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Decision Support

Integrated bank performance assessment and management planning using hybrid minimax reference point – DEA approach

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ABSTRACT

The purpose of assessing past performances and setting future targets for an organisation such as a bank branch is to find where the branch stands in comparison to its peers within the bank branch network and how to improve the efficiency of its operations relatively when compared to the best practice branches. However, future performance targets may be set arbitrarily by the head-office and thus could be unrealistic and not achievable by a branch. A hybrid minimax reference point-data envelopment analysis (HMRP-DEA) approach is investigated to incorporate the value judgements of both branch managers and head-office directors and to search for the most preferred solution (MPS) along the efficient frontier for each bank branch. The HMRP-DEA approach is composed of three minimax models, including the super-ideal point model, the ideal point model and the shortest distance model, which share the same decision and objective spaces, are different from each other only in their reference points and weighting schema, and are proven to be equivalent to the output-oriented DEA dual models. These models are examined both analytically and graphically in this paper using a case study, which provides the unprecedented insight into integrated efficiency and trade-off analyses. The HMRP-DEA approach uses DEA as an ex-post-facto evaluation tool for past performance assessment and the minimax reference point approach as an ex-ante planning tool for future performance forecasting and target setting. Thus, the HMRP-DEA approach provides an alternative means for realistic target setting and better resource allocation. It is examined by a detailed investigation into the performance analysis for the fourteen branches of an international bank in the Greater Manchester area.

1. Introduction

The model structures of data envelopment analysis (*DEA*) and multiple objective linear programming (*MOLP*) have much in common but *DEA* is directed to assessing past performances as part of management control function and *MOLP* to planning future performances (Cooper, 2004). *DEA* has been used to measure the relative efficiency of a set of bank branches that possess shared functional goals with incommensurate inputs and outputs (Chen, 1998; Avkiran, 1999; Soteriou and Stavrinides, 2000; Cook and Hababou, 2001; Lin et al., 2009). Recent applied research in this area highlights the gaps between research and practice in performance management and the needs to take into account the preferences of the decision makers in planning future performances (Yavas and Eisher, 2005; Duygun-Fethi and Pasiouras, 2009). The use of the classical *DEA* models proposed by Charnes, Cooper and Rhodes (*CCR*) (1978) and Banker, Charnes and Cooper (*BCC*) (1984), however, generate efficiency scores and target levels that do not take into account the decision maker (*DM*)'s preferences. Existing techniques that incorporate the *DM*'s preference information in *DEA* have been proposed such as the goal and target setting models of Golany (1988), Thanassoulis and Dyson (1992) and Athanassopoulos (1995, 1998), and weight restriction models including imposing bounds on individual weights (Dyson and Thanassoulis, 1988), assurance region (Thompson et al., 1990), restricting composite inputs and outputs, weight ratios and proportions (Wong and Beasley, 1990), and the cone ratio concept by adjusting the observed input–output levels or weights to capture value judgments to belong to a given closed cone (Charnes and Cooper, 1990; Charnes et al., 1994). However,

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the above techniques require *prior* preferences from the *DM* that are often subjective and require knowledge about what are achievable, which is not available *a priori* and needs to be explored.

Research on integrating *DEA* and *MOLP* has attracted increasing attentions to support both past performance assessment and future target setting in hybrid manners. For instance, Golany (1988) developed an interactive model combining both *DEA* and *MOLP* approaches to allocate a set of input levels as resources and to select the most preferred output levels from a set of alternative points on the efficient frontier. Post and Spronk (1999) combined the use of *DEA* and interactive goal programming to adjust the upper and lower feasible boundaries of the input and output levels.

A hybrid minimax reference point-*DEA* (*HMRP–DEA*) approach has been developed to incorporate the *DM*'s preference information into performance assessment and target setting without necessarily requiring *prior* judgments (Yang, 1999, 2001; Yang and Li, 2002; Wong, 2005; Yang and Wong, 2004; Wong et al., 2009; Yang et al., 2009; Yang and Xu, 2010). The *HMRP–DEA* approach is composed of three minimax reference point models, all equivalent to the output oriented *CCR* dual models and different from each other only in their reference points and weighting schema, which include the super-ideal point model, the ideal point model and the shortest distance model. The super-ideal point model is proven identical to the output-oriented *CCR* dual models under certain conditions and can be used to replace the latter to generate efficiency scores and *DEA* composite inputs and outputs. The ideal point model is designed to facilitate an interactive trade-off analysis process to search for *MPS*s for individual decision makers. The shortest distance model can be used to support a group management planning process for mapping a group *MPS* (*GMPS*) back to the feasible space of each branch to find its local *MPS* (*LMPS*) having realistic and achievable performance targets with both individual and group preferences taken into account. In the *HMRP-DEA* approach, the strengths of *DEA* and the minimax reference point models are combined to support past performance assessment and management planning for future target setting with preferences from *DM*s at both individual and group levels taken into account.

This paper reports an application of the *HMRP-DEA* approach to the performance assessment of 14 bank branches in the Greater Manchester area of a major retail bank in the UK. Data set is provided by the head-office and each bank branch is measured and assessed on the basis of the selected input and output variables as agreed by the head-office directors. The classical *DEA* is conducted for each branch based on the output oriented *DEA* models as well as the identical super-ideal point model, resulting in a set of technical input and output targets. If the manager of a branch is not satisfied with these targets, tradeoff analysis between the outputs can be conducted on the basis of the ideal point model to search for the *MPS* along the efficient frontier of the branch with the branch manager's preferences elicited interactively. A group management planning process is then illustrated where all branch managers and the head-office directors could collectively determine a *GMPS* as a group performance benchmark. Since the *GMPS* may not be achievable by individual branches, it needs to be mapped back to the local feasible solution space of a branch in order to find a *LMPS* of the branch, where preferences provided in terms of weights for the output variables are incorporated. Such a *LMPS* is expected to have realistic and achievable targets for each branch, which can reflect both the branch's and the organisation's preferences. It should be noted that a *MPS* is set on the basis of the preferences of an individual branch manager, whilst a *LMPS* takes into account both individual and group value judgements. Finally, the analytical and graphical investigation into the case study is conducted which provides a new insight into the integrated efficiency and trade-off analyses.

The rest of the paper is organised as follows. In Section 2, the *HMRP-DEA* approach is introduced by describing the three minimax models briefly. Section 3 reports its application to the performance assessment and management planning for target setting for the branches of a major bank in the UK. The paper concludes in Section 4.

2. Brief introduction to the hybrid minimax reference point-DEA approach

The hybrid minimax reference point-*DEA* (*HMRP-DEA*) approach has been developed (Yang et al., 2009) to support both past performance assessment and future target setting using three minimax equivalence models: the super-ideal point model, the ideal point model and the shortest distance model. The super-ideal point model is proven identical to the output-oriented *CCR* dual models under certain conditions and can be used to generate *DEA* efficiency score and the corresponding composite inputs and outputs. The ideal point model is used to support an interactive trade-off analysis process to generate individual *MPS*s, thereby setting realistic future targets with the individual *DM*'s preferences taken into account. The shortest distance model is designed to support a group negotiation process to set organizational or group targets, based on which to update local *MPS*s, with both the group's and individual members' preferences taken into account. In this section, these models will be introduced briefly.

Suppose an organisation has n decision making units (*DMUs*) ($j = 1,\ldots,n$), produces s outputs denoted by y_{rj} (the rth output of *DMU* j for $r = 1,\ldots,s$) and consumes m inputs denoted by x_{ij} (the ith input of *DMU* j for $i = 1,\ldots,m$). The output-orientated *CCR* dual model is given as follows (Charnes et al., 1994).

$$\begin{aligned}
\text{Max}\quad & h_o = \theta_{j_o} \\
\text{s.t.}\quad & \theta_{j_o} y_{rj_o} - \sum_{j=1}^{n} \lambda_j y_{rj} \leqslant 0 \quad r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_o} \quad i = 1,\ldots,m \\
& \lambda_j \geqslant 0 \text{ for all } j
\end{aligned} \tag{1}$$

In the output-orientated *CCR* dual model (1), for each observed DMU_0 an imaginary composite unit is constructed that outperforms DMU_0.λ_j is the reference weight for *DMU* j ($j = 1,\ldots,n$) and $\lambda_j > 0$ means that *DMU* j is used to construct the composite unit for DMU_0. The composite unit consumes at most the same inputs as DMU_0 and produces outputs that are at least equal to a proportion θ_{j_o} of the outputs of DMU_0. The parameter θ_{j_o} indicates by how much DMU_0 has to proportionally increase its outputs to become efficient. The inverse of θ_{j_o} is the efficiency score of DMU_0. The increase is employed concurrently to all outputs and results in a radial movement towards the envelopment surface (Charnes et al., 1994).

The following *super-ideal point model* provides the basis for interactive trade-off analysis.

$$\begin{aligned}
&\min \quad \theta \\
&\text{s.t.} \quad w_r(f_r^* - f_r(\lambda)) \leqslant \theta \quad r = 1, \ldots, s \\
&\qquad \lambda = [\lambda_1 \cdots \lambda_n]^T \in \Omega_{j_0} \\
&\qquad \Omega_{j_0} = \left\{ \lambda \left| \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ij_0}, i = 1, \ldots, m; \ \lambda_j \geqslant 0, j = 1, \ldots, n \right. \right\}
\end{aligned} \tag{2}$$

where $f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj}$ is defined as the rth objective (composite output) and λ_j the decision variable for the jth *DMU*. Model (2) is proven identical to the *CCR* dual model (1) with $\theta = F^{\max} - \theta_{j_0}$ and $e_0 = 1/(F^{\max} - \theta)$ being the optimal efficiency score under the following conditions: $w_r = 1/y_{rj_0}, f_r^* = y_{rj_0} F^{\max}, F^{\max} = \max_{1 \leqslant r \leqslant s}\{\bar{f}_{rj_0}/y_{rj_0}\}, \bar{f}_{rj_0} = f_r(\lambda^*) = \max_{\lambda \in \Omega_{j_0}} f_r(\lambda)$ (Yang et al., 2009). The name "super-ideal point model" is used because f_r^* is at least as good as the best feasible (ideal) value of the rth objective and the objective of model (2) is to find a feasible solution that is as close to the super-ideal point $[f_1^*, \ldots, f_s^*]^T$ in the objective space as possible.

In the same decision and objective spaces of the general *MOLP* problem for DMU_0 as for model (2), the *ideal point model* is given by (Yang et al., 2009)

$$\begin{aligned}
&\min \quad \theta \\
&\text{s.t.} \quad w_r(\bar{f}_{rj_0} - f_r(\lambda)) \leqslant \theta \quad r = 1, \ldots, s \\
&\qquad \lambda = [\lambda_1 \cdots \lambda_n]^T \in \Omega_{j_0}
\end{aligned} \tag{3}$$

where $w_r \geqslant 0$ ($r = 1, \ldots, s$) are parameters that can be systematically adjusted to locate any most preferred solution (*MPS*) by the *DM* (Steuer and Choo, 1983; Lightner and Director, 1981). An *MPS* is defined as an efficient solution that maximises the utility function of the *DM*. Since the explicit *DM* utility function is not known in general, an interactive trade-off analysis method, such as the gradient projection method (Yang, 1999; Yang and Li, 2002), can be used to find an *MPS*, as illustrated in the following section. The name "ideal point model" is used because $\bar{f}_{rj_0}$ is the best feasible (ideal) value of the rth objective and the objective of model (3) is to find a feasible solution that is as close to the ideal point $[\bar{f}_{1j_0}, \ldots, \bar{f}_{sj_0}]^T$ in the objective space as possible.

The *MPS* located for each *DMU* relative to its peers only takes into account the preferences at a branch or local level. In order to set a performance benchmark with the organisational or group preferences taken into account, a group *MPS* (*GMPS*) would need to be determined first. It is possible that a *GMPS* is assigned by picking up an existing efficient *DMU*. Alternatively, a mathematical procedure could be used to find a convex combination of the generated individual *MPS*s or existing efficient *DMU*s as a *GMPS*, which then needs to be mapped back to the feasible space of each local branch. Such a procedure is summarised as follows, which takes two steps.

The first step is to construct the weighted outputs and inputs of a *GMPS*. For example, let's consider the relative contribution of each *DMU* to the overall efficiency through the use of the following weighted sum as the rth output of the *GMPS*, denoted by

$$y_r^{GMPS} = \sum_{j=1}^{n} \bar{q}_{rj} \hat{y}_{rj} \quad r = 1, \ldots, s \tag{4}$$

where $\hat{y}_{rj}$ is the rth output of the *MPS* for the jth *DMU*. $\bar{q}_{rj}$ is a normalised marginal efficiency output, showing the proportion that the rth output of the jth *DMU* contributes to efficiency as compared to the rth output of the other *DMU*s, generated as follows.

$$\bar{q}_{rj} = q_{rj} / \sum_{j=1}^{n} q_{rj} \quad \text{with} \quad q_{rj} = \hat{y}_{rj} / \sum_{i=1}^{m} v_i x_{ij} \qquad r = 1, \ldots, s, j = 1, \ldots, n \tag{5}$$

where x_{ij} is the ith input for the jth *DMU* and v_i the weight for the ith input generated by solving the primal *DEA* model (Eq. (2) in Yang et al. (2009)). q_{rj} in Eq. (5) is referred to as marginal efficiency output and $\sum_{i=1}^{m} v_i x_{ij}$ as composite input in this paper. Similarly, the ith input of the *GMPS* can be calculated, denoted as x_i^{GMPS} for $i = 1, \ldots, m$.

A *GMPS* generated above may lie within, on or outside the efficient frontier of a *DMU* and thus may not necessarily be achievable by the *DMU*. As such, a *GMPS* needs to be mapped back to the feasible space of each *DMU* to achieve a local *MPS* (*LMPS*) for each *DMU*. The new local input and output targets could then be used as benchmark to align towards the organisation's or group's targets with both group and individual *DMs*' preferences taken into account.

A *LMPS* for each *DMU* could be generated as the one closest to the *GMPS* in the composite output space using the following shortest distance model (Yang, 2001; Yang et al., 2009),

$$\begin{aligned}
&\text{Min} \quad d \\
&\text{s.t.} \quad w_r(f_r^{GMPS} - f_r(\lambda)) \leqslant d \\
&\qquad - w_r\left(f_r^{GMPS} - f_r(\lambda)\right) \leqslant d, \quad r = 1, \ldots, s, \lambda \in \Omega_{j_0}
\end{aligned} \tag{6}$$

where $f_r^{GMPS} = \bar{\lambda} y_r^{GMPS}$ with $\bar{\lambda} = \min_{1 \leqslant i \leqslant m}\{x_{ij_0}/x_i^{GMPS}\}$. Note that w_r is the relative weight of the objective $f_r(\lambda)$, which can be assigned for each *DMU* individually.

3. Performance assessment and target setting for bank branches

The *HMRP–DEA* approach is applied to assess the past performances of the bank branch network of a major international bank, in particular 14 homogenous and comparable bank branches located within the region of Greater Manchester, England. Suggestions are required as to what these bank branches should do in future to improve their business performances in line with the *DMs*' or branch managers' preferences, in other words to search for the *MPS* on the efficient frontier and set future target levels for each branch both individually and as a group.

3.1. Problem description and past performance assessment

The business performance variables used in this study were selected in consultation with the performance management director of the bank's head office in London as well as the managers of the local branches in Greater Manchester area. In total, seven variables were selected for this study with five inputs and two outputs. The selection of the input and output variables is largely due to the management interests of the company and the availability of the recorded performance data. The two outputs are customer service and commercial income. Customer service is measured by the number of customers who rate the branch service as being satisfied. Commercial income is income generated by the relationship managers from selling mortgages, bank loans, insurance, and investment products.

The five inputs are business reviews, contacts, registrations, key performance indicators (KPI) and future value added (FVA). Business reviews look into providing financial reviews for business clients and are measured by the number of reviews completed and their effectiveness ratio. Contacts are the means of setting customer contact promises and gaining relevant information for future cross-selling such as commercial mortgages, vehicle finance and business loans, all of which are measured by the number of contact promises generated and the percentage that is already fulfilled. Registrations take account of various delivery channels and are measured by the number of internet and telephone banking accounts opened and activated for the customers. KPI looks into the saving and lending balances of the customer accounts and acts as a proxy to measure the liquidity and financial health of the bank branch. FVA sales indicates the amount of warm leads being converted into sales in the ensuing months, and includes commercial start-ups, account switches, money and wealth management products and services.

The data set consists of actual scores provided by the bank's customer relationship management system from one month in a recent year, which will be used to assess the performances of the bank branches as shown in Table 1. The application predominantly employs the use of Microsoft Excel and its Solver function to formulate and calculate the *DEA* and minimax reference point models. For the purpose of illustrating the application in this paper, the University bank branch is investigated in detail. The output-orientated *CCR* dual model (1) is run to find the efficiency scores for each bank branch. In addition, *VBA* codes were written using the Excel platform to automate and replicate the *DEA* calculation for all bank branches.

The *DEA* results in Table 2 show that seven out of the fourteen bank branches are considered to be efficient with a full efficiency score of 100%. However, these results should be taken with caution because only 14 branches were assessed whilst as many as five inputs and two outputs were used in the analysis. While this is what the bank management wanted to analyse, the rule of thumb is that the number of *DMU*s should be at least three times the total number of input and output variables. One way to satisfy this rule is to reduce the number of inputs and/or outputs using correlation analysis and delete highly correlated variables. Nevertheless, the main purpose of this paper is to

Table 1
Data set for performance measurement of bank branches.

DMU	Branch	Inputs					Outputs	
		Business					Customer	Commercial
		Review	Contacts	Registrations	KPI	FVA	Service	Income
1	Oldham Road	60	16	40	38	190	88	200
2	Trafford Park	60	20	50	39	225	88	91
3	King Street	47	30	39	29	228	102	111
4	Swinton	60	15	38	22	164	93	143
5	Royal Exchange	60	23	44	34	190	80	101
6	High Street	60	26	37	42	98	89	173
7	University	60	30	44	29	140	78	140
8	Clayton	60	30	25	31	130	98	155
9	Oxford Road	47	13	50	32	140	88	132
10	Stretford	51	27	34	28	115	80	130
11	Didsbury	56	21	42	26	108	80	134
12	Chorlton	58	27	16	29	82	94	137
13	Eccles	58	22	30	31	142	92	71
14	Salford	60	25	35	32	97	82	132

Table 2
DEA scores and *CCR* dual multipliers.

DMU	Branch	Efficiency (%)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Oldham Road	**100.0**	1.000													
2	Trafford Park	**81.5**			0.006						0.899			0.301		
3	King Street	**100.0**			1.000											
4	Swinton	**100.0**				1.000										
5	Royal Exchange	**74.2**			0.207	0.170					0.497			0.288		
6	High Street	**100.0**						1.000								
7	University	**89.5**	0.246			0.371								0.397		
8	Clayton	**100.0**								1.000						
9	Oxford Road	**100.0**									1.000					
10	Stretford	**95.6**	0.107							0.477	0.137			0.164		
11	Didsbury	**99.2**	0.090			0.265								0.577		
12	Chorlton	**100.0**												1.000		
13	Eccles	**95.7**				0.211					0.256			0.574		
14	Salford	**89.2**				0.120		0.302			0.011			0.563		

Table 3
Equivalence between *CCR* Dual Model and Super-Ideal Point Model.

DMU	DEA dual model		Minimax model									$\theta = F^{max} - \theta_{j_0}$
			Composite inputs and outputs									
	DEA score (%)	θ_{j_0}	F^{max}	θ	*x1*	*x2*	*x3*	*x4*	*x5*	*y1*	*y2*	
1	100.0	1.000	1.088	0.088	60.0	16.0	40.0	38.0	190.0	88.0	200.0	0.088
2	81.5	1.228	2.198	0.970	60.0	20.0	50.0	37.7	151.9	108.0	160.6	0.970
3	100.0	1.000	1.387	0.387	47.0	30.0	39.0	29.0	228.0	102.0	111.0	0.387
4	100.0	1.000	1.000	0.000	60.0	15.0	38.0	22.0	164.0	93.0	143.0	0.000
5	74.2	1.347	1.839	0.492	60.0	23.0	44.0	34.0	168.3	107.7	152.4	0.492
6	100.0	1.000	1.108	0.108	60.0	26.0	37.0	42.0	98.0	89.0	173.0	0.108
7	89.5	1.118	1.301	0.184	60.0	20.2	30.3	29.0	140.0	93.4	156.5	0.184
8	100.0	1.000	1.057	0.057	60.0	30.0	25.0	31.0	130.0	98.0	155.0	0.057
9	100.0	1.000	1.153	0.153	47.0	13.0	50.0	32.0	140.0	88.0	132.0	0.153
10	95.6	1.046	1.119	0.073	51.0	22.2	25.7	28.0	115.0	83.6	135.9	0.073
11	99.2	1.008	1.114	0.106	54.8	21.0	22.9	26.0	108.0	86.9	135.0	0.106
12	100.0	1.000	1.000	0.000	58.0	27.0	16.0	29.0	82.0	94.0	137.0	0.000
13	95.7	1.045	2.287	1.242	58.0	22.0	30.0	29.5	117.5	96.1	142.6	1.242
14	89.2	1.121	1.171	0.049	58.5	25.0	25.3	32.0	97.0	91.9	148.0	0.049

demonstrate how the integrated performance assessment and planning can be conducted in this area using the hybrid minimax reference point–DEA approach.

The *DEA* efficiency scores show some expected results. The branches of King Street, Oldham Road, Oxford Road and High Street in the Manchester city centre tend to do very well, with the exception of the Royal Exchange branch. This is due to higher resources and more specified sales force made available to the city centre branches. The King Street branch is the largest and the main office for the bank's operations in Greater Manchester and is located right inside the financial district of the city, while the Oxford Road, Oldham Road and High Street branches are highly accessible and well located in high traffic flow areas in the city. The inefficient branches: Stretford, Didsbury, Eccles and Salford branches are all located in smaller towns that are at the outskirts of the city whose customer bases are mostly pensioners and home-dwellers. The inefficient University branch, on the other hand, serves largely the student community, while the Trafford Park branch mainly for the industries and factories.

The efficiency analysis can also be conducted using the super-ideal point model identical to the output-oriented *CCR* dual model. Running the equivalent super-ideal point model (2) leads to the results shown in Table 3, where θ_{j_0} is equal to $1/e_0$ with e_0 being the optimal efficiency score for each *DMU* shown in the third column of Table 2. It is clear from Table 3 that the equivalence of $\theta = F^{max} - \theta_{j_0}$ holds with θ generated using the super-ideal point model.

Table 3 also shows the target levels in terms of composite inputs and outputs for all the bank branches, generated using model (2). Each inefficient branch has its own benchmarking composite *DMU* made up of other efficient *DMUs*. The composite *DMUs* for all the inefficient branches have improved input and output targets. However, these target values are technically generated by the *DEA* model without any consideration of the *DM*'s preferences. Hence, in this case such input and output target values may not be preferred by the individual branch managers or the directors of the head-office.

3.2. Local target setting via interactive tradeoff analysis

The interactive gradient projection method (Yang, 1999; Yang and Li, 2002; Yang et al., 2009) is used to facilitate the tradeoff analysis, which is based on the ideal point model to elicit tradeoff information progressively from the local *DMs* (branch managers) to search for a *MPS* for each branch. However, it should be noted that other interactive multi-objective optimisation techniques could also be employed to locate individual *MPSs* if preferred.

Before the interactive trade-off analysis procedure can be started, Eq. (9) in Yang et al. (2009) needs to be solved for each composite output of the observed DMU_0 to generate an output payoff table. Table 4 shows the range of possible output values when each composite output of every branch is maximised. Note that the efficient branches of Swinton and Chorlton have the same maximum and minimum values in the payoff table, showing that there are no possible tradeoffs between the outputs of either Swinton or Chorlton as for each of them maximising y_1 and y_2 leads to the same set of solutions. This means that the preferences of the branch managers of Swinton and Chorlton do not matter since it is not possible to further improve either customer service or commercial income, given the current levels of inputs and outputs for all branches in the reference set. The other efficient branches can sacrifice one of the outputs to increase the other output, while both the outputs of the inefficient branches can be further improved.

Starting solution: For illustration purpose, the trade-off analysis procedure is demonstrated for the University branch, which is an inefficient branch. The maximum feasible value of the first composite output of the University branch is shown as $\bar{f}_{17}$= 101.51, while the maximum feasible value of the second composite output of the University branch is $\bar{f}_{27} = 156.51$, or $\bar{f}_7 = [101.51, 156.51]^T$, which are used in the next step of the procedure.

The *DEA* efficient solution for the University Branch, shown in Table 3, is taken as the starting point. The parameters of the starting solution are thus given as follows:

- $f_1^0 = 93.4, f_2^0 = 156.5 - \delta$ with δ being a small positive number to ensure $f_2^0 < 156.5$ that is the maximum value of the second output, or $\bar{f}_{27} = 156.5$, so that $w_2^0 < \infty$.
- $w_r^0 = 1/(\bar{f}_{r7} - f_r^0), r = 1, 2$.
- λ^0 = [0,0,0.008,0.243,0,0,0,0.736,0.019,0,0,0,0,0]^T generated by solving model (3).
- $f(\lambda^0)$ = [97.21,152.20]^T and N^0 = [0.691,0.309]^{T}generated using Eq. (21) in Yang et al. (2009).

Table 4
Payoff table for maximum outputs of all branches.

DMU	Branch	Max o1		Max o2		Maximum values		Minimum values	
		y1	*y2*	*y1*	*y2*	*y1*	*y2*	*y1*	*y2*
1	Old ham Road	95.75	146.20	88.00	200.00	95.75	200.00	88.00	146.20
2	Trafford Park	108.03	160.58	88.00	200.00	108.03	200.00	88.00	160.58
3	King Street	102.00	111.00	69.17	153.94	102.00	153.94	69.17	111.00
4	Swinton	93.00	143.00	93.00	143.00	93.00	143.00	93.00	143.00
5	Royal Exchange	107.75	152.36	89.25	185.75	107.75	185.75	89.25	152.36
6	High Street	98.61	144.43	89.00	173.00	98.61	173.00	89.00	144.43
7	University	101.51	142.06	93.40	156.51	101.51	156.51	93.40	142.06
8	Clayton	103.61	141.78	94.43	156.33	103.61	156.33	94.43	141.78
9	Oxford Road	88.00	132.00	67.82	152.13	88.00	152.13	67.82	132.00
10	Stretford	89.53	124.10	79.03	141.92	89.53	141.92	79.03	124.10
11	Didsbury	89.11	132.41	86.85	135.05	89.11	135.05	86.85	132.41
12	Chorlton	94.00	137.00	94.00	137.00	94.00	137.00	94.00	137.00
13	Eccles	96.13	142.64	89.25	162.36	96.13	162.36	89.25	142.64
14	Salford	96.00	140.70	90.79	149.11	96.00	149.11	90.79	140.70

First interaction: In the first interaction, the output of customer service is treated as the reference objective and the optimal indifference tradeoff for a unit change of customer service (f_1) is given by $df^0 = [1,2.24]^T$ using Eq. (27) in Yang et al. (2009). This means that if customer service is sacrificed by 1 unit from 97.21 to 96.21, the commercial income will be increased by 2.24 units from 152.20 to 154.44. However, suppose the branch manager of the University branch does not agree with this initial optimal indifference tradeoff, and instead proposes a new set of indifference tradeoff, for example as shown in Table 5, which implies that a decrease of 1 unit of customer service f_1 should lead to an increase of 3.50 unit of commercial income f_2. The new set of indifference tradeoff would then be used to approximate the marginal rate of substitution with $M^0 = [1,1/3.5]^T$ using Eq. (25) in Yang et al. (2009).

The projection of the gradient M^0 onto the tangent plane of the efficient frontier at $f(\lambda^0)$ is calculated using Eq. (28) in Yang et al. (2009) to find the tradeoff direction of the branch manager with $\Delta\bar{f} = [-0.06, 0.13]^T$, which means that the commercial income should be improved at the expense of customer service to improve the utility of the University branch manager. In order to calculate tradeoff size, the maximum step size is determined at $\alpha^0_{max} = 31.99$ using Eq. (30) in Yang et al. (2009), which is used to construct the step size table in terms of one-tenth of the maximum step size ($C_\alpha = 10$) as shown in Table 6. At this stage, suppose the University branch manager sets a minimum lower bound for customer service f_1 at 96.00, which exceeds at $\alpha = 0.7$. Hence, the size-step could be set at $\alpha = 0.6$.

A more precise step size could be set as in Table 7, where each step means the one-hundredth value of the maximum step size ($C_\alpha = 100$). Using the same minimum lower bound of 96.00 for customer service f_1, the step size is found to be $\alpha = 0.62$. This allows the weighting vector to be updated as $w^1 = [1.00,3.36]^T$ using Eq. (29) in Yang et al. (2009). (see Table 8)

Using the updated weights, the following new solution is found:

- $\lambda^1 = [0.117,0,0,0.296,0,0,0,0.429,0,0,0,0.163,0,0]^T$
- $f(\lambda^1) = [95.23,154.64]^T$, $N^1 = [0.775,0.757]^T$.

At this stage, the interactive search process moved onto another facet of the efficient frontier since the new normal vector N^1 is not in parallel with the previously identified normal vector N^0. This can also be shown by the over sacrifice of customer service from the expected

Table 5
First optimal indifference tradeoffs for university branch.

Optimal indifference tradeoff for 1 unit change of f1	
Original	(97.21, 152.20) ⇔ (97.21–1,152.20 + 2.24)
New	(97.21,152.20) ⇔ (97.21–1,152.20 + 3.50)

Table 6
Tradeoff Step Size for University Branch $C_\alpha = 10$).

C@=10		
@	f1	f2
0	97.21	152.20
0.1	97.01	152.63
0.2	96.82	153.06
0.3	96.63	153.49
0.4	96.43	153.92
0.5	96.24	154.36
0.6	96.05	154.79
0.7	95.86	155.22
0.8	95.66	155.65
0.9	95.47	156.08
1.0	95.28	156.51

Table 7
Tradeoff step size for university branch C_α = 100).

C@=100		
@	f1	f2
0.60	96.05	154.79
0.61	96.03	154.83
0.62	96.01	154.87
0.63	95.99	154.92
0.64	95.97	154.96
0.65	95.95	155.00
0.66	95.93	155.04
0.67	95.91	155.09
0.68	95.89	155.13
0.69	95.87	155.17
0.70	95.86	155.22

Table 8
Second optimal indifference tradeoff for university branch.

Optimal indifference tradeoff for 1 unit change of f1	
Original	(95.23, 154.64) ⇔ (95.23–1,154.65 + 1.02)
New	(95.23, 154.64) ⇔ (95.23–1,154.65 + 3.00)

96.01 shown in Table 7 to the actual 95.23 with the slightly less improvement of commercial income of 154.64 than the expected 154.87 shown in Table 7.

Second interaction: The optimal indifference tradeoff vector at $f(\lambda^1)$ for a unit change of f_1 is given as $df^1 = [1, 1.02]^T$. If the *DM* still does not agree with this new optimal indifference tradeoff of sacrificing 1 unit of customer service for an improvement of 1.02 units of commercial incomer, a new set of indifference tradeoffs can be provided by the *DM*, for example as shown in Table 8 where the sacrifice of 1 unit of customer service is supposed to lead to an increase of 3 units of commercial income, whereby the marginal rate of substitution of the *DM* at this point is approximated as $M^1 = [1, 0.33]^T$. The new gradient projection is calculated as $\Delta\bar{f}^1 = [-0.32, 0.33]^T$ and the maximum step size as $\alpha^1_{\max} = 5.68$, showing that it is still preferred to improve commercial income at the expense of customer service from their currently suggested levels. Suppose the branch manager changes the minimum lower bound on f_1 from 96.00 to 95.00. Such a change is allowed and is actually part of the learning process to find the lowest realistic score for customer service without unnecessary sacrifice. This is one of the main features of the interactive trade-off procedure and it can help the *DM* to set realistic targets. The trade-off step size $\alpha = 0.12$ is then generated to provide an updated weighting vector $w^2 = [1, 3.95]^T$.

Using the newly updated weights, a new efficient solution is generated as follows:

- $\lambda^2 = [0.133, 0, 0, 0.305, 0, 0, 0, 0.378, 0, 0, 0, 0.191, 0, 0]^T$
- $f(\lambda^2) = [95.01, 154.86]^T$, $N^2 = 1.276[0.775, 0.757]^T$.

At this stage, the interactive search process moved along the same facet of the efficient frontier as the last interaction while searching for the *MPS* because N^2 is in parallel with N^1.

Third interaction and the generated MPS: Suppose the manager of the University branch agrees with the optimal indifference tradeoff at λ^2 with a unit change of f_1 exactly offset by 1.02 unit of f_2, so $df^3 = [1, 1.02]^T$. Then, the interactive process terminates, and based on the value judgements of the manager of the University branch, $f(\lambda^2)$ is determined as the *MPS*, which is used as the target output values for the University branch. The corresponding target input values for the University branch are given by $x_i^{MPS} = \sum_{j=1}^{n} \lambda_j^2 x_{ij}$.

The interactive process started with a relatively high target level for customer service and a low commercial income of $[97.21, 152.20]^T$. During the interactive process, customer service is progressively sacrificed under the careful guidance of the *DM* to achieve better commercial income in an informed and calculated manner, leading to a more realistic and preferable *MPS* at $[95.01, 154.86]^T$.

Comparing the *MPS* targets with the *DEA* composite targets, the University branch manager set a slightly higher target level on customer service than on commercial income, as the *MPS* output target levels are given by $[95.01, 154.86]^T$ but the *DEA* target output levels are $[93.40, 156.51]^T$. Therefore, based on the branch manager's preferences, more emphasis is placed on customer service than commercial income compared with the *DEA* target output levels. Table 9 shows that customer service and commercial income should be increased by

Table 9
DEA and *MPS* target values for university branch.

	Inputs					Outputs	
	Business					Customer	Commercial
	Review	Contacts	Registrations	KPI	FVA	Service	Income
Evaluated unit	60.00	30.00	44.00	29.00	140.00	78.00	140.00
DEA targets	60.00	20.21	30.26	29.00	140.00	93.40	156.51
Improvement	0.00	9.79	13.74	0.00	0.00	15.40	16.51
%	**0.0**	**32.6**	**31.2**	**0.0**	**0.0**	**19.7**	**11.8**
MPS targets	60.00	23.19	29.40	29.00	140.00	95.01	154.86
Improvement	0.00	6.81	14.60	0.00	0.00	17.01	14.86
%	**0.0**	**22.7**	**33.2**	**0.0**	**0.0**	**21.8**	**10.6**

Table 10
MPS input and output target values for all branches.

DMU	Branch	Inputs					Outputs	
		Business					Customer	Commercial
		Review *x1*	Contacts *x2*	Registrations *x3*	KPI *X4*	FVA *X5*	Service *y1*	Income *y2*
1	**Oldham Road**	**60.00**	**16.00**	**40.00**	**34.63**	**182.53**	**90.00**	**186.13**
2	***Trafford Park***	***60.00***	***76.05***	***41.99***	***38.24***	***789.06***	***90.04***	***197.37***
3	**King Street**	**47.00**	**15.64**	**32.64**	**29.00**	**162.07**	**75.09**	**146.20**
4	Swinton	60.00	15.00	38.00	22.00	164.00	93.00	143.00
5	***Royal Exchange***	***60.00***	***17.30***	***40.99***	***34.00***	***190.00***	***93.00***	***180.54***
6	**High Street**	**59.67**	**26.00**	**28.54**	**35.36**	**98.00**	**94.99**	**155.20**
7	***University***	***60.00***	***23.19***	***29.40***	***29.00***	***140.00***	***95.07***	***154.86***
8	**Clayton**	**60.00**	**24.42**	**25.00**	**31.00**	**122.10**	**94.99**	**156.12**
9	**Oxford Road**	**46.88**	**13.00**	**47.07**	**31.63**	**140.00**	**84.95**	**135.04**
10	***Stretford***	***51.00***	***20.45***	***23.32***	***28.00***	***115.00***	***80.00***	***140.93***
11	***Didsbury***	***54.93***	***21.00***	***23.33***	***26.00***	***108.00***	***87.00***	***134.92***
12	Chorlton	58.00	27.00	16.00	29.00	82.00	94.00	137.00
13	***Eccles***	***58.00***	***20.90***	***29.66***	***31.00***	***142.00***	***90.00***	***161.67***
14	***Salford***	***58.49***	***25.00***	***25.29***	***32.00***	***97.00***	***92.00***	***147.93***

Data in italics shows inefficient DMUs and data in bold shows different target values (MPS) as compared to DEA calculations.

21.8% and 10.6% from the current levels to the *MPS*, respectively. On the other hand, contacts and registrations can be further decreased by 22.7% and 33.2%.

By using the interactive tradeoff analysis procedure for the other bank branches, similar tradeoff preferences can also be elicited from the respective branch managers and consequently the *MPS* for each branch can be found with a corresponding set of target levels, as shown in Table 10. For the Swinton and Chorlton branches, the outputs of customer service and commercial income could not be further improved as shown in the maximum composite output payoff table. As for the other efficient branches of Oldham Road, King Street, High Street, Clayton and Oxford Road, due to the preferences of the branch managers, the *MPS*s provide new sets of efficient solutions or target values that lie on the efficient frontiers and are different from the original data values. Likewise, the inefficient branches of Trafford Park, Royal Exchange, University, Stretford, Didsbury, Eccles and Salford have different *MPS* target values compared to the *DEA* composite target values due to the different preferences of the branch managers.

3.3. Group management planning

The individual *MPS*s were generated by individual branch managers independently and may not necessarily be preferred by the director of the head office or other branch managers. Thus, a *GMPS* needs to be determined that should in a sense be preferred by all managers and the director. In practice, the director of the head office could select one of the efficient branches for all the 14 branches to benchmark against. Alternatively, Eq. (4) could be used to calculate the levels of outputs as a starting point for the director and the group of branch managers to decide a *GMPS* in a negotiation process.

As shown in the third column of Table 11, the composite input in Eq. (5) is calculated using the input weights generated from the output oriented *CCR* primal model and the original input values shown in Table 1. As shown in columns 4–7 of Table 11, the marginal efficiency output and normalised marginal efficiency output are calculated using Eq. (5) and the output levels of the individual *MPS* values given in Table 10. The generated output levels of the *GMPS* are shown in the last two columns of Table 11.

It is possible that the *GMPS* output targets may be higher than the targets set by an individual branch, which means that the *GMPS* point may be an infeasible solution for the branch. On the other hand, if the *GMPS* output targets are lower than the *MPS* output targets of a

Table 11
GMPS output values.

DMU	Branch	Composite input	Mariginal efficiency output		Normalised marginal efficiency output		GMPS	
			y1	*y2*	*y1*	*y2*	*y1*	*y2*
1	Oldham Road	1.00	90.00	186.13	0.08	0.09	89.86	156.74
2	Trafford Park	1.23	73.33	160.75	0.06	0.08		
3	King Street	1.00	75.09	146.20	0.06	0.07		
4	Swinton	1.00	93.00	143.00	0.08	0.07		
5	Royal Exchange	1.35	69.05	134.05	0.06	0.07		
6	High Street	1.00	94.99	155.20	0.08	0.08		
7	University	1.12	84.98	138.52	0.07	0.07		
8	Clayton	1.00	94.99	156.12	0.08	0.08		
9	Oxford Road	1.00	84.95	135.04	0.07	0.07		
10	Stretford	1.05	76.51	134.78	0.06	0.07		
11	Didsbury	1.01	86.32	133.87	0.07	0.07		
12	Chorlton	1.00	94.00	137.00	0.08	0.07		
13	Eccles	1.04	86.12	154.71	0.07	0.08		
14	Salford	1.12	82.05	131.94	0.07	0.06		

Table 12
Organisational weights and *LMPS* of university branch.

Weights			GMPS			Local MPS		
w1	0.67		y1	89.86		y1	90.24	
w2	0.33		y2	156.74		y2	155.98	
LMPS	Branch	x1	x2	x3	x4	x5	y1	y2
7	University	60.0	20.0	36.9	29.0	140.0	90.2	156.0

branch, then the *GMPS* may be an inefficient solution for the branch. In order to investigate this issue, the *GMPS* should be mapped back to the local feasible space of each branch to find a *LMPS* for each branch.

The *LMPS* procedure requires group preference information provided by the head-office of the bank. Aligning to the organisational goals and priorities, weights can be assigned to reflect the relative importance of the outputs or objectives. As shown in Table 12, customer service is seen by the bank to be twice more important than commercial income. Then, using the assigned weights and the *GMPS* as the reference point, the *LMPS* for the University branch can be generated using the shortest distance model (6) as shown in Table 12. The *LMPS* results show that the target output levels for customer service and commercial income for the University branch are 90.24 and 155.98 respectively and the target input values for business reviews, contacts, registrations, KPI and FVA are 60.00, 20.0, 36.91, 29.00 and 140.0, respectively.

As discussed above, however, it is possible that the calculated *LMPS* may not lie on the efficient frontier. Therefore, it is necessary to test whether the *LMPS* are efficient by solving the *CCR dual* model or the super-ideal point model with the *LMPS* of the observed *DMU* included as an additional *DMU* in the reference set. The resultant efficient *LMPS* points are shown in Table 13. It is clear from Table 13 that four out of the fourteen branches' *LMPS*s are inefficient solutions. Here, the efficient *LMPS* points provide new realistic target input and output values that each individual branch should benchmark against, which take account of both individual branch manager preferences as well as the preferences of the group and the head-office.

A full analysis of the target levels and areas for improvement of the University branch is shown in Table 14. The original output targets technically generated by the *CCR* model are 93.40 and 156.51 for customer service and commercial income. In this case, customer service and commercial income could be improved by 19.7% and 11.8% from its current levels, while input targets could also be improved by 32.6%

Table 13
Efficient *LMPS* point for all bank branches.

DMU	Branch	Inputs					Outputs		
		Business					Customer	Commercial	Test of
		Review *x1*	Contacts *x2*	Registrations *x3*	KPI *x4*	FVA *x5*	Service *y1*	Income *y2*	Efficiency %
1	Oldham Road	57.5	16.0	40.0	30.1	161.6	91.4	159.4	98.4%
2	Trafford Park	59.6	18.2	36.8	31.0	156.6	93.8	163.7	95.8%
3	King Street	47.0	18.7	34.0	29.0	176.1	80.8	138.7	100%
4	Swinton	60.0	15.0	38.0	22.0	164.0	93.0	143.0	100%
5	Royal Exchange	59.5	17.6	37.0	30.1	158.8	93.1	162.4	96.5%
6	High Street	58.2	25.1	27.3	35.1	98.0	89.9	156.7	100%
7	University	59.6	20.0	30.2	29.0	140.0	92.6	156.3	100%
8	Clayton	57.8	22.2	25.0	31.0	121.2	90.4	155.7	100%
9	Oxford Road	46.7	13.0	43.3	31.1	140.0	81.0	139.0	100%
10	Stretford	51.0	21.4	23.8	28.0	115.0	81.2	139.4	100%
11	Didsbury	54.8	21.0	22.9	26.0	108.0	86.9	135.0	100%
12	Chorlton	58.0	27.0	16.0	29.0	82.0	94.0	137.0	100%
13	Eccles	58.0	20.2	30.0	30.9	136.0	90.9	158.5	98.9%
14	Salford	57.6	25.0	22.5	32.0	97.0	90.8	149.1	100%

Table 14
DEA, MPS and *LMPS* target values for university branch.

	Inputs					Outputs	
	Business					Customer	Commercial
	Review	Contacts	Registrations	KPI	FVA	Service	Income
Evaluated unit	60.00	30.00	44.00	29.00	140.00	78.00	140.00
DEA composite unit	60.00	20.21	30.26	29.00	140.00	93.40	156.51
Improvement	0.00	9.79	13.74	0.00	0.00	15.40	16.51
%	**0.0**	**32.6**	**31.2**	**0.0**	**0.0**	**19.7**	**11.8**
MPS targets	60.00	23.19	29.40	29.00	140.00	95.01	154.86
Improvement	0.00	6.81	14.60	0.00	0.00	17.01	14.86
%	**0.0**	**22.7**	**33.2**	**0.0**	**0.0**	**21.8**	**10.6**
LMPS efficient targets	59.58	19.99	30.23	29.00	140.00	92.64	156.35
Improvement	0.42	10.01	13.77	0.00	0.00	14.64	16.35
%	**0.7**	**33.4**	**31.3**	0.0	**0.0**	**18.8**	**11.7**

and 31.2% for contacts and registrations. However, the branch manager of the University branch realises that the branch should focus on improving customer service while slightly sacrificing commercial income. This view is reflected in the preferences provided in the interactive trade-off analysis procedure with the *MPS* set at 95.01 and 154.86 for customer service and commercial income respectively. Therefore, if the value judgements of the branch manager are considered only, the University branch should improve the output levels of customer service and commercial income by 21.8% and 10.6%, while the input levels of contacts and registrations should be decreased by 22.7% and 33.2%.

On the other hand, by taking into account the preferences of the group director and the other branch managers, the *LMPS* efficient targets for the University branch are set at 92.64 and 156.35, leading to a possible improvement of 18.8% and 11.7% for customer service and commercial income with the input levels of business review, contacts and registrations decreased by 0.7%, 33.4% and 31.3% respectively. Apparently, both the *LMPS* target output and input levels are lower than the *DEA* target levels. This implies that after considering the views of the branch managers, the head-office may be more concerned with the cost efficiency of minimising input levels than with profit efficiency of output maximisation, resulting in lower target output levels and lower target inputs as well.

3.4. Interpretation of efficient frontier, efficiency score and trade-off analysis

The *MOLP* model for the University branch is constructed from Table 1 as follows.

$$\max f = \begin{bmatrix} f_1(\lambda) = 88\lambda_1 + 88\lambda_2 + 102\lambda_3 + 93\lambda_4 + 80\lambda_5 + 89\lambda_6 + 78\lambda_7 + 98\lambda_8 + 88\lambda_9 \\ \quad +80\lambda_{10} + 80\lambda_{11} + 94\lambda_{12} + 92\lambda_{13} + 82\lambda_{14} \\ f_2(\lambda) = 200\lambda_1 + 91\lambda_2 + 111\lambda_3 + 143\lambda_4 + 101\lambda_5 + 173\lambda_6 + 140\lambda_7 + 155\lambda_8 \\ \quad +132\lambda_9 + 130\lambda_{10} + 134\lambda_{11} + 137\lambda_{12} + 71\lambda_{13} + 132\lambda_{14} \end{bmatrix}$$

$$\text{s.t. } \lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}]^T \in \Omega_7$$

$$\Omega_7 = \left\{ \lambda \left| \begin{array}{l} 60\lambda_1 + 60\lambda_2 + 47\lambda_3 + 60\lambda_4 + 60\lambda_5 + 60\lambda_6 + 60\lambda_7 + 60\lambda_8 + 47\lambda_9 \\ \quad +51\lambda_{10} + 56\lambda_{11} + 58\lambda_{12} + 58\lambda_{13} + 60\lambda_{14} \leqslant 60 \\ 16\lambda_1 + 20\lambda_2 + 30\lambda_3 + 15\lambda_4 + 23\lambda_5 + 26\lambda_6 + 30\lambda_7 + 30\lambda_8 + 13\lambda_9 \\ \quad +27\lambda_{10} + 21\lambda_{11} + 27\lambda_{12} + 22\lambda_{13} + 25\lambda_{14} \leqslant 30 \\ 40\lambda_1 + 50\lambda_2 + 39\lambda_3 + 38\lambda_4 + 44\lambda_5 + 37\lambda_6 + 44\lambda_7 + 25\lambda_8 + 50\lambda_9 \\ \quad +34\lambda_{10} + 42\lambda_{11} + 16\lambda_{12} + 30\lambda_{13} + 35\lambda_{14} \leqslant 44 \\ 38\lambda_1 + 39\lambda_2 + 29\lambda_3 + 22\lambda_4 + 34\lambda_5 + 42\lambda_6 + 29\lambda_7 + 31\lambda_8 + 32\lambda_9 \\ \quad +28\lambda_{10} + 26\lambda_{11} + 29\lambda_{12} + 31\lambda_{13} + 32\lambda_{14} \leqslant 29 \\ 190\lambda_1 + 225\lambda_2 + 228\lambda_3 + 164\lambda_4 + 190\lambda_5 + 98\lambda_6 + 140\lambda_7 + 130\lambda_8 + 140\lambda_9 \\ \quad +115\lambda_{10} + 108\lambda_{11} + 82\lambda_{12} + 142\lambda_{13} + 97\lambda_{14} \leqslant 140 \\ \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14} \geqslant 0 \end{array} \right. \right\}$$

Using the methods and procedures investigated by Yang and Xu (2010), we can generate the dual objective space for the University branch. The values of the variables and objectives for the typical points in the objective space are as shown in Table 15.

Table 15
Typical points in dual objective space for university branch.

Composite *DMU*	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	f_1	f_2
1	0.74														64.84	147.37
2		0.62													54.76	56.62
3			0.61												62.63	68.16
4				0.85											79.39	122.07
5					0.74										58.95	74.42
6						0.69									61.45	119.45
7							1.00								78.00	140.00
8								0.94							91.68	145.00
9									0.88						77.44	116.16
10										1.04					82.86	134.64
11											1.05				83.81	140.38
12												1.00			94.00	137.00
13													0.94		86.06	66.42
14														0.91	74.31	119.63
15			0.21	0.27								0.59			101.51	142.06
16				0.27								0.19	0.57		94.97	104.62
17				0.11									0.86		89.21	76.91
18			0.08										0.87		87.26	69.77
19	0.25			0.37								0.4			93.40	156.51
20				0.25				0.72	0.04						97.14	152.34
21			0.02	0.23				0.76							97.27	152.05
22	0.03			0.24				0.73							96.50	153.34
23	0.7											0.08			69.29	151.39
24	0.21			0.17							0.66				87.47	155.52
25	0.61										0.23				71.66	151.99
26	0.71					0.04									66.79	150.49

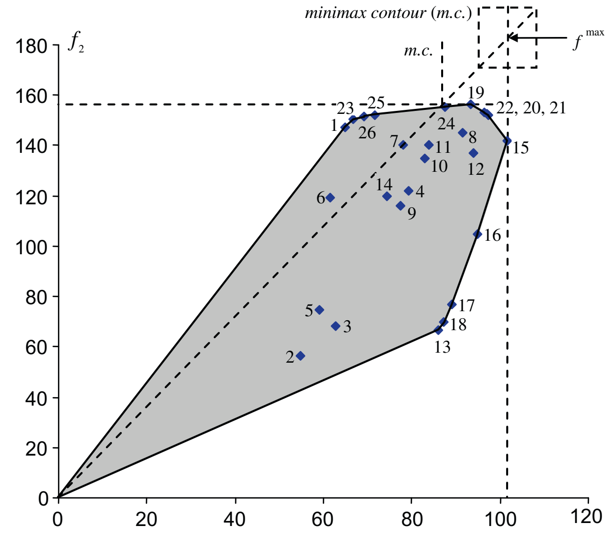

Fig. 1. The dual objective space for university branch.

Note that the values of the variables shown in Table 15 are different from the dual weights shown in Table 2. The former are the values of the valuables at the typical points in the objective space of the MOLP problem defined at the beginning of this section, which are used to draw the dual objective space for University branch as shown in Fig. 1. Also note that solutions 1–14 stand for the 14 original scaled *DMU*s and solutions 15–26 are composite *DMU*s in the objective space for University branch. The feasible dual objective space is the shaded area shown in Fig. 1. The data envelope for the University branch is composed of the following line segments: $\overline{0,1}$, $\overline{1,23}$, $\overline{23,26}$, $\overline{26,25}$, $\overline{25,24}$, $\overline{24,19}$, $\overline{19,22}$, $\overline{22,20}$, $\overline{20,21}$, $\overline{21,15}$, $\overline{15,16}$, $\overline{16,17}$, $\overline{17,18}$, $\overline{18,13}$ and $\overline{13,0}$. One interesting phenomenon is that there are many composite *DMU*s on the data envelope. The super-ideal point f^{max} is also shown in Fig. 1. The minimax contour is a rectangle with f^{max} as its centre and its southwest and northeast corners on the line emitting from the origin through point seven which stands for the University branch.

The efficient frontier for the University branch is enlarged as shown in Fig. 2, which is composted of the following line segments: $\overline{19,22}$, $\overline{22,20}$, $\overline{20,21}$ and $\overline{21,15}$. Solving the super-ideal point model (2) for the University branch is to expand the minimax contour until it just touches the feasible objective space, leading to point 19 which is the same as the *DEA* composite solution for the University branch. Note from Table 15 that points 15, 19, 20, 21 and 22 all represent composite *DMU*s, each composed of three original *DMU*s. This explains why the *DEA* composite solution for the University branch is composed of 0.246 of *DMU* 1, 0.371 of *DMU* 4 and 0.397 of *DMU* 12, although none of these three *DMU*s are on the efficient frontier for the University branch and there are only two outputs.

Another observation is that the *DEA* score of the University branch shown in Table 2 is defined by $e_7 = \overline{0,7}/\overline{0,7'}$ where point 7′ is the southwest corner of the optimal minimax contour with $7' = [87.2, 156.51]^T$. Indeed, $e_7 = \overline{0,7}/\overline{0,7'} = \sqrt{78^2 + 140^2}/\sqrt{87.2^2 + 156.51^2}$=0.895, the same result as shown in Table 2. However, point 7′ is infeasible, and also point 19 rather than point 7′ was suggested as the *DEA* composite unit for the University branch, as shown in Table 3. Thus, the efficiency measure is inconsistent with the target setting. As such, the technical efficiency score as investigated by Yang and Xu (2010) should be used as the efficiency measure for the University branch, given by

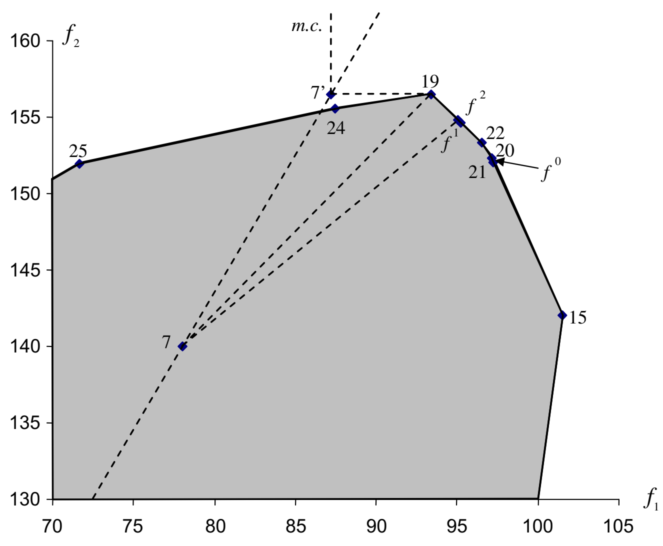

Fig. 2. Efficient frontier for university branch.

Table 16
Over-estimation of *DEA* efficiency score.

Inefficient *DMU*	e_i	TES_i	Over estimation
2	0.815	0.636	0.179
5	0.742	0.688	0.054
7	0.895	0.877	0.018
10	0.956	0.956	0
11	0.992	0.957	0.035
13	0.957	0.618	0.339
14	0.892	0.892	0

$$TES_7 = \overline{0,7}/(\overline{0,7}+\overline{7,19}) = \sqrt{78^2+140^2}\Big/\left(\sqrt{78^2+140^2}+\sqrt{(93.4-78)^2+(156.51-140)^2}\right) = 0.877$$

So $e_7 > TES_7$. In other words, the original *DEA* score for the University branch is over-estimated by 0.018. Similarly, we can calculate the technical efficiency scores for all the inefficient *DMU*s. As summarised in Table 16, out of the seven inefficient *DMU*s only e_{10} for *DMU* 10 and e_{14} for *DMU* 14 are not over-estimated.

The interactive trade-off process discussed in Section 3.2 is also shown in Fig. 2. The process was started at $f^0 = [97.21, 152.20]^T$, which lies on the line segment $\overline{20,21}$ with its normal vector $\vec{N}_{\overline{20,21}} = [0.29, 0.13]^T$ in parallel with $\vec{N}^0$ as given in Section 3.2. The process was then moved to $f^1 = [95.23, 154.64]^T$ and finally terminated at $f^2 = [95.01, 154.86]^T$, both on the line segment $\overline{19,22}$ with its normal vector $\vec{N}_{\overline{19,22}} = [3.17, 3.1]^T$ in parallel with $\vec{N}^1$ and $\vec{N}^2$. The preferred efficiency score (Yang and Xu, 2010) for the University branch is given by

$$PES_7 = \overline{0,7}/(\overline{0,7}+\overline{7,f^2}) = \sqrt{78^2+140^2}\Big/\left(\sqrt{78^2+140^2}+\sqrt{(95.01-78)^2+(154.86-140)^2}\right) = 0.878$$

4. Concluding remarks

This paper reported the application of the *HMRP-DEA* approach to the performance analysis and target setting of the 14 branches of a major retail bank in Greater Manchester of England. The *HMRP-DEA* approach incorporates the *DM*s' preference information in an interactive fashion without necessarily requiring prior value judgments, and it combines the strengths of DEA as an ex-post-facto evaluation tool for assessing past performances and the minimax reference point models as an ex-ante management planning tool for future target setting. In the *HMRP-DEA* approach, each bank branch is regarded to be unique by itself and different branches can have different sizes and targets for specific market segments.

The super-ideal point model was used to conduct efficiency analysis in the same way as the output oriented *CCR* dual model as the two models are identical under certain conditions, whilst the former leads to the construction of a generic *MOLP* formulation from which different *MOLP* solution schema could be applied to search for efficient solutions. In particular, the ideal point model was used to design an interactive tradeoff analysis procedure based on the gradient projection to locate the *MPS* along the efficient frontier for each *DMU*, which can maximise the *DM*'s implicit utility function. As such, the targets identified through this procedure are not only technically achievable but also have the decision maker's value judgements taken into account in a progressive fashion. The group management planning process is based on the shortest distance model and leads to the further improvement of the input and output target levels with both the individual and group preferences taken into account. The graphical investigation of the case study provided the unprecedented insight into the integrated efficiency and trade-off analyses. This investigation shows that the *HMRP-DEA* approach has the potential to be applied to other types of performance assessment and management planning where both individual and group decision makers' preferences need to be taken into account.

Note, however, that the current research is limited to the CCR model with the assumption of constant returns to scale. In many real world problems, variable return to scale (VRS) may be a more appropriate assumption. Research has been undertaken to generalise the procedure investigated in this paper to VRS cases. From a practical point of view, this case study was conducted with the researchers as facilitators and the bank managers as decision makers. This is probably one way for real life applications of the methodology with consultants equipped with appropriate knowledge and software tools acting as facilitators. The computational requirements for implementing this methodology mainly include a linear optimiser, a database and a graphical user-friendly interface. Preliminary efforts have been made to develop such a decision support system (DSS) (Wong et al., 2009).

This paper is a first step towards validating the newly developed method for integrated performance assessment and planning (Yang et al., 2009) using a real life performance management problem together with the performance management director and branch managers of a corporation in London and Manchester, supported by the researchers. The principle and procedure of the integrated performance management and planning is well received by the practitioners. Due to the early stage development of the software tool used in the case study, however, the practitioners were not supposed to complete the whole process without support from the researchers. However, this should not be a deferring factor for the application of the method, because performance planning for a large corporation is strategic decision making and it is not unusual to get independent consultants involved to support it. On the other hand, with the ever increasing power and versatility of modern computing and graphical human–machine interface design, we believe that the seemingly complicated procedure can be made easy and applicable by real decision makers with little support from consultants eventually, because most of the calculations can be done automatically, except for the interaction for preference elicitation from the decision maker. With the use of a DSS (Wong et al., 2009), for example, the main focus of the decision maker can be directed to providing indifference trade-offs or marginal rates of substitutions using domain specific knowledge in a natural and meaningful way. To achieve wide applications of the method, however, more work is needed to develop powerful software tools with user-friendly interfaces.

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