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Theory and Methodology

A multi-period, multiple criteria optimization system for manpower planning

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Abstract: This paper discusses a multi-period, multiple criteria optimization system for manpower supply forecasting. The system is used to identify recruitment and promotion strategies for managing the enlisted force of the U.S. Navy. With the criteria modeled as trajectories of goal values over the multiple time periods, the system uses the interactive augmented weighted Tchebycheff method as its solution procedure. Illustrative computer results are presented.

Keywords: Multiple criteria optimization, manpower planning, accession planning, trajectory optimization, Tchebycheff procedure

1. Introduction

There is a rich, though diffuse, literature describing the use of optimization techniques in manpower planning. Some of this work has been reviewed by Price, Martel, and Lewis (1980), Grinold and Marshall (1977), and Vajda (1978). Frequently, problems in manpower planning entail the manipulation of *multiple objectives*. Because goal programming was the first widely known technique capable of dealing with large-scale multiple criteria optimization problems, many of these problems have been cast in the mold of goal programming (see Price and Piskor (1972)). A considerable body of goal programming applications is contained in Charnes, Cooper, and Niehaus (1972); Bryant and Niehaus (1978); and Clough, Lewis, and Oliver (1974). Also, some new ideas concerning the goal programming implementation of manpower planning problems are given in Charnes et al. (1984).

One class of manpower planning problems receiving fairly intensive treatment is that of recruitment or *accession planning* (see Bres, et al. (1980); Rowe and Silverman (1982); Van Nunen and Wessels (1978); Kleinman (1978); Young and Abodunde (1979); Grinold and Marshall (1978); and Niehaus (1979)). In accession planning, we are concerned with the selection of a recruitment

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schedule over multiple time periods, which best meets goals pertaining to promotion opportunity, salary expenditures, desired levels of experience in the workforce, and requirements for manpower in each of the planning periods. In this paper, we discuss a prototype model used to examine the accession planning problem in a military system.

Generally speaking, our accession planning problem is classified as a multiple criteria *trajectory optimization* problem because (a) the problem spans T time periods, (b) we wish to monitor the performance of k criteria in each of the T time periods, and (c) there is a goal level of achievement for each objective in each time period. For some references on trajectory optimization, see Kallio, Lewandowski, and Orchard-Hays (1980) and Wierzbicki (1980 and 1982).

Using the terminology of trajectories, it is said that the time path of goal levels for a given objective forms a *trajectory* over the T time periods. By the same token, for each solution, there is a *trajectory* of criterion values for each objective over the T time periods. The challenge, then, in trajectory optimization is to find the solution whose k *criterion value trajectories* most closely match the k *goal trajectories*.

Rather than using goal programming on our manpower problem, we have chosen to employ the augmented weighted Tchebycheff procedure of Steuer and Choo (1983). The appeal of the Tchebycheff approach is that it generates multiple solution candidates at each iteration, does not ask the user to specify weights, and enables one to converge to nonextreme final solutions if so desired. Further details about the reasons why the Tchebycheff method was selected are discussed in Section 5.

2. Structure of the model

With reference to the prototype model, this section describes the complexities which characterize base entry manpower systems, like those which exist in both the enlisted and officer forces of the U.S. Navy. In the prototype model we examine recruitment and promotion policies of a force with 11 length of service (LOS) categories and three paygrades. Individuals in their n-th year of service are in LOS category n. Retirement is mandatory for all individuals after completion of 11 periods of service. Entry into the force can take place only into the lower two paygrades, with fixed proportions of the recruits going to paygrades 1 and 2. Let us assume that these fixed proportions are specified in the form of the *recruit assignment vector*

$$g = \begin{bmatrix} 0.586 \\ 0.222 \\ 0.000 \end{bmatrix} \tag{2.1}$$

which states, for instance, that 22.2% of the new recruits survive their first year and end it in paygrade 2.

The status of the force at any point in time can be represented by a *personnel inventory matrix*. A typical personnel inventory matrix is given in Table 1. For instance, the number of people in their 5th year of service is 23 899 of which 4 892 are in paygrade 3.

There are four types of flows of personnel in the force that will be of interest to us: losses from the force, recruitments, promotions, and demotions. Recruitments and promotions are decision variables to be determined by the model in response to the trajectories of goals placed upon the

Table 1
Personnel inventory matrix (at time 0)

LOS	Paygrade		
	1	2	3
1	167 010	63 270	0
2	16 954	75 337	1 518
3	1 997	49 685	3 266
4	398	30 747	4 328
5	163	18 844	4 892
6	92	11 530	5 149
7	55	7 054	5 220
8	34	4 315	5 180
9	21	2 640	5 077
10	13	1 615	4 937
11	8	988	4 778

Table 2
Transition matrix for estimating demotions and losses from the force

From paygrade	To paygrade		
	1	2	3
1	0.320	0.000	0.000
2	0.004	0.634	0.000
3	0.000	0.000	0.960

system. In contrast, losses from the force and demotions are calculated from a *transition matrix*. A typical transition matrix is given in Table 2. For instance, applying the transition matrix to the 23 899 individuals who are in their 5th LOS, there would be 75 demotions from paygrade 2 to paygrade 1 and 7 128 losses from the force during the next time period. While the prototype model can provide forecasts for any number of periods, in this paper it is illustrated with seven time periods.

There are seven trajectories of goals in the model. The trajectories are typically in conflict with one another, making it impossible to attain all goal trajectories simultaneously. The seven goal trajectories are as follows:

Salary Expenditures (1 goal trajectory). Associated with each paygrade/LOS trajectory is a salary which is paid to each person in that category. The matrix of salaries used in the model is given in Table 3. For instance, the annual salary of each person in paygrade 3 in the 5th LOS is $27 200. The objective here is to minimize salary expenditures while conforming closely to the shape of the goal trajectory over the seven time periods.

Strength-of-Force (2 goal trajectories). Each of paygrade 2 and 3 is given a strength-of-force goal trajectory. The objective here is to maximize the strength-of-force in each of the two paygrades while conforming to the shapes of the trajectories.

Promotion Opportunity (2 goal trajectories). These two goal trajectories specify that, of all the persons remaining in a grade after the losses from the force and demotions have been accounted for, a given number should be promoted from paygrade 1 to 2 and from paygrade 2 to 3. The objective here is to maximize the number of promotions in each category while conforming to the shapes of the trajectories.

Mean Length of Service (2 goal trajectories). For each paygrade it is desirable to retain a relatively high level of experienced personnel in order to maintain (or increase) productivity and performance. One measure of experience is the average LOS in each paygrade. A mean LOS goal trajectory for each of paygrades 2 and 3 is specified over each of the time periods. The objective here is to maximize the mean LOS of people in each of paygrades 2 and 3 while conforming to the shapes of the trajectories.

The seven goal trajectories used in the model are given in the *goal trajectory matrix* of Table 4.

Table 3
Salary matrix

LOS	Paygrade		
	1	2	3
1	11000	14000	24000
2	11100	14500	24800
3	11200	15000	25600
4	11300	15500	26400
5	11400	16000	27200
6	11500	16500	28000
7	11600	17000	28800
8	11700	17500	29600
9	11800	18000	30400
10	11900	18500	31200
11	12000	19000	32000

3. Model formulation

The purpose of the model described in Section 2 is to determine a recruitment and promotion strategy that most closely conforms to the individual goal values as well as to the shapes of their trajectories. To achieve this effect, we define seven unrestricted variables, one for each goal trajectory.

Table 4
Goal trajectory matrix

Time Period	Salaries (10^6)	Strength-of-force 2	Strength-of-force 3	Promotions to 2	Promotions to 3	Mean LOS 2	Mean LOS 3
1	8000	266023	44345	45000	9000	2.4	6.3
2	8000	268642	44766	45000	9000	2.4	6.4
3	8000	271261	45187	45000	9000	2.4	6.5
4	8000	274080	45688	45000	9000	2.5	6.6
5	8000	282381	47072	45000	9000	2.5	6.7
6	8000	290933	48498	45000	9000	2.6	6.8
7	8000	299744	49967	45000	9000	2.7	6.9

min	α-var assoc with salary expenditures goal trajectory
max	α-var assoc with paygrade 2 strength-of-force goal trajectory
max	α-var assoc with paygrade 3 strength-of-force goal trajectory
max	α-var assoc with promotions into paygrade 2 goal trajectory
max	α-var assoc with promotions into paygrade 3 goal trajectory
max	α-var assoc with mean paygrade 2 LOS goal trajectory
max	α-var assoc with mean paygrade 3 LOS goal trajectory

s.t.	Salary constraints
	Paygrade strength constraints
	Promotion opportunity constraints
	Maximum promotion opportunity constraints
	Mean length of service constraints
	Total strength in final period constraint

Figure 1. MOLP formulation

We call these variables α-variables. The α-variable associated with salary expenditures is defined to represent the *maximal* period overdeviation from the salary expenditure goal trajectory. The other six α-variables are defined to represent the respective *minimal* period overdeviations from the other six goal trajectories. Since overdeviations are *bad* with regard to the salary expenditures goal trajectory, but *good* with regard to the other goal trajectories, we will attempt to minimize the first α-variable and maximize each of the other six. Notice that these seven α-variables are all we need to control deviations from the goal trajectories. The overall structure of the multiple objective linear programming (MOLP) formulation of the prototype model is now given in Figure 1.

The notation used in the model formulation, and in the specification of the constraints and objectives is as follows:

- k number of objectives,
- p number of paygrades,
- N number of length of service (LOS) categories,
- T number of time periods,
- e (column) vector of ones of appropriate dimension,
- u_j j-th unit (column) vector of appropriate dimension,
- $y_j(t)$ promotions into paygrade j during period t,
- $r(t)$ recruits during period t,
- g $p \times 1$ recruit assignment vector,
- M_n $p \times p$ transition matrix for the n-th LOS category (where, in the prototype model, matrices M_1 through M_{10} are as given in Table 2, and M_{11} is a matrix of zeros because everyone must retire after eleven LOS periods),
- $I(t)$ $N \times p$ personnel inventory matrix at end of period t,
- $I^n(t)$ n-th row of $I(t)$,
- S $N \times p$ salary matrix,
- R $T \times k$ goal trajectory matrix,
- m maximal promotions vector (m_j specifies the fraction of paygrade $j-1$ personnel eligible for promotion in each time period),
- τ total strength at end of last period (i.e., sum of the elements in $I(T)$),
- $\circ$ operator denoting the multiplication of corresponding matrix elements followed by the summation of all of the resulting product terms.

Note that the $I^n(t)$ are not explicit model variables, but are values generated recursively from the *personnel flow equation* given in the Appendix. The constraints and objectives of the model are specified as follows.

Salary constraints

$$S \circ \frac{I(t-1)+I(t)}{2} - \alpha_1 \leqslant R_{t1}, \quad t=1,\dots,T.$$

By relating the actual salary expenditures to the individual salary expenditure goals, these constraints define the α-variable associated with the salary expenditures goal trajectory.

Paygrade strength constraints

$$e^T I(t) u_j - \alpha_j \geqslant R_{tj}, \quad j = 2, 3, \quad t = 1, \ldots, T.$$

These constraints relate the actual number of people in paygrades 2 and 3 to the individual strength-of-force goals for paygrades 2 and 3. In this way, they define the α-variables associated with the two strength-of-force goal trajectories.

Promotion opportunity constraints

$$y_j(t) - \alpha_{2+j} \geqslant R_{t,(2+j)}, \quad j = 2, 3, \quad t = 1, \ldots, T.$$

These constraints relate to the individual promotion opportunity goals the actual number of people who start time period t in paygrade $j-1$ (and who do not leave the system during the period) that are promoted to paygrade j.

Maximum promotion opportunity constraints

$$y_j(t) - m_j \sum_{n=1}^{N} I^n(t-1) M_n u_{j-1} \leqslant 0, \quad j = 2, 3, \quad t = 1, \ldots, T.$$

These constraints state that no more than $(m_j \times 100)$% of the people who start a period in paygrade $j-1$ (and who do not leave the system during the period) can be promoted to paygrade j. In the model, the m_j values come from the *maximal promotions vector*

$$m = \begin{bmatrix} 0.000 \\ 0.700 \\ 0.080 \end{bmatrix}. \tag{3.1}$$

The $I^n(t-1)$ terms are generated from the *personnel flow equation* given in the Appendix.

Mean length of service constraints

$$\frac{\sum_{n=1}^{N} (n - 1/2) I^n(t) u_j}{\sum_{n=1}^{N} I^n(t) u_j} - \alpha_{4+j} \geqslant R_{t,(4+j)}, \quad j = 2, 3; \quad t = 1, \ldots, T.$$

These constraints relate to the individual length of service goals the average number of years that the people in paygrades 2 and 3 have been in the system. The linearized versions of these constraints that are used in the model are

$$\sum_{n=1}^{N} \left(n - 1/2 - R_{t,(4+j)}\right) I^n(t) u_j - \alpha_{4+j} \geqslant 0, \quad j = 2, 3, \quad t = 1, \ldots, T.$$

Linearizing the mean length of service goals essentially turns them into goals for the maximization of man-years of experience. Due to the fact that the denominator terms are of roughly the same magnitude in all years, the linear form of constraints has been considered adequate.

Total strength in final period constraint

$$\sum_{n=1}^{N} I^n(T) e = \tau.$$

This constraint is used as a precaution to preserve the overall structure of the force at horizon cut-off. There are, of course, other ways to control model behavior at the end of the horizon, including the addition of extra time periods or special constraints. Since Navy and Congressional budget planners typically think foremost of the total size of the force when examining the manpower plan, this constraint is considered to be the most appropriate way of dealing with the difficulty on this application.

Objectives

Using the α-variables associated with the goal trajectories, we have

$\min\{\alpha_1 = z_1\}$	salary expenditures
$\max\{\alpha_2 = z_2\}$, $\max\{\alpha_3 = z_3\}$	total strength
$\max\{\alpha_4 = z_4\}$, $\max\{\alpha_5 = z_5\}$	promotions
$\max\{\alpha_6 = z_6\}$, $\max\{\alpha_7 = z_7\}$	mean length of service

With $T = 7$ time periods, $p = 3$ paygrades, and $N = 11$ LOS categories, the prototype MOLP has 64 constraints (seven salary, 14 paygrade strength, 14 promotion opportunity, 14 maximum promotion opportunity, 14 mean LOS, and one total strength in final period) and 28 variables (fourteen $y_j(t)$, seven $r(t)$, and 7 unrestricted α variables).

4. Augmented weighted Tchebycheff procedure

Details about the interactive augmented weighted Tchebycheff procedure are given in Steuer and Choo (1983) and Steuer (1986). We only summarize the Tchebycheff procedure here. However, before beginning, it is necessary to introduce some notation and terminology.

Consider the MOLP

$$\begin{aligned} \max \quad & \{c^1 x = z_1\}, \\ & \vdots \\ \max \quad & \{c^k x = z_k\}, \\ \text{s.t.} \quad & x \in S. \end{aligned}$$

Let C be the $k \times n$ matrix whose rows are the c^i. Let $Z \subset \mathbb{R}^k$ be the set of all *feasible* criterion vectors where $z \in Z$ if and only if there exists an $x \in S$ such that $z = Cx$. Then, $\bar{z} \in Z$ is *nondominated* if and only if there does not exist a $z \in Z$ such that $z_i \geqslant \bar{z}_i$ for all i and $z_i > \bar{z}_i$ for at least one i. We call the set of nondominated criterion vectors the *nondominated set*.

The idea of the Tchebycheff procedure is to sample a sequence of progressively smaller subsets of the nondominated set until we locate a solution close enough to being optimal to terminate the decision process. The sampling is accomplished by repetitively solving the *augmented weighted Tchebycheff program*

$$\begin{aligned} \min \quad & \left\{ \alpha + \rho \sum_{i=1}^{k} w_i \right\}, \\ \text{s.t.} \quad & \alpha \geqslant \lambda_i w_i, \qquad i = 1, \ldots, k, \\ & w_i = z_i^{**} - z_i, \qquad i = 1, \ldots, k, \\ & z = Cx, \\ & x \in S, \end{aligned}$$

for different λ's, $\lambda \in \Lambda$ where (a) $\Lambda = \{\lambda \in \mathbb{R}^k \mid \lambda_i > 0, \sum_{j=1}^{k} \lambda_i = 1\}$, (b) each z_i^{**} is perturbed to be a value slightly larger than the maximal value of its corresponding z_i over S, and (c) ρ is a sufficiently small positive number. Letting P be the *sample size*, t the *number of iterations*, and r the *Λ-reduction factor*, the Tchebycheff algorithm is specified as follows:

Step 1. Let $h = 0$. Compute z^{**} and let $(l_i^{(1)}, \mu_i^{(1)}) = (0, 1)$ for all i.

Step 2. Let $h = h + 1$ and form the subset of weighting vector space

$$\Lambda^{(h)} = \left\{ \lambda \in \mathbb{R}^k \mid \lambda_i \in \left(l_i^{(h)}, \mu_i^{(h)}\right), \sum_{i=1}^{k} \lambda_i = 1 \right\}.$$

Step 3. Obtain $2P$ dispersed λ-vectors from $\Lambda^{(h)}$ and for each one, solve the augmented weighted Tchebycheff program.

Step 4. Display the P most different of the resulting criterion vectors and have the decision-maker select the most preferred designating it $z^{(h)}$.

Step 5. In the light of any insights gained by viewing the solutions of Step 4, the user is permitted at this point to make any adjustments to the model and its goal trajectories that may seem appropriate.

Step 6. If $h < t$ and any adjustments were made to the model in Step 5, recompute the z^{**} ideal criterion vector and go to Step 7. If $h = t$, let $x^{(h)}$ be the inverse image of $z^{(h)}$, and stop with $(x^{(h)}, z^{(h)})$ as the final solution.

Step 7. Using $\lambda^{(h)}$ whose components are given by

$$\lambda_i^{(h)} = \frac{1}{\left| z_i^{**} - z_i^{(h)} \right|} \left[\sum_{j=1}^{k} \frac{1}{\left| z_j^{**} - z_j^{(h)} \right|} \right]^{-1}$$

tighten the interval bounds as follows:

$$\left(l_i^{(h+1)}, \mu_i^{(h+1)}\right) = \begin{cases} (0, r^h) & \text{if } \lambda_i^{(h)} - \dfrac{r^h}{2} < 0, \\ (1 - r^h, 1) & \text{if } \lambda_i^{(h)} + \dfrac{r^h}{2} > 1, \\ \left(\lambda_i^{(h)} - \dfrac{r^h}{2}, \lambda_i^{(h)} + \dfrac{r^h}{2}\right) & \text{otherwise,} \end{cases}$$

in which r^h is r raised to the h-th power.

Step 8. Now form the next smallest subset of weighting vector space

$$\Lambda^{(h+1)} = \left\{ \lambda \in \mathbb{R}^k \mid \lambda_i \in \left(l_i^{(h+1)}, \mu^{(h+1)}\right)_i, \sum_{i=1}^{k} \lambda_i = 1 \right\}$$

and go to Step 2.

Some comments are now in order about the implementation of the above Tchebycheff algorithm on the accession planning application of this paper. First, we have been working with an upper bound of five on the number of iterations t because of the tedium involved with additional iterations.

Second, we have been using sample sizes P of between 10 and 15. On a first pass, users seem to be able to quickly eliminate about half of the solutions as being unacceptable for one reason or another. Then, the user is left with about 'seven plus or minus two' solutions (see Miller, 1956) from which to make a most preferred selection. This does not appear to cause difficulties.

Third, the Λ-reduction factor r, which enables us to reduce weighting vector space, is chosen so that the final iteration $[l_i, \mu_i]$ interval widths are between 0.1000 and 0.1500. In laboratory experiments such as the one reported in Steuer and Choo (1983), it has been found that convergence to within one or two percent of the optimum can usually be achieved with such final interval widths.

Fourth, because the first accession planning objective is a minimization objective, the augmented weighted Tchebycheff program that is solved in Step 3 is

$$\begin{aligned} \min \quad & \left\{ \alpha + \rho \sum_{i=1}^{k} w_i \right\}, \\ \text{s.t.} \quad & \alpha \geqslant \lambda_i w_i, && i = 1, \ldots, 7, \\ & w_i = z_i - z_i^{**}, && i = 1, \\ & w_i = z_i^{**} - z_i, && i = 2, \ldots, 7, \\ & z = Cx, \\ & x \in S. \end{aligned}$$

By solving the augmented Tchebycheff program using a commercial-grade code, we note that we can use the Tchebycheff method with accession planning MOLPs of any size.

Fifth, we make use of computer graphics at the computer/user interface to help manage the amount of trajectory information generated at each iteration and facilitate its comparison with the goal trajectories. How this has been done is shown in the numerical illustration of Section 6.

Sixth, in many planning exercises, a decision maker may start out with only fuzzy ideas about the confines of his or her feasible region and what the exact goal trajectories should be for the different objectives. Consequently, it is anticipated that a user will gain ideas and insights about the planning problem as the groups of solutions generated at each iteration are examined. Thus, Step 5 is very important because it is this step that makes the solution procedure a *system*. It is this step that enables a user to iteratively refine the constraints and update the goal trajectories as more is learned about what is, and is not, possible in the problem, and how the objectives tradeoff against one another.

5. Why Tchebycheff method was selected

We now comment about the reasons why we settled on the Tchebycheff method on this application. While we do not deny that personal preference may be involved, our adoption of the Tchebycheff method was motivated by the following:

(1) *No weights*. In our initial efforts to formulate the accession planning problem, we used goal programming. In large goal programming problems there are typically many deviational variables. Because of the uncertainties involved in trying to assign weights to these variables, a considerable burden is often placed upon the process of setting up a goal programming model for operational use. This burden is avoided in the Tchebycheff method because the user never has to specify weights.

(2) *No solution point 'jumping' problems*. On this application, it is a client requirement that we be able to conveniently explore solutions in a neighborhood of a given solution. In an attempt to explore nearby solutions with the goal programming approach, we would make small changes to the weights. Sometimes this made no difference; the same solution would be returned. Other times, a drastically different solution would result. We never knew what to expect. Such unpredictable solution point 'jumping' behavior, as illustrated in Steuer (1986, Chapter 10), is a consequence of GP's use of the weighted L_1-metric and its affinity for returning extreme point nondominated criterion vectors. The Tchebycheff method, on the other hand, uses the L_∞-metric to sample the nondominated set. This lends to the Tchebycheff procedure the property that small changes in the weights lead to small changes in the solution, and large changes in the weights lead to large changes

in the solution. Furthermore, this capability enables the Tchebycheff procedure to converge to nonextreme final solutions if that is where the best solution lies. Thus, the Tchebycheff method provides the user with the desired degree of control over being able to 'steer' the solution, and explore nearby solutions in a neighborhood of a given solution.

(3) *Multiple solutions*. Another client requirement is that the solution procedure allow a user to quickly grasp the size and range of the nondominated set. Whereas we get only one solution at a time in goal programming, with the Tchebycheff method, a group of dispersed solution alternatives is generated by presentation to the user at each iteration. This is done so that a user can rapidly come to understand the limits of what can be accomplished with the feasible region and the types of tradeoffs at one's disposal. This is also necessary to facilitate the model refinement and goal trajectory adjustment processes that are anticipated, particularly in the early iterations, in Step 5 of the algorithmic procedure.

For these reasons, we settled on the Tchebycheff method as the interactive solution procedure to be employed.

6. Computer results

We now illustrate the implementation of the augmented weighted Tchebycheff procedure on the prototype accession planning model. Along with a matrix generator for preparing MOLP input decks, we employ the AUTOPAKM package of Steuer, Dominey, and Whisman (1985) which calls the MINOS code of Murtagh and Saunders (1980) and Preckel (1980) as a subroutine to perform the $2P$ augmented weighted Tchebycheff optimizations of Step 3 in one job submission. Also, the LAMBDA and FILTER codes from the ADBASE package of Steuer (1983) are used for the λ-vector generation and filtering chores of Steps 3 and 4. Step by step we have the following:

1. Let $P = 10$, $t = 4$, and $r = 0.5000$ which means that the final iteration $[l_i, \mu_i]$ interval width will be 0.1250.

2. After reading in the recruit assignment vector (2.1), the maximal promotions vector (3.1), and the data in the matrices of Tables 1, 2, 3, 4, and 5, the matrix generator produces an MOLP input deck and stores it in MOLP1.

3. Using the scheme of setting z_i^{**} to be the smallest number larger than the maximal value of z_i over S that is divisible by 1000, we obtain $z^{**} = (-3000, 252300, 8000, -2000, 5000, 8000, 25000)$.

4. Using LAMBDA and FILTER, we generate 20 dispersed λ-vectors from $\Lambda^{(1)}$ and store them in LAM1. Then, AUTOPAKM reads MOLP1 and LAM1, and performs the 20 augmented weighted Tchebycheff optimizations associated with the different λ-vectors in LAM1. The resulting criterion vectors are written to ZVECT1.

5. Reading ZVECT1, FILTER then determines the 10 most different of the criterion vectors in ZVECT1 and displays them as in Figure 2. Note that the positive numbers in the table mean above goal and the negative numbers mean below goal. After reviewing Figure 2 and *trajectory graph displays* (as in Figure 3) of whatever solutions the user wishes to examine in detail, let us assume that solution 1–4 is selected as the most preferred.

6. Assume that after studying Figure 2 and the trajectory graphs, the decision-maker decides to make the following changes to the model:

(a) Replacement of the initial salary goal trajectory with a salary goal trajectory that increases linearly from 7 900 to 8 100 over the seven time periods.

(b) Placement of an upper bound of 340 000 on the number of recruits $r(t)$ during each period $t = 1, \ldots, 7$ to reflect the capacity of the Navy's recruiting organization.

(c) Placement of an upper bound of 11 000 and a lower bound of 4 000 on the number of people $y_3(t)$ promoted from paygrade 2 to paygrade 3 in each period.

The revised model is now stored in MOLP2.

7. Because the model has been modified, we solve for a new z^{**} vector and obtain $z^{**} = (-1000, 17000, 8000, -2000, 3000, -12000, 4000)$ With the new z^{**}, we compute

$$\lambda^{(1)} = \begin{bmatrix} 0.2477 \\ 0.0116 \\ 0.0390 \\ 0.6257 \\ 0.0527 \\ 0.0071 \\ 0.0162 \end{bmatrix}.$$

Then, we store in LAM2 20 dispersed λ-vectors

Criterion Vector	Salaries (10^6)	Strength-of-Force 2	Strength-of-Force 3	Promotions 2	Promotions 3	Mean LOS 2	Mean LOS 3
1-1	2,808	76,686	-26,711	-2,427	-7,383	-0.34	0.21
1-2	107	-49,935	5,848	-38,950	-1,726	-0.09	-0.30
1-3	1,080	34,094	-20,818	-33,071	-8,999	-0.11	-1.21
1-4	82	-6,079	1,837	-2,428	-2,087	-0.17	-0.25
1-5	17,907	207,577	-2,448	-44,985	-8,999	-1.01	-1.28
1-6	22,718	241,284	-33,429	-44,983	-8,237	-1.16	0.53
1-7	2,118	-206,234	-35,157	-33,468	-8,532	-1.05	0.54
1-8	40,385	239,545	-43,118	-12,164	-9,000	-1.53	2.92
1-9	107,270	252,460	-43,118	-42,074	-9,000	-1.79	2.92
1-10	2,613	1,260	7,084	-2,427	4,443	-0.32	0.96

Figure 2. Criterion vector display of the first iteration

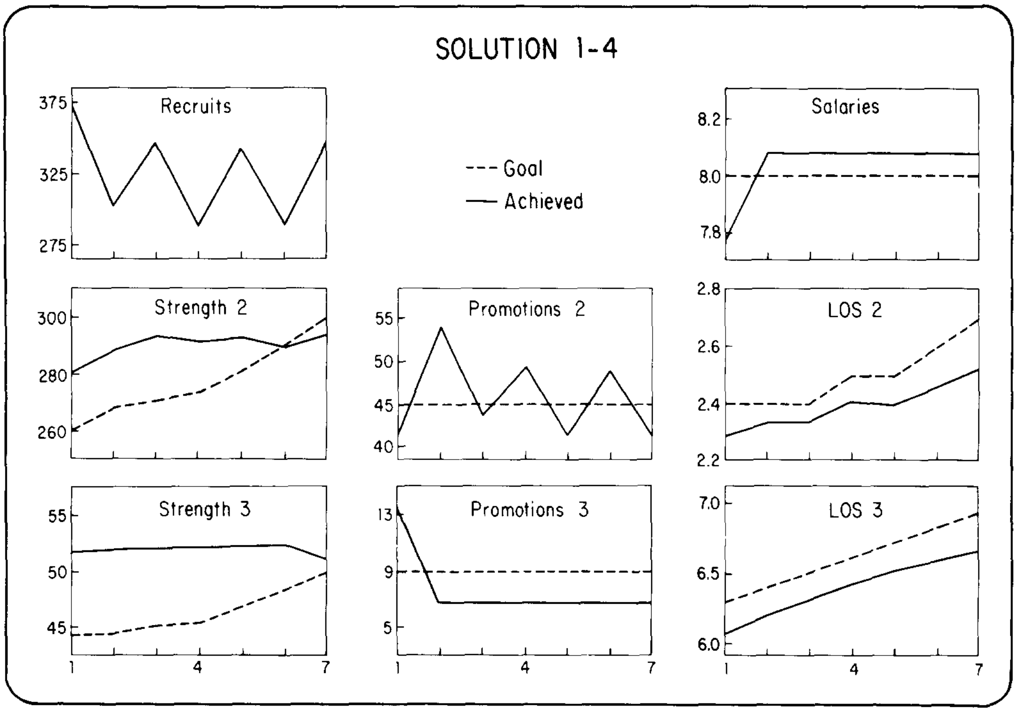

Figure 3. Trajectory Graph Display of Solution 1–4

Criterion Vector	Salaries (10^6)	Strength-of-Force 2	Strength-of-Force 3	Promotions 2	Promotions 3	Mean LOS 2	Mean LOS 3
1-4	82	-6,079	1,837	-2,428	-2,087	-0.17	-0.25
2-1	286	164	-10,184	-6,366	-5,000	-0.10	0.24
2-2	-196	-2,239	-11,597	-2,789	-5,000	-0.16	0.02
2-3	293	-7,276	1,745	-5,143	-893	0.00	-0.51
2-4	312	5,202	-10,609	-3,243	-5,000	-0.11	-0.31
2-5	499	12,426	1,275	-2,420	-4,620	-0.22	-0.46
2-6	137	29	-19,540	-9,693	-5,000	-0.08	-0.13
2-7	408	-5,257	6,906	-2,486	-4,999	-0.14	-0.41
2-8	193	16,062	-19,540	-2,486	-5,000	-0.15	-0.13
2-9	137	10,458	-19,540	-3,014	-5,000	-0.09	-0.13
2-10	49	-16,579	7,083	-2,835	2,000	-0.18	-0.60

Figure 4. Criterion vector display of the second iteration

from the intervals

$$[l^{(2)}, \mu^{(2)}] = \begin{bmatrix} 0.0000, 0.5000 \\ 0.0000, 0.5000 \\ 0.0000, 0.5000 \\ 0.3757, 0.8757 \\ 0.0000, 0.5000 \\ 0.0000, 0.5000 \\ 0.0000, 0.5000 \end{bmatrix}.$$

8. After reading MOLP2 and LAM2, AUTO-PAKM performs 20 augmented weighted Tchebycheff optimizations. Displaying the 10 most different of the resulting criterion vectors we have the 2nd iteration display of Figure 4, and so forth.

Thus we see the advantages, and perhaps the necessity, of presenting multiple solutions, the use of computer graphics, and being able to update the model (as in Step 5 of the algorithm) to trajectory optimization. They make it possible to absorb large amounts of multiple criteria information at each iteration, and they enable us to converge to a final problem definition while, at the same time, converging to a final solution of the problem as a whole.

Appendix

In the constraint specifications of Section 3, the rows of the personnel inventory matrices are recursively generated from the *personnel flow equation*

$$I^n(t) = (1 - \delta_{n1}) I^{n-1}(t-1) M_{n-1} + \sum_{j=2}^{3} H_j^n y_j(t) + \delta_{n1} g^T r(t)$$

in which (i) δ_{n1} is the *Kronecker delta* (i.e., $\delta_{n1} = 1$

Table 5
Promotion distribution matrices

H_2			H_3		
0.000	0.000	0.000	0.000	0.000	0.000
−0.894	0.894	0.000	0.000	−0.239	0.239
−0.091	0.091	0.000	0.000	−0.284	0.284
−0.011	0.011	0.000	0.000	−0.187	0.187
−0.002	0.002	0.000	0.000	−0.116	0.116
−0.001	0.001	0.000	0.000	−0.071	0.071
−0.001	0.001	0.000	0.000	−0.044	0.044
0.000	0.000	0.000	0.000	−0.027	0.027
0.000	0.000	0.000	0.000	−0.016	0.016
0.000	0.000	0.000	0.000	−0.010	0.010
0.000	0.000	0.000	0.000	−0.006	0.006

when $n = 1$ and $\delta_{n1} = 0$ when $n \neq 1$), (ii) g is the $p \times 1$ recruit assignment vector, and (iii) H_j^n is the n-th row of the LOS × paygrade *promotion distribution* matrix H_j. The H_j matrices used in this paper are given in Table 5.

For instance, the (4, 2) and (4, 3) elements of H_3 specify that of all the people promoted into paygrade 3 in a given year, 18.7% of them will be in the fourth LOS category.

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