the workforce but little movement in within-group formality rates.

[29]The reduction in labor market exit, while beyond the confines of our model, could be consistent with less voluntary exit from the formal sector due to the rise in its average wage level.

[30]Our results match evidence on minimum wage effects on job flows and employment in the US (Dube et al., 2016;

Table 10. Data: Employment effects of the minimum wage, 1996–2012

	Marginal effect (s.e.)
Panel A. Cross-sectional household survey (PNAD)	
Log population size	-0.050 (0.041)
Labor force participation rate	-0.001 (0.016)
Employment rate	-0.003 (0.010)
Formal employment share	-0.005 (0.024)
Panel B. Longitudinal household survey (PME)	
Transition rate nonformal-formal	-0.045* (0.024)
Transition rate formal-nonformal	-0.026** (0.012)
Panel C. Linked employer-employee data (RAIS)	
Log mean hours worked	-0.011 (0.031)
Firm entry rate	0.002 (0.017)
Firm exit rate	0.001 (0.007)

Notes: Table shows predicted marginal effects with standard errors in parentheses evaluated at the worker-weighted mean across Brazil's 27 states from 1996 to 2012. Underlying regressions are variants of equation (4) with polynomial degree $N = 2$ including year effects and linear state-year trends. * = significant at the 10% level, ** = 5%, *** = 1%. Results for the PNAD and PME data are weighted by the appropriate sample weights. Source: PNAD, PME, RAIS.

7 Conclusion

What are the effects of the minimum wage on inequality? The answer to this question depends crucially on the microstructure of the labor market. To make this point, we developed a flexible equilibrium model in the spirit of Burdett and Mortensen (1998) to quantify the effects of a minimum wage increase on the distribution of wages, employment, and other macroeconomic outcomes in Brazil between 1996 and 2012. We showed that the joint distribution of AKM fixed effects provides valuable information that helps discipline the model predictions about the effects of a minimum wage and, conversely, that the minimum wage shapes the dispersion of AKM fixed effects in a way that helps explain Brazil's remarkable inequality decline over this period.

Applying our theory to the case of Brazil, where the minimum wage increased by 119 percent in real terms between 1996 and 2012, we find that the change in the wage floor had large effects throughout the distribution. We find that the policy induced a 11 log points decline in the variance of wages, affecting workers up to the 80th percentile. At the same time, our results imply a muted nonemployment response and small efficiency gains from the policy. The effects of the minimum wage are mediated by a lower firm productivity pay premium and lower returns

Cengiz et al., 2017) as well as firm dynamics in the UK and Hungary (Draca et al., 2011; Harasztosi and Lindner, 2017).

to worker ability. We present empirical evidence from administrative linked employer-employee data and household surveys in support of these findings. While our estimated effects are striking in many ways, sensitivity analysis of our model shows that the effects of the minimum wage on inequality could have been much smaller and the effects on employment could have been more negative under different labor market conditions.

These insights point to fruitful avenues for future research. First, it would be interesting to quantify spillovers of Brazil's formal sector minimum wage into the informal economy, which is not directly constrained by the policy. Second, our finding of large spillovers complements recent empirical work that identifies minimum wage effects by comparison to a carefully selected control group. Third, it is worth revisiting the contribution of other labor market policies and institutions, including unions, unemployment benefits, and non-compete agreements—all of which affect only a small worker share directly but may lead to sizable equilibrium effects—towards inequality trends in other countries. Finally, while we have stopped short of optimal policy analysis, our results will be an important ingredient for any such venture.

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Appendix For Online Publication

We structure the additional materials as follows: Data (Appendix A), Theory (Appendix B), Estimation (Appendix C), and Empirics (Appendix D).

A Data Appendix

This appendix provides details on the datasets used in Section 2 and throughout the paper, including subsections on data description (Appendix A.1), summary statistics (Appendix A.2), and the wage distribution by year (Appendix A.3).

A.1 Dataset description

Administrative linked employer-employee data (RAIS). Our main data source is the *Relação Anual de Informações Sociais (RAIS)*, a linked employer-employee register by the Brazilian Ministry of Labor (*Ministério do Trabalho*, or *MTb*). We use the RAIS microdata with person and firm identifiers covering the period 1992–2012 available to us under a confidentiality agreement with MTb. We also use a version of the same data going back to 1988, which we accessed through Brazil's Institute of Applied Economic Research (*Instituto de Pesquisa Econômica Aplicada*, or *IPEA*) at the Brazilian Institute of Geography and Statistics (*Instituto Brasileiro de Geografia e Estatística*, or *IBGE*).

Firms' survey response is mandatory, and misreporting is deterred through audits and threat of fines. Collection started in 1986, with coverage becoming near universal from 1994 onward. The data contain detailed information on job characteristics, with 73 million formal sector employment spells recorded in 2012. Although reports are annual, we observe for every job spell the date of accession and separation in addition to average monthly earnings. We keep for each worker the highest-paid among each year's longest employment spells. As Brazil's minimum wage is set in terms of monthly earnings, henceforth we interchangeably refer to this income concept as "earnings" or "wages." Unless otherwise noted, we keep only workers who earn at least the legal minimum wage to make the data consistent with our model and, following the argument in Andrews et al. (2008) and much of the follow-up literature, keep firms with ten or more employees in a given year to mitigate limited-mobility bias in our estimates.[31]

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[31]We reproduced the estimation and our quantitative results using targets obtained without imposing these restrictions and obtained very similar results.

We devise our own cleaning procedure for these data, starting with the raw text files, benefiting from guidance by the data team at IPEA. Our cleaning procedure consists of three stages. The first stage reads in and standardizes the format of the raw data files that were transmitted to us at the region-year level, saving a set of compatible region-year files. The second stage reads in all region files within a year and applies a set of cleaning and recoding procedures to the data to make them consistent within each year, saving a set of yearly files. The third stage reads in all yearly files and applies a set of cleaning procedures to the data to make them consistent across years. Whenever possible, we use the official crosswalks provided by IBGE to convert industry (IBGE, CNAE 1.0, and CNAE 2.0 classifications), occupation (CBO 1994 and CBO 2002), and municipality codes (IBGE classification).

Cross-sectional household survey data (PNAD). A substantial fraction of Brazil's working-age population is not formally employed and hence not covered by the RAIS. To address this gap, we complement our analysis using data from the *Pesquisa Nacional por Amostra de Domicílios (PNAD)*, a nationally representative annual household survey. Respondents are asked to produce a formal work permit (*Carteira de Trabalho e Previdência Social assinada*). Following Meghir et al. (2015), we classify as informal all self-employed and those in remunerated employment without a work permit.

The PNAD data collection consists of a double-stratified sampling scheme by region and municipality, interviewing a representative of households in Brazil. The survey asks the household head to respond on behalf of all family members and report a rich set of demographic and employment-related questions. In particular, the survey asks a question about whether the respondent holds a legal work permit. We use the answer to this survey question to identify individuals as working in the formal or in the informal sector. Survey questions regarding income and demographics of the respondent household members are comparable to the US March Current Population Survey (CPS). We keep only observations that satisfy our selection criteria and have non-missing observations for labor income, whose variable definition we harmonize across years.

The raw microdata are publicly available for download starting from 1996 at `ftp://ftp.ibge.gov.br/Trabalho_e_Rendimento/`. For basic cleaning, starting with the raw data in text format, we use the standardized cleaning procedures adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at `http://www.econ.puc-rio.br/datazoom/english/`

`index.html`. From there, we apply a set of procedures to clean and recode key variables used in our analysis.

Longitudinal household survey data (PME).   We also use a second household survey, the *Pesquisa Mensal de Emprego (PME)*, conducted in Brazil's six largest metropolitan regions. The advantage of this dataset is that it features for every respondent two continuous four-month interview spells separated by eight months. Starting in 2002, this short panel component allows us to compute transition rates of workers between all employment states. For presentation purposes, we label formal sector workers as "employed," and pool informal sector workers and the unemployed under the label "nonemployed." We distinguish between the disaggregated categories in our empirical analysis of minimum wage effects later.

The raw microdata are publicly available for download starting from March 2002 at `ftp: //ftp.ibge.gov.br/Trabalho_e_Rendimento/`. For basic cleaning, starting with the raw data in text format, we use the standardized cleaning procedures adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at `http://www.econ.puc-rio.br/datazoom/ english/index.html`. From there, we apply a set of procedures to clean and recode key variables used in our analysis, similar to the procedures that we applied to the PNAD data.

A.2   Summary statistics

Overview of the three datasets.   While each of the three datasets—RAIS, PNAD, and PME—is geared at slightly different subpopulations and labor market questions, together they provide a holistic picture of Brazil's labor market. We restrict attention to male workers of age 18–49 to avoid issues related to female labor force participation and retirement. Table 11 presents summary statistics for this worker group from the three datasets. The RAIS data show that between 1996 and 2012, Brazil experienced an 18 log points increase in mean formal sector wages while the standard deviation declined by 19 log points—a striking compression visualized in Figure 20 of Appendix A.3. While the age distribution remained stable, there was a significant increase in educational attainment over this period. Using the PNAD survey data, we confirm congruent trends in the formal sector wage distribution. Relative to the formal sector, informal wages are initially characterized by lower levels but similar dispersion. Throughout 2012, the informal sector wage distribution saw an increase in its mean accompanied by mild compression. At the same time, the employment rate remained stable while the formal employment share rose by eight

percentage points. Consistent with the increase in formality, the longitudinal PME data show a slight rise in the inflow rate into formal employment and a decline in the outflow rate.

Table 11. Summary statistics from three main datasets, 1996 and 2012

	1996		2012	
Panel A. Linked employer-employee data (RAIS)	Mean	St.d.	Mean	St.d.
Age	31.69	8.37	32.05	8.51
Years of education	7.78	3.92	10.73	3.18
Real wage (log BRL 2012, formal sector)	7.02	0.86	7.20	0.67
Observations	16,308,762		28,578,057	
	1996		2012	
Panel B. Cross-sectional household survey (PNAD)	Mean	St.d.	Mean	St.d.
Real wage (log BRL 2012, formal sector)	7.01	0.81	7.13	0.62
Real wage (log BRL 2012, informal sector)	6.26	0.81	6.56	0.78
Employment rate	0.95		0.95	
Formal employment share	0.68		0.76	
Observations	74,487		86,031	
	2002		2012	
Panel C. Longitudinal household survey (PME)	Mean	St.d.	Mean	St.d.
Transition rate nonemployed-employed	0.08		0.10	
Transition rate employed-nonemployed	0.05		0.04	
Observations	94,280		121,211	

Notes: Statistics are for males of age 18–49. Real wage is average (RAIS) or usual (PNAD) monthly earnings. Respondents are classified as employed if they are a domestic worker, employee, or self-employed. Formal employment is defined as being employed and having a legal work permit. Transition rates are conditional on initial labor market status, divided into employed (formal) and nonemployed (unemployed + informal). See Figures 18–19 and Tables 12–13 in Appendix ?? for further details. Source: RAIS, PNAD, PME.
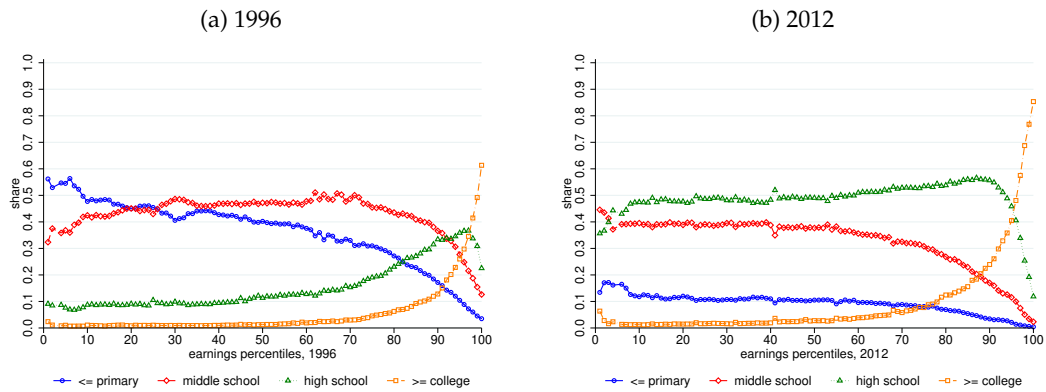
Summary statistics for RAIS data. Figure 18 shows mean values of basic descriptive variables— monthly earnings in multiples of the minimum wage, years of education, age in years, and job tenure in years—throughout the earnings distribution in 1996 (panel (a)) and in 2012 (panel (b)). Zooming in on educational attainment, which increased significantly over this period, Figure 19 shows the distribution of eduaction degrees grouped into individuals with primary school or lower levels of education, middle school, high school, and college or higher levels of education for 1996 (in panel (a)) and for 2012 (in panel subfig: RAIS summary stats-2-B).

Figure 18. RAIS cross-sectional summary statistics, 1996 and 2012

(a) 1996

(b) 2012
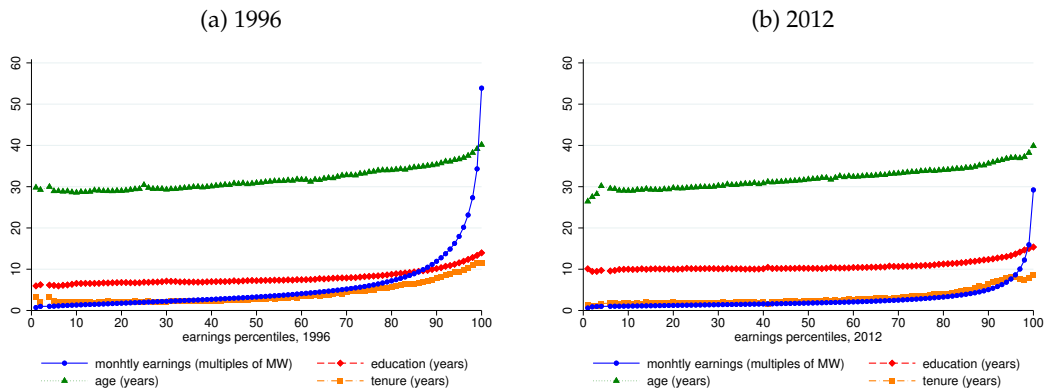
Notes: Figure shows mean monthly earnings ("wages"), years of education, age, and tenure across wage percentiles for 1996 in panel (a) and for 2012 in panel (b). All statistics are for adult male workers of age 18–49. Source: RAIS.

Figure 19. RAIS education degree shares, 1996 and 2012

(a) 1996

(b) 2012

Notes: Figure shows shares of education degrees across wage percentiles for 1996 in panel (a) and for 2012 in panel (b). All statistics are for adult male workers of age 18–49. Source: RAIS.

Summary statistics for PNAD data. Table 12 presents summary statistics on the PNAD data.

Table 12. Summary statistics for cross-sectional household survey (PNAD)

	# Workers	Real wage (formal)		Real wage (informal)		Employment rate	Formal share
		Mean	Std. dev.	Mean	Std. dev.		
1996	74,487	7.01	0.81	6.26	0.81	0.95	0.68
1997	78,731	7.02	0.79	6.26	0.82	0.94	0.68
1998	79,060	7.03	0.78	6.26	0.81	0.93	0.67
1999	81,230	6.97	0.77	6.21	0.79	0.93	0.66
2000	--	--	--	--	--	--	--
2001	89,102	6.93	0.74	6.20	0.81	0.93	0.66
2002	90,855	6.90	0.73	6.19	0.81	0.93	0.66
2003	91,490	6.84	0.71	6.12	0.77	0.92	0.67
2004	94,526	6.85	0.69	6.15	0.77	0.94	0.68
2005	97,348	6.89	0.67	6.19	0.77	0.93	0.68
2006	97,757	6.94	0.66	6.25	0.76	0.94	0.69
2007	95,598	6.97	0.65	6.30	0.78	0.94	0.71
2008	93,677	7.00	0.65	6.35	0.76	0.95	0.72
2009	95,170	7.02	0.63	6.36	0.76	0.94	0.73
2010	--	--	--	--	--	--	--
2011	84,910	7.07	0.62	6.51	0.75	0.95	0.76
2012	86,031	7.13	0.62	6.56	0.78	0.95	0.76

Notes: Table shows summary statistics on wages, employment rates, and formal employment shares between 1996 and 2012. All statistics are for adult male workers of age 18–49. Real wages are measured in 2012 BRL and in logs. Surveys are not available for census years 2000 and 2010. Source: PNAD.

**Summary statistics for PME data.** Table 13 presents summary statistics on the PME data.

Table 13. Summary statistics for longitudinal household survey (PME)

	# Workers	Transition rate employed-nonemployed	Transition rate employed-nonemployed
2002	94,280	0.08	0.05
2003	140,734	0.09	0.06
2004	146,847	0.08	0.05
2005	154,159	0.08	0.05
2006	153,646	0.08	0.04
2007	154,338	0.09	0.05
2008	150,104	0.10	0.05
2009	149,762	0.10	0.04
2010	150,443	0.10	0.04
2011	145,012	0.11	0.04
2012	121,211	0.10	0.04

Notes: Table shows summary statistics on transition rates between employment (formal) and nonemployment (nonformal + unemployed) between March 2002 and December 2012. All statistics are for adult male workers of age 18–49. Source: PME.

A.3 Wage distribution by year

Figure 20. Data: Wage distribution by year, 1996–2012
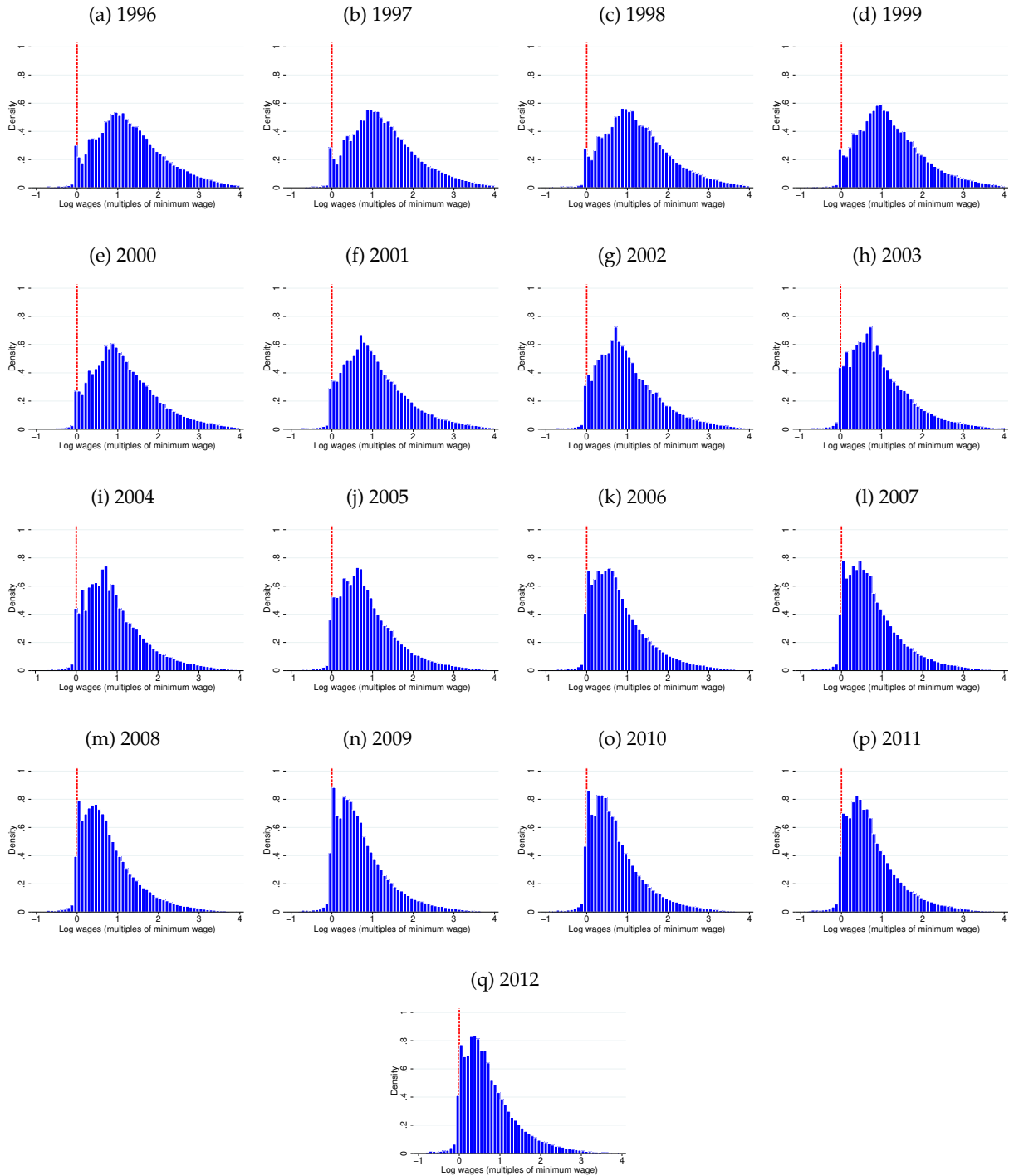
Notes: Each panel shows the wage histogram based on 60 equi-spaced bins for population of male workers aged 18–49 for one year between 1996 and 2012. Source: RAIS.

B   Theory Appendix

This appendix provides details on the model outlined in Section 3, including subsections on the equilibrium definition (Appendix B.1), equilibrium characterization (Appendix B.2), and proof of the proposition (Appendix B.3).

B.1   Equilibrium definition

Definition. A *search equilibrium with a minimum wage* consists of a set of firms' wage and vacancy posting policies $(w_\theta(p), v_\theta(p))$, wage offer distribution $F_\theta(w)$, employment level function $l_\theta(w, v)$, unemployment rate u_θ, aggregate vacancy mass V_θ, contact rate q_θ, reservation wage policy R_θ, transition rates $(\lambda_\theta^u, \lambda_\theta^e)$, and flow value of nonemployment b_θ subject to a minimum wage w^{min}:

1. *Worker optimality:* Given transition rates, the separation rate, the flow value of nonemployment, and the offer distribution, the reservation wage policy solves workers' problem;

2. *Firm optimality:* Taking as given the employment level function and wage offer distribution, wage and vacancy policies solve firms' problem;

3. *Enforcement:* No worker accepts, or else no firm posts, a wage below the minimum wage;

4. *Labor market aggregation:* The unemployment rate is consistent with transition rates and the separation rate, the aggregate vacancy mass is consistent with firms' vacancy posting policies, transition rates are determined through the aggregate matching function, and the employment level function and the wage offer distribution are consistent with wage and vacancy posting policies.

B.2   Equilibrium characterization

Given a mass of posted vacancies v and a wage w, the size of a firm in a market θ, $l_\theta(w, v)$, evolves according to the Kolmogorov forward equation:

$$\dot{l}_\theta(w, v) = -\left(\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))\right) l_\theta(w, v) + vq_\theta \left(\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta} + \frac{(1 - u_\theta) s_\theta}{u_\theta + (1 - u_\theta) s_\theta} G_\theta(w)\right) \quad (5)$$

The separate terms are explained as follows. Out of a firm's current workforce l_θ, a fraction δ_θ exit to nonemployment and a fraction $s_\theta \lambda_\theta^u (1 - F_\theta(w))$ quit to better-paying employers. At rate q_θ,

each of the firm's vacancies contacts a worker, and a share $u_\theta / (u_\theta + s_\theta(1 - u_\theta))$ of those workers is nonemployed while the remainder are currently employed. Nonemployed workers accept all wage offers in equilibrium, while a fraction $G_\theta(w)$ of employed workers accept an offer of wage w. In the stationary equilibrium, equation (5) must equal zero such that

$$l_\theta(w, v) = vq_\theta \frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))} \left(\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta} + \frac{(1 - u_\theta) s_\theta}{u_\theta + (1 - u_\theta) s_\theta} G_\theta(w) \right).$$

Recall also that $u_\theta = \delta_\theta / (\delta_\theta + \lambda_\theta^u)$ and $G_\theta(w) = F_\theta(w) / (1 + \kappa_\theta (1 - F_\theta(w)))$ in the stationary economy. Substituting and simplifying, for given vacancy and wage policies (w, v), a firm's stationary equilibrium size is given by

$$l_\theta(w, v) = vq_\theta \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))} \right)^2 \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u}. \tag{6}$$

Define the piece rate $\tilde{w}_\theta$ such that $w_\theta = \theta \tilde{w}_\theta$. We can write the problem of firm p in market θ as

$$\max_{\tilde{w} \geq R_\theta / \theta, v} \left\{ \theta(p - \tilde{w}) \times vq_\theta \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}))} \right)^2 - c_\theta(v) \right\}.$$

Imposing our assumed functional form for the cost of vacancy creation, $c_\theta(v) = c_\theta v^{1 + c_1} / (1 + c_1)$, the first-order conditions with respect to vacancies and piece rates are

$$v_\theta(p) = \left((p - \tilde{w}_\theta(p)) \frac{q_\theta}{c_\theta} \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \theta \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta [\tilde{w}_\theta(p)])} \right)^2 \right)^{\frac{1}{c_1}} \tag{7}$$

$$1 = (p - \tilde{w}_\theta(p)) \frac{2 s_\theta \lambda_\theta^u f_\theta(\tilde{w}_\theta(p))}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}_\theta(p)))}. \tag{8}$$

Note that we can write equation (7) as

$$v_\theta(p) = \left\{ \frac{q_\theta}{c_\theta} \pi_\theta(p) \right\}^{\frac{1}{c_1}},$$

where

$$\pi_\theta(p) = \max_{\tilde{w} \geq R_\theta / \theta} \left\{ (p - \tilde{w}) \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \left[\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}))} \right]^2 \right\}.$$

Since $\pi_\theta(p)$ is increasing in productivity and $c_\theta, c_1 > 0$, it follows that $v_\theta'(p) > 0$. That is, more productive employers create more jobs. As in Burdett and Mortensen (1998), a single-crossing property of the profit function with respect to productivity and wages for a given vacancy policy

implies that the optimal wage policy, $\tilde{w}_\theta(p)$, is strictly increasing in productivity. Similarly, the equilibrium wage offer distribution has no mass points.

Since the wage policy is strictly monotone in productivity, the distribution of wage offers is given by

$$F_\theta\left(w_\theta\left(p\right) \right) = M_\theta \int_{\underline{p}_\theta}^{p} \frac{v_\theta\left(p'\right)}{V_\theta} d\Gamma_\theta\left(p'\right), \tag{9}$$

where total vacancies equal

$$V_\theta = M_\theta \int_{\underline{p}_\theta}^{\overline{p}} v_\theta\left(p'\right) d\Gamma_\theta\left(p'\right), \tag{10}$$

and the job finding rate of workers is implicitly defined by

$$\lambda_\theta^u = \left(h_\theta\left(\frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u} + \frac{s_\theta \lambda_\theta^u}{\delta_\theta + \lambda_\theta^u} \right) \right)^{-\alpha} V_\theta^\alpha, \tag{11}$$

while firms' contact rate is given by $q_\theta = \left(\lambda_\theta^u\right)^{\frac{\alpha-1}{\alpha}}$.

B.3 Proof of the proposition

Under the assumption that labor market parameters are the same across worker types and that the minimum wage is initially low enough to be nonbinding, equilibrium wages can be written as

$$w(p,\theta) = \theta\left[p - \int_{\underline{p}_\theta}^{p} \left[\frac{1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)}{1 + \kappa_\theta^e\left(1 - \Gamma_0(x)\right)} \right]^2 dx \right] \tag{12}$$

Part 1.   We want to show that an increase in the minimum wage raises all wages in markets where it becomes binding. Differentiating equation (12) with respect to the minimum wage gives that

$$\frac{\partial w(p,\theta)}{\partial w^{\min}} = \left[\frac{1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)} \right]^2 > 0,$$

which establishes the first part of the proposition.

Part 2.   Consider a market where the minimum wage is binding. Differentiating equation (12) with respect to productivity gives that the productivity-pay gradient is given by

$$\frac{\partial w(p,\theta)}{\partial p} = \theta 2\kappa_\theta^e \gamma_0(p)\left[1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)\right] \int_{\underline{p}_\theta}^{p} \left(\frac{1}{1 + \kappa_\theta^e\left(1 - \Gamma_0(x)\right)} \right)^2 dx.$$

Differentiating this equation with respect to the minimum wage gives that

$$\frac{\partial\left(\frac{\partial w(p,\theta)}{\partial p}\right)}{\partial w^{\min}} = \theta 2\kappa_\theta^e \gamma_0(p)\left[1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)\right]\left(-\frac{1}{\theta}\right)\left(\frac{1}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)}\right)^2 < 0.$$

Hence, the firm productivity-pay gradient falls with the minimum wage.

Part 3. Consider markets where the minimum wage is binding. Differentiating equation (12) with respect to ability gives that the ability-pay gradient is given by

$$\frac{\partial w(p,\theta)}{\partial\theta} = p - \int_{\underline{p}_\theta}^{p}\left[\frac{1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)}{1 + \kappa_\theta^e\left(1 - \Gamma_0(x)\right)}\right]^2 dx - \frac{w^{\min}}{\theta}\left[\frac{1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)}\right]^2$$

Differentiating this equation with respect to the minimum wage gives that

$$\begin{aligned}
\frac{\partial\left(\frac{\partial w(p,\theta)}{\partial\theta}\right)}{\partial w^{\min}} &= \frac{1}{\theta}\left[\frac{1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)}\right]^2 - \frac{1}{\theta}\left[\frac{1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)}\right]^2 \\
&\quad - \frac{w^{\min}}{\theta}\left[1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)\right]^2 (-2)\left(-\kappa_\theta^e\gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)\frac{1}{\theta}\left[\frac{1}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)}\right]^3 \\
&= -\frac{2w^{\min}}{\theta^2}\left[1 + \kappa_\theta^e\left(1 - \Gamma_0(p)\right)\right]^2 \kappa_\theta^e\gamma_0\left(\frac{w^{\min}}{\theta}\right)\left[\frac{1}{1 + \kappa_\theta^e\left(1 - \Gamma_0\left(\frac{w^{\min}}{\theta}\right)\right)}\right]^3 < 0.
\end{aligned}$$

Hence, in markets where the minimum wage is binding, the worker ability-pay gradient falls with the minimum wage. $\square$

C Estimation Appendix

This appendix provides details on the estimation procedure of Section 4, including subsection on the algorithm that we use to solve and estimate the model (Appendix C.1), details of the estimatino routine (Appendix C.2), how targeted auxiliary moments are computed (Appendix C.3), as well as additional results on the model identification (Appendix C.4).

C.1 Solution algorithm

Define $J_\theta(p)$ as the vacancy-weighted cdf of hiring firms in market θ. Because of the wage policy is monotone in productivity, $J_\theta(p) = F_\theta(w_\theta(p))$ so that $f_\theta(w_\theta(p)) = J'_\theta(p)/w'_\theta(p)$ and $v_\theta(p) = V_\theta/M_\theta \times J'_\theta(p)/\gamma_\theta(p)$. Substituting this into the first-order conditions (7)–(8),

$$J'_\theta(p) = \frac{M_\theta \gamma_\theta(p)}{V_\theta} \left((p - w_\theta(p)) \frac{(\lambda_\theta^u)^{-\frac{1-\alpha}{\alpha}}}{c_\theta} \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \theta \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - J_\theta(p))} \right)^2 \right)^{\frac{1}{c_1}} \quad (13)$$

and

$$w'_\theta(p) = (p - w_\theta(p)) \frac{2 s_\theta \lambda_\theta^u J'_\theta(p)}{\delta_\theta + s_\theta \lambda_\theta^u [1 - J_\theta(p)]} \quad (14)$$

Equations (13)–(14) is a system of two first-order differential equations. Given reduced form estimates of δ_θ, s_θ and λ_θ^u from the data, the required mass of vacancies based on equation (11) is $V_\theta = (\lambda_\theta^u)^{\frac{1}{\alpha}} h_\theta \left(\frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u} + \frac{s_\theta \lambda_\theta^u}{\delta_\theta + \lambda_\theta^u} \right)$. Taking V_θ and the finding rates as given, we solve the system (13)–(14) subject to the boundary conditions

$$w_\theta \left(\underline{p}_\theta \right) = \frac{\max \left\{ \phi_\theta, w^{min} \right\}}{\theta}, \quad \text{and} \quad \lim_{p \to \underline{p}_\theta} J_\theta(p) = 0$$

to obtain $J_\theta(p)$ and $w_\theta(p)$. A solution requires that the cost of posting vacancies c_θ is such that the optimal vacancy policy aggregates to $\lim_{p \to \bar{p}} J_\theta(p) = 1$. We find such a c_θ by guessing an initial c_θ, solving the problem, and checking whether the condition $\lim_{p \to \bar{p}} J(p) = 1$ holds. If it does not, we update c_θ until convergence. In order to subsequently evaluate the impact of a rise in the minimum wage, we take the estimated cost parameter c_θ as given and instead find the job finding rate λ_θ^u that ensures that $\lim_{p \to \bar{p}} J(p) = 1$ holds.

C.2 Details of the estimation routine

In practice, we sample one million parameter configurations using Sobol sequences, which efficiently distribute points in a space. After finding the criterion-minimizing parameter configuration, we simulate four million individuals at monthly frequency under the estimated parameter values. Identically to our treatment of the data, we then aggregate employment data to the annual level by retaining the highest-paid among all longest employment spells that an individual held during a given year. Recall that the solution to the equilibrium system of differential equations (13)–(14) in Section C.1 yields a wage policy and a steady-state mass of workers hired by each firm of a given productivity p firm operating in ability market θ. We further decompose this wage policy into a "worker component" and a "firm component" of pay following the AKM procedure used in Alvarez et al. (2018). Notice that this procedure avoids the need to simulate a large number of individual wage histories to estimate AKM on model-generated data, which substantially speeds up estimation. In an earlier version of this paper, we simulated a large number of worker histories and found that this procedure produced virtually indistinguishable results.

Finally, and again as woth the data, we estimate the AKM regression on five-year windows of these simulated work histories, dropping individual-years at firms with less than ten employees and those whose wage plus noise falls below the minimum wage. As we demonstrate below, the great majority of the noise is reflected in the AKM residual, with a relatively small share inflating the variance of the estimated worker AKM fixed effects, which makes us confident that our estimation routine correctly informs the underlying parameters of the model.

C.3 How targeted auxiliary moments are computed

We compute these moments in the following way. First, we estimate firm and worker AKM fixed effects based on the AKM regression (1) $y_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$ on the 1996–2000 RAIS data. Following an empirical literature which has suggested that the AKM two-way fixed effect decomposition is not well identified for small firms, we restrict attention to firms that hire at least 10 employees (and the workers that work for these firms). We impose the same size threshold in the model.[32] Based on the estimates, we group workers into types based on AKM worker fixed effects, and compute a set of auxiliary moments summarizing worker mobility by worker decile.

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[32] This criterion drops slightly less than 20 percent of individual-years, both in the data and in the model under our estimated parameter values. We have verified that our results do not substantively change with a less stringent threshold.

Categorizing workers' underlying structural type based on their estimated AKM worker fixed effect works to the extent that the estimated AKM worker fixed effects are monotonic in underlying ability, which we find that our estimated model satisfies. Specifically, we compute the following set of auxiliary moments characterizing labor market flows by worker groups:

1. We estimate the monthly separation rate as the average rate of leaving formal employment for at least one month: $\hat{\delta}_\theta = \mathbb{E}(\text{nonemployed}_{t+1}|\text{employed}_t, \theta)$.

2. We estimate the job hazard from nonemployment, $\hat{\lambda}_\theta^u$, by tracking workers for up to 24 months after leaving a formal sector job, and estimating via nonlinear least squares the following proportional hazard model: $\log \mathbb{P}(\# \text{ months until reentry} \geq t|\theta) = t \times \log(1 - \lambda_\theta^u)$.

3. We map the rate of upward mobility across AKM firm fixed effects ranks into the effective speed of climbing the job ladder, κ_θ. To this end, we exploit the model restriction $G_\theta(w) = F_\theta(w)/(1 + \kappa_\theta(1 - F_\theta(w)))$ using nonparametric density estimates of the AKM firm effects distribution, $\mathcal{G}_\theta(\hat{\alpha}_j)$, and AKM firm effects starting distribution from nonemployment, $\mathcal{F}_\theta(\hat{\alpha}_j)$, to estimate $\hat{\kappa}_\theta$.[33] Combined with our estimate of the separation rate, we obtain the job-to-job mobility parameter of interest using the model relation $\hat{\lambda}_\theta^e = \hat{\kappa}_\theta^e \times \hat{\delta}_\theta$ and hence $\hat{s}_\theta = \hat{\lambda}_\theta^e/\hat{\lambda}_\theta^u$.

4. We infer workers' reservation wage as the lowest accepted wage out of nonemployment, $\hat{\mathcal{R}}_\theta = \min\{w_\theta\}$.[34]

In practice, we group workers by deciles of AKM worker fixed effects and linearly interpolate the estimated type-specific mobility moments to allow for more types in our structural estimation.

C.4   Identification

To verify that the grids for possible parameter values we pick cover approximately the right domain for each parameter, Figure 21 plots the distribution of different moments across the range of possible parameter values that we consider, together with the empirical moment in red.

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[33] In a previous version of this paper, we obtained similar estimates for the mean value of κ_θ using two additional model-consistent methods: one from a job duration regression and the other by comparing the wage distribution of new hires to that of the population.

[34] To control for outliers, we estimate $\mathcal{R}_\theta$ by trimming the lowest one percent of observations by θ-group. As a robustness checked, we have repeated this procedure for lower (0.1 percent) and higher (3 percent) trims.

Figure 21. Estimated relative and absolute vacancy posting cost c_θ across worker abilities
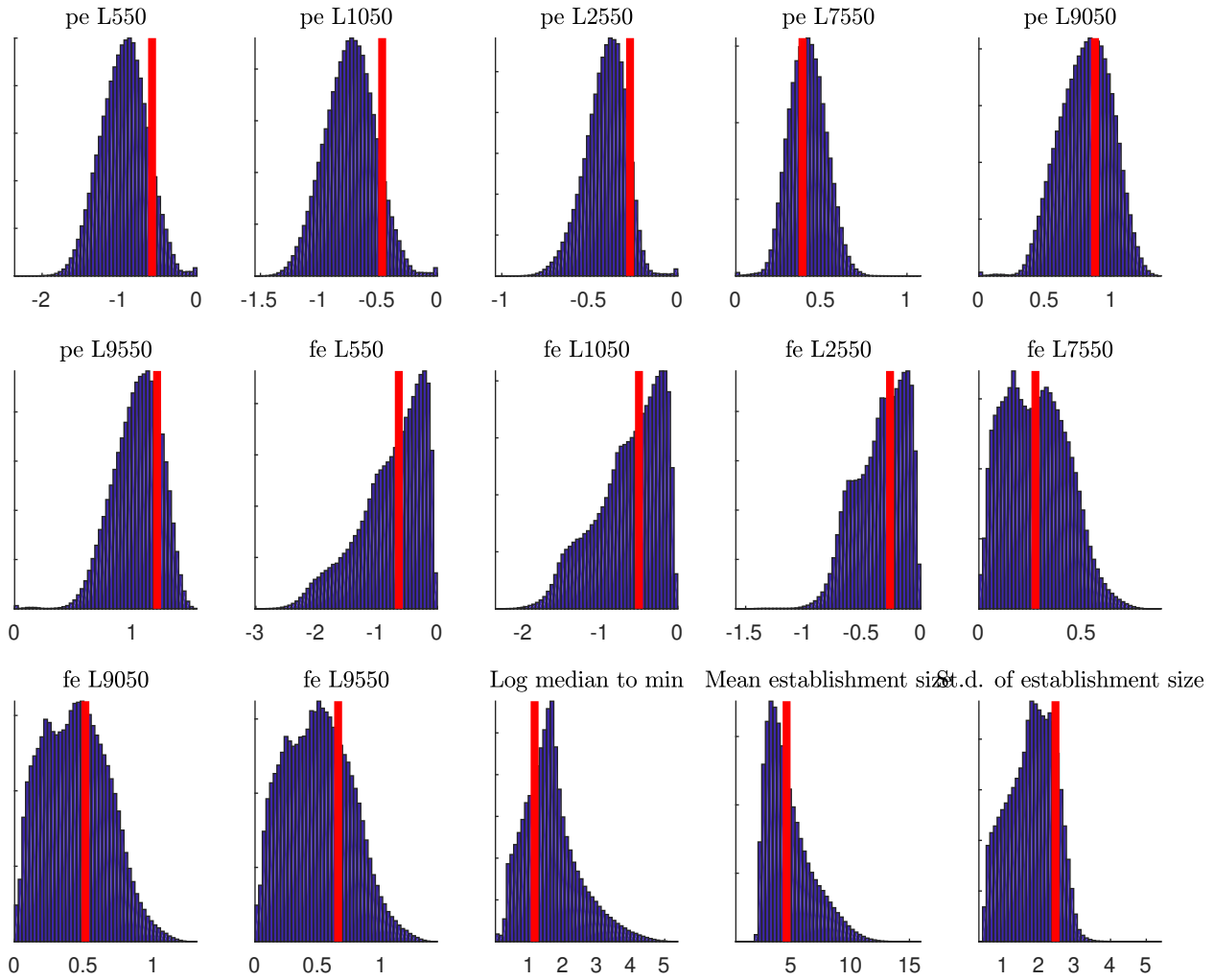
Notes: Moments across 1mn evaluations of the model randomly distributed according to Sobol sequences in the 11-dimensional parameter space.

Figure 22 plots how the distance between the targeted moments in the model and in the data varies with each of the structural parameters of the model.[35] Each small light blue dot in the "clouds" denotes the distance between the targeted moments in the model and data for one particular random draw of parameters. The solid blue line traces out the minimum distance across a range of random draws of the other parameters, holding the parameter of interest fixed at its value on the x-axis. Because of how we sample points (essentially at random), any systematic variation in the distance between the model and the data with the parameter on the x-axis may be viewed as the marginal effect of that parameter, since the other parameters vary at random in the back-

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[35]To save space, we omit the parameter M_0, the relative mass of firms, in these graphs. However, this parameter is clearly identified off average firm size in the data and none of our results are sensitive to its value.

ground. Although the behavior of the minimum distance suggests that some parameters are less than perfectly identified, reassuringly the minimum distance increases monotonically both to the left and right of our point estimates for the key parameters governing the effects of the minimum wage, θ^{50} and γ_2.

Figure 22. Model: Minimum distance to target
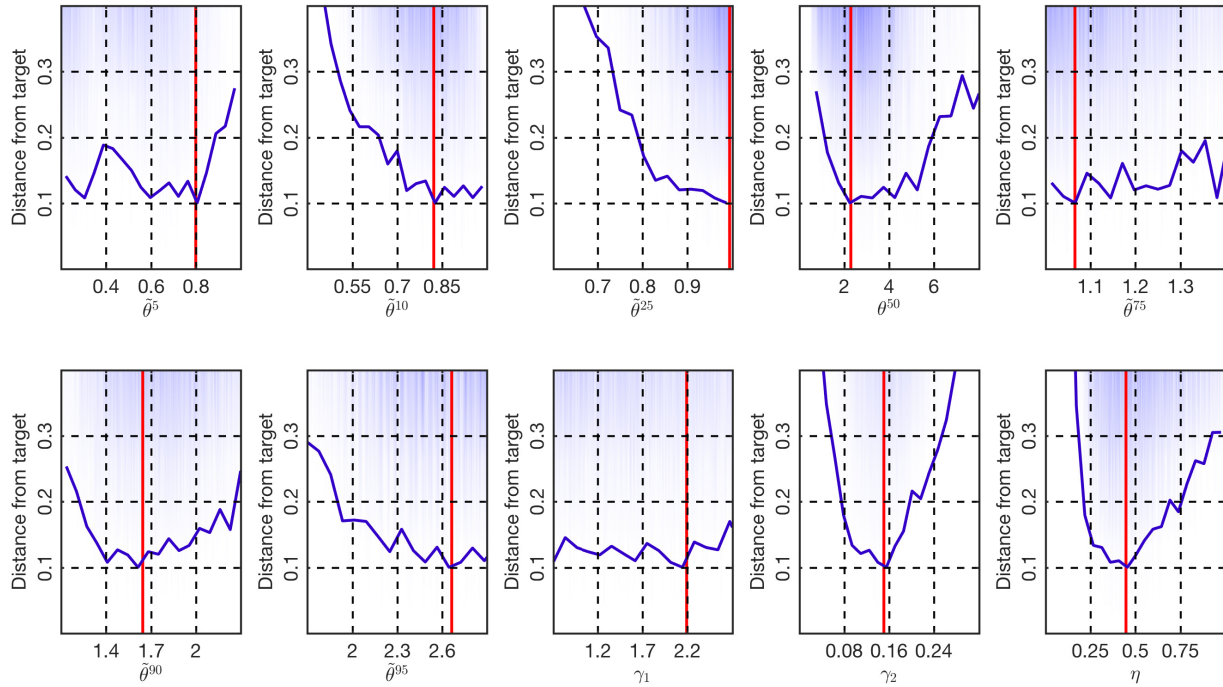
Notes: Graph shows evaluations of the estimation criterion (**??**) for 5 million random draws of parameters from a Sobol sequence. Each blue dot represents one model evaluation, the bold blue line represents the minimum distance to the target, and the red vertical line represents the estimated parameter value that minimizes the distance function. Source: simulations.

D   Empirics Appendix

This appendix provides further empirical evidence related to the facts presented in Section 6, including subsections on the evolution of the Kaitz index by state over time (Appendix D.1), additional results on spillover effects identified off regional variation (Appendix D.2), evidence supporting the empirical identifying assumption of our empirical strategy (Appendix D.3), and details on the hours distribution and its relation to the bindingness of the minimum wage (Appendix D.4).

D.1   Evolution of the Kaitz index by state

Figure 23. Data: Evolution of the Kaitz index by state, 1996–2012
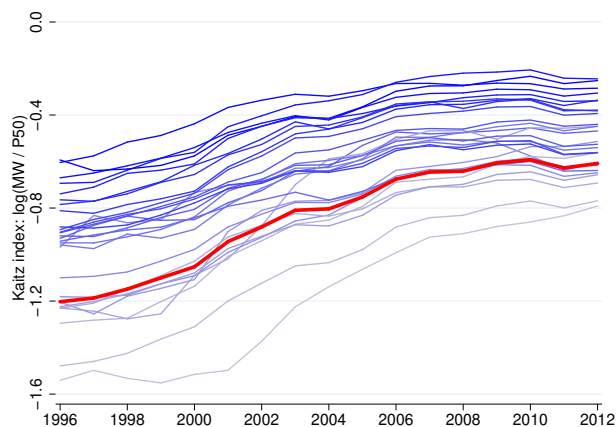
Notes: Kaitz index is defined as $kaitz = \log\left(\text{minimum wage}\right) - \log\left(\text{median wage}\right)$. Each blue line markets one of Brazil's 27 states. Red line marks weighted mean across states. Source: RAIS.

D.2 Additional results on spillover effects identified off regional variation

Figure 24. Data: Wage percentile ratios across Brazilian states over time, 1996–2012
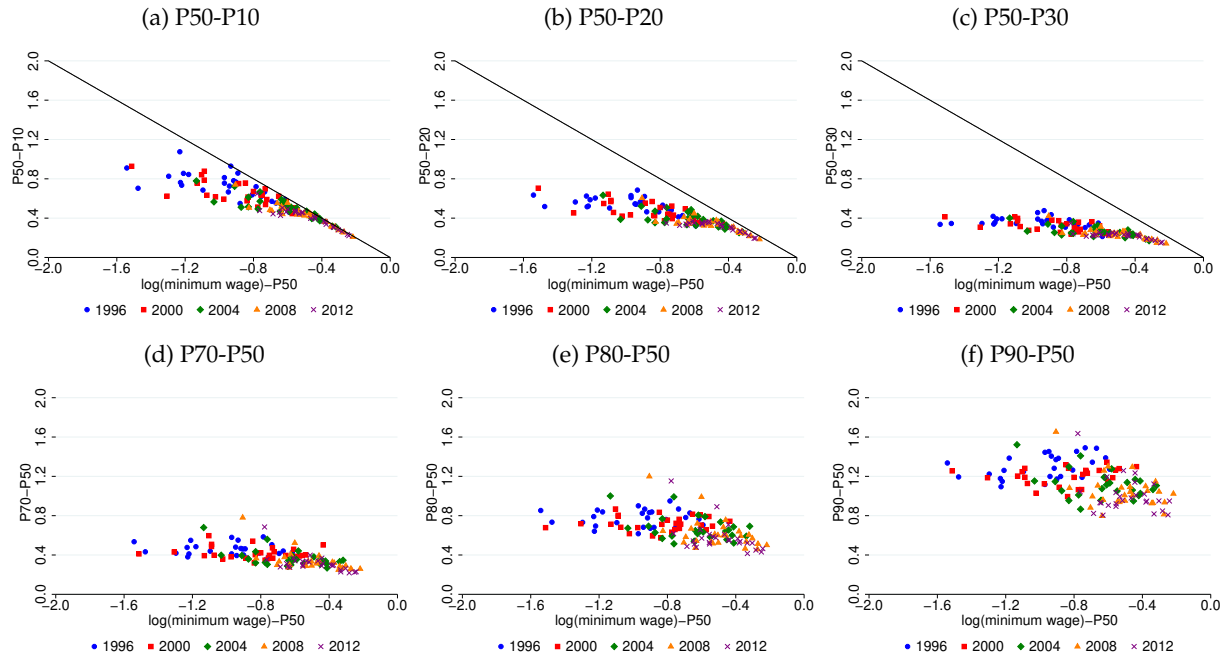
Notes: Figure plots different wage percentile ratios against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one state-year combination for each of Brazil's 27 states. Source: RAIS.

Figure 25. Data: Wage percentile ratios across Brazilian microregions over time, 1996–2012
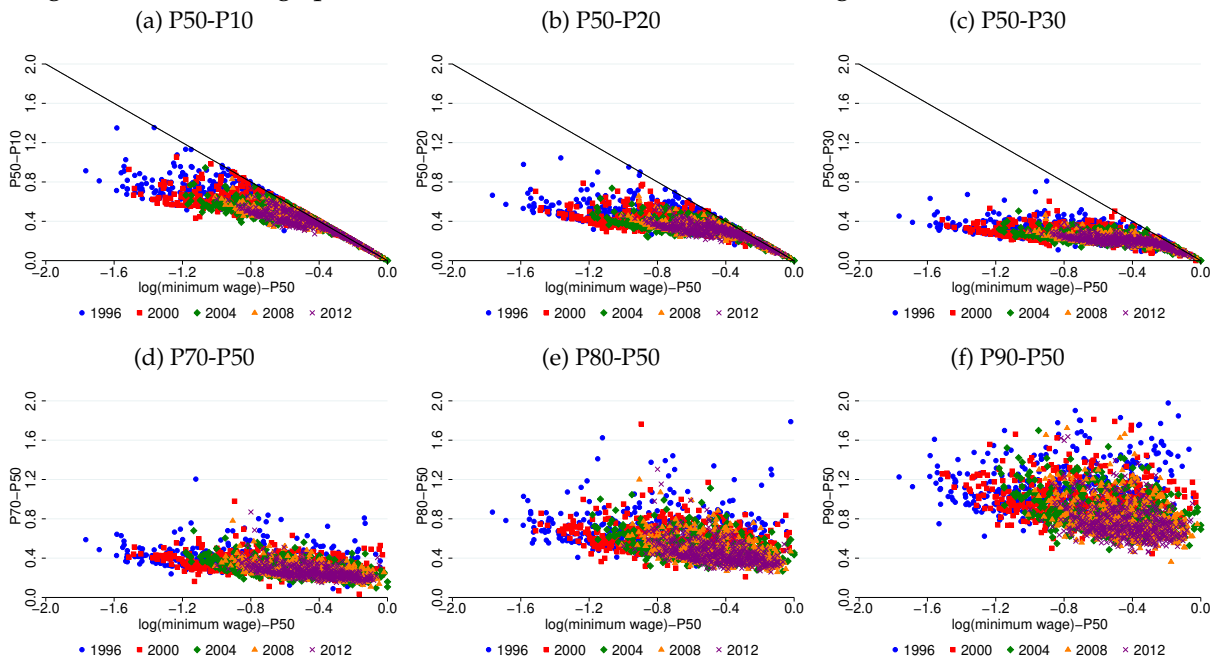

Notes: Figure plots different wage percentile ratios against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one state-microregion combination for each of Brazil's 556 microregions. A small number of outliers are dropped for presentation purposes. Source: RAIS.

D.3 Evidence supporting the empirical identifying assumption

This section provides support for our empirical identifying assumption in Section 6, namely that cross-state variation in the "centrality" of the wage distribution is not systematically related to the shape of the "underlying" wage distribution in Brazil. Although the assumption is not literally testable, we here provide two proxy tests for the assumption.

We show as a first test of our identifying assumption, namely that Brazilian states share a "latent wage distribution," the resemblance of the wage wage distribution in one of Brazil's poorest states in 1996 and that of one of its richest states in 2012 under the higher minimum wage. Figure 26 shows histograms of wages for the state of *Maranhão*, the second poorest in Brazil, and for the state (federal district) of *Distrito Federal*, the richest in Brazil, in 1996 and 2012. Both states see pronounced compression in their distribution over time as the minimum wage increases. We note the striking similarity in the shape of the wage distribution between panel (a) showing the poor state Maranhão in 1996 and panel (d) showing the rich state Distrito Federal in 2012.

Figure 26. Data: Wage histograms for a poor versus rich state of Brazil, 1996 and 2012

(a) Poor state (Maranhão), 1996   (b) Poor state (Maranhão), 2012

(c) Rich state (Distrito Federal), 1996   (d) Rich state (Distrito Federal), 2012
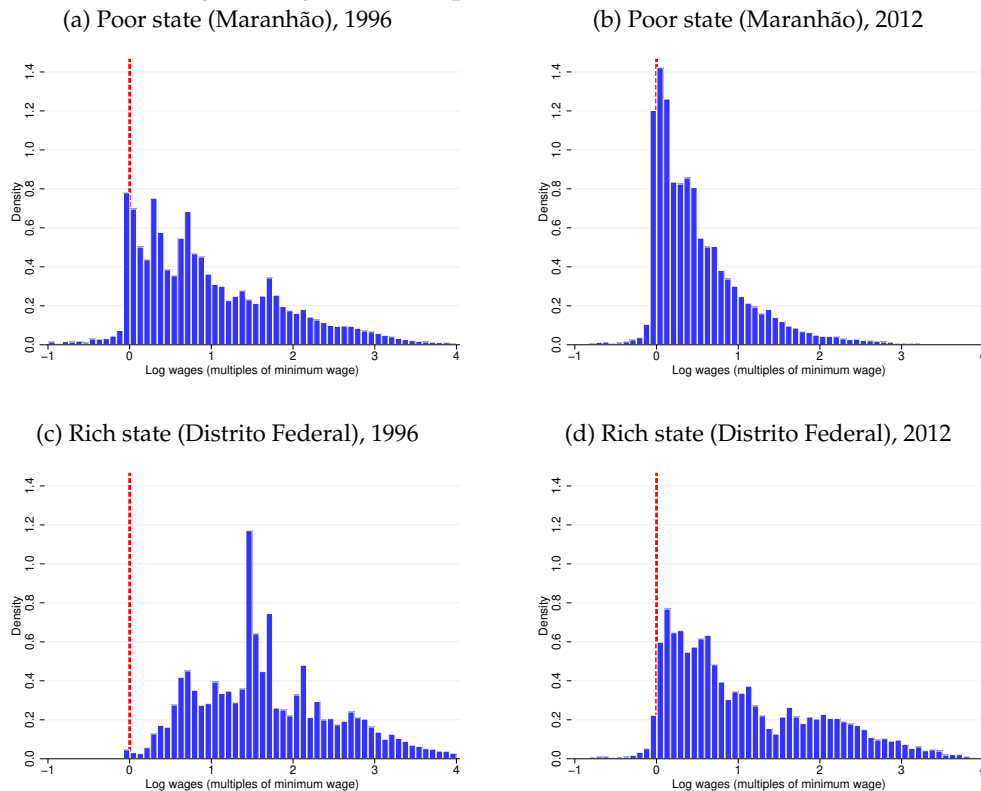
Notes: Figure shows wage histograms for the state of Maranhão, the second poorest in Brazil, and in the state (federal district) of Distrito Federal, the richest in Brazil, in 1996 and 2012. Source: RAIS.

We now demonstrate as a second test of our identifying assumption that the upper tail of the

wage distribution is invariant to the level of the "effective minimum wage." Figure 27 shows that the relation between various upper-tail wage percentile ratios and the median wages across states from 1996 to 2000 is mostly flat, consistent with a shared "latent distribution" across states that is differentially uncovered by the federal minimum wage.

Figure 27. Data: Upper-tail inequality versus median earnings across states, 1996–2000

| (a) P60-P40 | (b) P70-P50 | (c) P80-P60 |
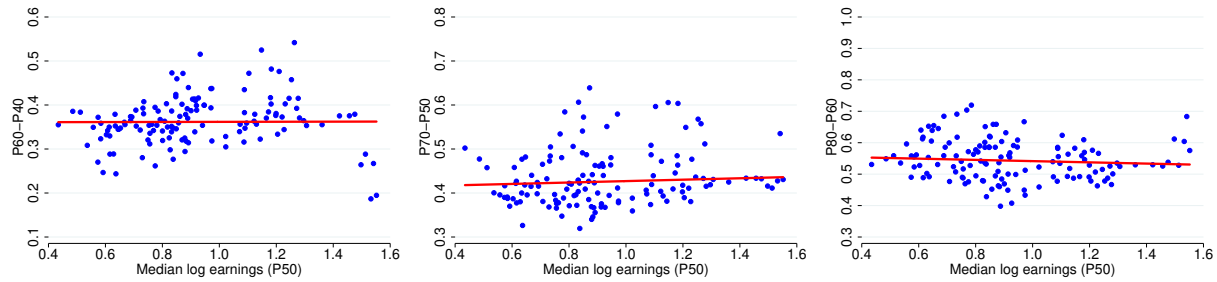
Notes: Blue dots represent state-year observation. Red line represents worker-weighted linear fit. Specification with no state dummies or state trends. Source: RAIS.

D.4 Hours distribution and its relation to the bindingness of the minimum wage

Most adult males in Brazil's formal sector work in a full-time contract, defined as either 40 work hours (spread across 5 days) or 44 work hours (spread across 6 days) per week. Figure 28 shows the raw distribution of hours for this population for 1996–2000 in panel (a), and for 2008–2012 in panel (b). In the initial period, around 74 percent of all workers work 44 hours per week and another 15 percent work 40 hours per week, constituting around 89 percent of workers in full-time employment. Only arounnd five percent of all employees are in employment arrangements with less than 35 contractual work hours per week.[36] Furthermore, the comparison between panels (a) and (b) show that there is no evidence over time—as the minimum wage increases—of a shift towards shorter work weeks in the aggregate. In contrast, there was a small reduction in the share of workers with 30 hour contracts that in the aggregate shifted toward 44 hour contracts.

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[36] According to the Bureau of Labor Statistics a higher share, around 12 percent of employees, works part-time (less than 35 hours) in the US in 2017.

Figure 28. Contractual hours distribution

(a) 1996–2000

(b) 2008–2012
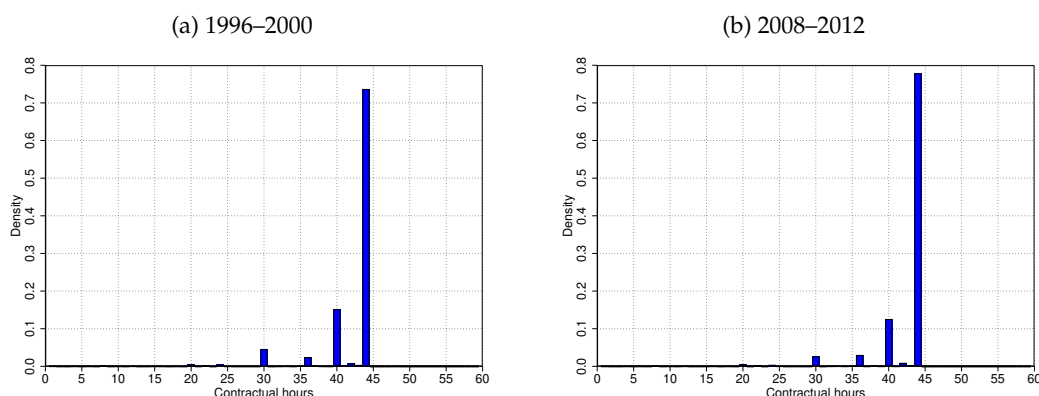
Notes: Figure shows density of contractual work hours for period 1996–2000 in panel (a) and for 2012 in panel (b). All statistics are for adult male workers of age 18–49. A very small fraction of observations reporting more than 60 hours has been dropped from the graphs. Source: RAIS.

Looking beyond the aggregate statistics, there is little systematic covariation between the relative bindingness of the minimum wage and work hours across Brazilian states. Figure 29 shows that both the share of full-time employment in panel (a) and the mean number of hours in panel (b) stay constant as the minimum wage increases between 1996 and 2012. While in 1996 there is a weak systematic negative relationship between the full-time worker share and the bindngnes of the minimum wage, measured by the Kaitz index, this appears almost entirely driven by transitory fluctuations and permanent state-specific heterogeneity, rather than a correlation with the rising minimum wage over time within states.

Figure 29. Relation between contractual hours and bindingness of the minimum wage

(a) Share of workers in full-time employment

(b) Mean contractual hours
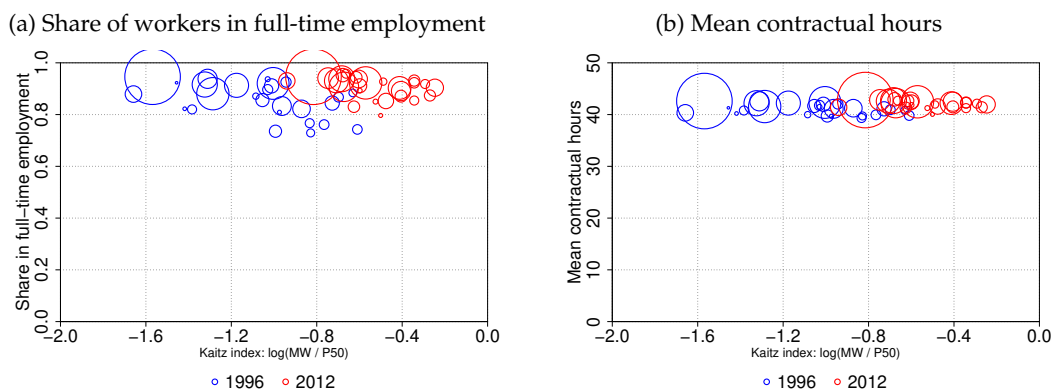
Notes: Notes: Figure shows for male workers of age 18–49 the share in full-time employment, defined as working 40 hours or more (panel (a)), and the mean number of contractual work hours (panel (b)), against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, across states in 1996 and 2012. Area of circles is proportional to population size. Source: RAIS.

To formalize some of these results and as a robustness check to our econometric specification

finding insignificant employment and hours effects of the minimu wage in Section 6.2 we repeated a state-level variant of regression equation (4) with the share of full-time workers a the dependent variable. Our findings indicate a robustly insignificant relation between the incidence of the minimum wage and hours worked in all specifications that we ran.