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Capital utilization, economic growth and convergence

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Abstract

Optimal decisions by economic agents regarding the utilization of capital lead to empirically plausible speeds of convergence in one-sector models of economic growth. The relationship between depreciation and capital utilization plays a crucial role in slowing down convergence to the steady state. Cross-country differences in the extent to which the capital utilization decision is internalized along the transition path may lead to differences in convergence rates, even for countries with similar initial and terminal conditions. Finally, by assuming a constant depreciation rate and full capital utilization, standard growth models may be overstating the magnitude of the steady-state equilibrium.

JEL Classification: D9; E0; E2.

Keywords: Economic growth; Capital utilization; Speed of convergence; Depreciation; User cost

1. Introduction

This paper examines the implications of capital utilization for the dynamics of growth and convergence. The concept of capital utilization as an optimal decision is not new to the macroeconomics literature and, in fact, dates back to the early work

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of Keynes (1936). In developing the notion of 'user cost,' Keynes pointed out that 'user cost constitutes the link between the present and the future. For in deciding his scale of production an entrepreneur has to exercise a choice between using up his equipment now or preserving it to be used later on… .'[1] This observation captures both the essence and importance of capital utilization for the dynamics of growth. A more intense utilization of the existing capital stock would cause higher wear and tear and, as a consequence, increase depreciation costs. This in turn would affect new investment in the future.[2]

Although capital accumulation has been assigned a central role in explaining economic growth, little attention has been paid to the implications of the capital utilization decision for the dynamics of long-run growth.[3] Defining capital utilization as the speed or intensity with which a given stock of capital equipment is operated (for example, the 'workweek' of capital), we can identify two important channels through which capital utilization affects the intertemporal growth path of an economy. First, the flow of output depends not only on the existing stock of capital, but also on the flow of services derived from it, through the firms' decision on the intensity (say, length of time) with which that capital stock must be used. Therefore, the capital utilization decision provides the firm with an extra margin to change output. Second, the rate of depreciation depends on the degree of utilization of the capital stock, and is therefore endogenously determined. Specifically, the higher the rate of capital utilization, the higher will be the associated wear and tear of the capital stock, and the higher will be the rate of depreciation. This is in sharp contrast to the existing growth literature, which treats the rate of depreciation as a constant and assumes that the flow of capital services is a *constant* proportion of the underlying capital stock. A constant depreciation rate implies a zero marginal cost of capital utilization, and therefore it is always optimal for the agent to fully utilize capital. In contrast, in the capital utilization model, optimal behavior by the economic agent causes the marginal cost of utilization to change along with the marginal product of the underlying capital stock being accumulated. This affects not only the rate at which the economy is approaching the steady-state equilibrium, but also the transitional path of new investment and hence future output, as the marginal benefits must be weighed against the marginal costs.

The debate on convergence in the growth literature has mainly revolved around two issues. The first is the speed of convergence, i.e., the rate at which the gap between a country's current and steady state per-capita output is being closed. The second is the nature of the convergence path, and concerns cross-country differences in growth rates and standards of living and whether these differences show tendencies to diminish or increase over time. Numerical calculations based on the

[1] Keynes (1936, pp. 69–70); also quoted in Greenwood et al. (1988).

[2] This insight has been used by Lucas (1970), Smith (1970), Taubman and Wilkinson (1970), Calvo (1975), and Oi (1981) to understand and explain investment behavior and business cycles, and more recently, has found wider application in the context of the real business cycle literature; see Greenwood et al. (1988), Basu (1996), Burnside et al. (1996), and Wen (1998).

[3] A few notable exceptions are Betancourt and Clague (1981), Rumbos and Auernheimer (2001) and Dalgaard (2003).

standard one-sector neoclassical Ramsey model and the two-sector Lucas (1988) endogenous growth model suggest fairly high speeds of convergence, between 7 and 10 percent. However, empirical estimates of the speed of convergence contrast sharply to that implied by theory. Estimates obtained in the recent influential works of Barro and Sala-i-Martin (1992, 2004), Mankiw et al., (1992), and Sala-i-Martin (1994, 1996) fall in the range of 2–3 percent, thereby implying much slower adjustment to the steady state than implied by theoretical models. The theoretical literature has attempted to reconcile this discrepancy between models and evidence by introducing various additional sources of sluggishness that reduce the implied speed of convergence. For example, Ortiguera and Santos (1997) introduce convex adjustment costs of installing capital in a two-sector endogenous growth model with physical and human capital. Eicher and Turnovsky (1999a, b) and Turnovsky (2004) introduce a second capital good, in the form of knowledge or public capital, into a generalized version of the neoclassical model. These modifications, however, are not without limitations. The adjustment cost framework has been subject to criticisms regarding its empirical relevance (Kydland and Prescott, 1982), and the introduction of multiple capital goods or sectors often lead to analytically complex models (Turnovsky, 2000). However, the fact remains that standard *one-sector* models of growth, without the above additional sources of sluggishness, generate implausibly fast speeds of convergence.

By introducing capital utilization as an optimal decision, this paper attempts to develop a simple framework for analyzing the dynamics of growth and convergence consistent with empirical evidence. Therefore, this reduces the need to incorporate additional sources of sluggishness like adjustment costs or multiple capital goods. However, our aim is not to criticize the above modifications, but to provide a much simpler alternative based on optimal choice that helps us resolve the issue at hand.

In a recent paper, Dalgaard (2003) has shown that the presence of capital utilization dampens the speed of convergence in the neoclassical Solow growth model, and it would be instructive at this point to highlight the value-added of this paper.[4] This paper serves both as a complement and an extension of the insight provided by Dalgaard (2003). We embed the optimal choice of capital utilization in an intertemporal optimizing framework that employs a general production structure from which the neoclassical Ramsey growth model, the 'AK' type endogenous growth model, and the more recent non-scale or 'semi-endogenous' growth model emerge as special cases.[5] We show that once the capital utilization decision is taken into account, these one-sector growth models can generate speeds of convergence that are in line with recent empirical evidence. Since the non-scale growth model is generic in nature, we use it as a useful benchmark against which the predictions of the neoclassical Ramsey and endogenous growth models can be evaluated. Typically,

[4]Another recent contribution is that of Rumbos and Auernheimer (2001), who also introduce endogenous capital utilization in a variant of the neoclassical Solow growth model. However, their analysis has to rely on convex adjustment costs to obtain slow speeds of convergence.

[5]See Jones (1995) for an early version of the non-scale or 'semi-endogenous' growth framework. Recent contributions include Eicher and Turnovsky (1999a, b). Turnovsky (2003b) provides a survey of the three generations of growth models.

the calculated speeds of convergence from standard endogenous growth models are much higher than the ones obtained from their neoclassical or non-scale counterparts. Our numerical analysis, however, shows that under capital utilization, the endogenous growth model yields speeds of convergence that lie not only within the empirically plausible range, but are also *lower* than those in the neoclassical or non-scale models. Further, the relationship between the marginal cost of capital utilization and the speed of convergence turns out to be exactly opposite in the two models.

Another significant contribution of this paper is to illustrate how economies with *similar* initial conditions may approach a *common* long-run equilibrium at *different* speeds of adjustment, depending on their underlying differences in the costs and benefits of capital utilization along the transition path. Therefore, internalizing the capital utilization decision is not only important for the speed of convergence, but also for the convergence path. Finally, we also show that by not allowing for depreciation to be sensitive to capital utilization, the standard growth models may be overstating the magnitude of the long-run equilibrium. The equilibrium values of the endogenous depreciation and capital utilization rates generated by our numerical simulations are consistent with their corresponding empirical estimates, thereby underscoring the importance of capital utilization for economic growth.

2. Capital utilization and depreciation: the empirical evidence

Empirical studies on capital utilization have been based on the Keynesian proposition that higher utilization induces higher depreciation, or 'user cost,' on the margin. Early evidence on this proposition comes from Foss (1963), who notes that the capital stock in U.S. manufacturing has been idle most of the time, even in periods of economic prosperity. Marris (1964) provides similar evidence for the U.K., and argues that much of the observed idleness reflects the ex ante intention to leave capital idle, owing to higher expected operating costs.[6] More recently, Foss (1981a, b, 1995) finds that the average workweek of capital in the U.S. increased about 25 percent between 1929 and 1976, and by 8.1 percent over 1976–1988. Similar upward trends have also been documented by Nadiri and Rosen (1969), Taubman and Gottschalk (1971), Orr (1989), and Beaulieu and Mattey (1998). These findings have recently been complemented by Imbs (1999), who reports a low correlation of 0.12 between the growth in capital services and the underlying capital stock in the U.S.

Estimates of capital utilization also vary across time and industries. For example, Shapiro (1986) and Orr (1989) report that the average workweek of capital in U.S. manufacturing over the 1952–1982 period was slightly above 50 h/week (out of a maximum of 168 h), with a corresponding capital utilization rate of about 30 percent. On the other hand, Beaulieu and Mattey (1998) estimate the average workweek to be

[6]Further confirmation comes from Christensen and Jorgenson (1969), who find that the rental price on capital includes both the cost of interest as well as the cost of depreciation that is induced by its usage.

97 h over the period 1974–1992, yielding a capital utilization rate of about 58 percent. They also report a great deal of variation across industries, with apparel having the lowest utilization rate of 26.5 percent, while petroleum refining had the highest rate of 93.5 percent. Therefore, it seems difficult to reconcile this evidence with the implicit theoretical assumption that capital services are proportional to the underlying capital stock.

Empirical evidence on capital utilization rates across countries is sparse, but the few studies that do exist document (i) a positive correlation between capital utilization and economic development, and (ii) significant variation in cross-country utilization rates. For example, Kim and Watson (1974) use data for Pakistan, South Korea and the U.S. to find that the rate of capital utilization increases with per-capita income. Betancourt and Clague (1981) studies data from India, Japan, Israel, and France and report that the rate of capital utilization is positively related to the proportion of firms using shift work. Mayshar and Halevy (1997) report an European Commission survey of 24,000 companies in ten European countries conducted in 1991, where the correlation between income per worker and the rate of capital utilization was not only positive, but quite high, at 0.57. Using survey data from 1971–1972, Bautista et al. (1981) find that capital utilization rates in Colombia, Israel, and Malaysia were 42.6, 38.4, and 54.6 percent, respectively. In a similar study for ten European Community (EC) countries, Anxo et al. (1995) report a large variation in utilization rates across Europe. For example in 1989, Germany had the lowest capital utilization rate at 31.5 percent, and Belgium had the highest at 45.8 percent, while the average for all of Europe was about 39 percent. They also report much higher capital utilization rates in U.S. manufacturing industries than in Europe. Imbs (1999) constructs time series on capital and labor utilization rates for ten OECD countries based on a cost-minimization model, and finds a downward trend for the capital utilization rate in Germany, Canada, Australia, Japan, and the U.S.

The endogeneity of the capital utilization and depreciation rates can be viewed as two sides of the same coin. Therefore, the theoretical assumption of a constant depreciation rate is also a debatable issue. Epstein and Denny (1980) use aggregate U.S. manufacturing data to implement an econometric model of endogenous capital utilization and depreciation and are able to strongly reject the standard assumption of a constant depreciation rate. Similar evidence has been provided by Kollintzas and Choi (1985); see Nadiri and Prucha (1996) for a survey of this literature. Abadir and Talmain (2001) estimate time-varying depreciation rates for Japan, Canada, Germany, and the UK. On another vein, Ambler and Pacquet (1994) have suggested that depreciation rates may be subject to stochastic shocks, such as the effects of weather and natural disasters on physical capital and infrastructure. Finally, in a recent study, Gylfason and Zoega (2001) explore the relationship between depreciation and growth using data from the World Bank. In a sample of 85 countries for the 1965–1998 period, they find a positive correlation between depreciation and per-capita income growth. This finding is consistent with the findings of a positive correlation between per-capita income and capital utilization discussed earlier.

3. The analytical framework

We begin by outlining a canonical model of a growing economy with N identical agents, each of whom have an infinite planning horizon. Each agent maximizes lifetime utility from consumption and also produces output, which can be costlessly transformed into a consumption good and investment. The agent supplies one unit of labor at each point of time and optimally chooses the rate of consumption, investment, and the rate of capital utilization, given the stock of available capital at that instant. The labor force, or population (N) grows exponentially at the steady rate n, and the economy experiences labor-augmenting technological progress at an exogenous rate g. There is no government in this economy.

3.1. Production

Production at any instant takes place by means of the flow of services derived from the available capital stock at that instant. In other words, the agent's production and investment decisions take place through the following two channels. First, at any instant, the optimal choice of investment determines the rate of accumulation of new capital. Second, given the available stock of capital, each agent chooses the rate of utilization of that stock. Following Taubman and Wilkinson (1970) and Calvo (1975), we can define the rate of utilization, u, as the intensity (say, the number of hours per day) with which the available stock of capital is utilized. We also assume that the rate of depreciation of capital is an increasing function of the rate of its utilization. In other words, the agent must weigh the marginal benefits of utilizing capital against the marginal costs of higher depreciation. As we will show later, a direct consequence of this is that the agent may find it optimal not to fully utilize capital.

The generic production function for an individual agent, indexed by i, can be written as

$$Y_i = AK_i^s + (K_i^s)^{\alpha}K^{\varepsilon}, \quad A\geqslant 0, 0<\alpha<1, \varepsilon\geqslant 0, \tag{1}$$

where $K_i^s = uK_i$ represents the flow of capital services derived from the available capital stock by agent i, and Y_i denotes the corresponding flow of output produced. Production is also subject to a positive externality through the existence of an economy-wide aggregate stock of capital, K, which is related to the individual stocks by

$$K_i = \frac{K}{EN},$$

where E is an exogenous labor productivity parameter, and EN is a measure of the 'effective' labor force.[7] The spillover effect of the aggregate capital stock on the individual agent can be motivated by appealing to Romer (1986).

[7] The rate of technological progress, therefore, is $\dot{E}/E = g$.

The production structure described in (1) is sufficiently general to yield a variety of one-sector growth models. For example, setting $A = 0$, $\varepsilon = 0$, and aggregating across the effective labor force reduces (1) to the neoclassical production function, where the possibility of sustained long-run growth arises only in the presence of exogenous factors like population growth and technological progress:

$$Y = (uK)^{\alpha}(EN)^{1-\alpha}, \tag{2a}$$

where $Y_i = Y/EN$. On the other hand, when $A > 0$ and $\varepsilon = 0$, (1) generates short-run transitional dynamics through the behavior of the second term on the right-hand side, but in the long-run converges to the 'AK' type linear technology, thereby giving rise to the possibility of sustained endogenous growth. The aggregate production function in this case is

$$Y = AuK + (uK)^{\alpha}(EN)^{1-\alpha}. \tag{2b}$$

The equilibrium properties and the possibilities of sustained long-run growth from this type of production structure have been explored by Jones and Manuelli (1990); also see Barro and Sala-i-Martin (2004). Finally, when $A = 0$ and $\varepsilon > 0$, we get the one-sector counterpart of the 'semi-endogenous' or 'non-scale' growth model, as described in Eicher and Turnovsky (1999a) and Turnovsky (2000):

$$Y = (uK)^{\sigma_K}(EN)^{\sigma_N}, \tag{2c}$$

where $\sigma_K = \alpha + \varepsilon$, and $\sigma_N = 1 - \alpha$.[8] The aggregate returns to scale for this model is $\sigma_K + \sigma_N \equiv 1 + \varepsilon$ and, for long-run stable growth, must be constrained in the following manner:[9]

$$\sigma_K + \sigma_N < 1 + \sigma_N.$$

The rate of depreciation of the capital stock, δ, is sensitive to the choice of capital utilization. As capital is utilized more intensively, its rate of depreciation increases according to[10]

$$\delta(u) = \bar{\mathrm{d}}u^{\phi}, \quad \phi > 1, \quad \bar{d} > 0, \quad 0 \leqslant \delta(u) \leqslant 1, \tag{3}$$

where

$$\delta'(u) > 0, \delta''(u) > 0.$$

The parameter ϕ measures the elasticity of depreciation with respect to the rate of capital utilization.[11] In this framework, the marginal depreciation cost of capital

[8]It is straightforward to see that the neoclassical model emerges as a special case of (2c), when $\varepsilon = 0$ and $\sigma_K + \sigma_N = 1$.

[9]For a detailed description of the stability properties of a one-sector non-scale growth model, see Turnovsky (2000, pp. 518–522).

[10]The 'depreciation-in-use' function in (3) is a standard specification and has been used by Greenwood et al. (1988), Finn (1995), and Burnside and Eichenbaum (1996) in the context of business cycles, and by Imbs (1999), Rumbos and Auernheimer (2001), and Dalgaard (2003) in the context of the neoclassical growth model.

[11]$\phi = u\delta'(u)/\delta(u)$.

utilization, $\delta'(u)$, is variable.[12] This is in contrast to the implicit assumption in the growth literature, where $\delta'(u) = 0$.

3.2. Consumer optimization

The agent maximizes lifetime utility from consumption according to

$$\int_0^\infty \frac{1}{\gamma}(C_i)^\gamma e^{-(\beta-n-\gamma g)t}\,dt, \quad -\infty<\gamma<1, \quad \beta-n-\gamma g>0, \quad n\geqslant 0, \quad g\geqslant 0, \tag{4}$$

where $C_i = C/EN$ denotes consumption per effective worker. In performing the optimization, the agent is constrained by the following flow budget constraint

$$\dot{K}_i = Y_i - [\delta(u) + n + g]K_i - C_i. \tag{5}$$

The agent chooses the rates of consumption and capital utilization to maximize (4) subject to (5), given (1) and (3). The optimality conditions, after aggregating across the effective labor force, are

$$C^{\gamma-1}(EN)^{-\gamma} = \lambda, \tag{6a}$$

$$A + \sigma_K(uK)^{\sigma_K-1}(EN)^{\sigma_N} = \bar{d}\phi\, u^{\phi-1}, \tag{6b}$$

$$\beta - n - \gamma g - \frac{\dot{\lambda}}{\lambda} = Au + \sigma_K u^{\sigma_K} K^{\sigma_K-1}(EN)^{\sigma_N} - \bar{d}u^{\phi}, \tag{6c}$$

$$\dot{K} = AuK + (uK)^{\sigma_K}(EN)^{\sigma_N} - \bar{d}u^{\phi}K - C, \tag{6d}$$

$$\lim_{t\to\infty} \lambda K e^{-\beta t} = 0. \tag{6e}$$

The interpretation of the optimality conditions (6a)–(6e) is standard. Eq. (6a) equates the marginal utility of consumption to the marginal utility of wealth, λ, which is also the co-state variable for the above problem. Eq. (6b) determines the optimal rate of capital utilization by equating its marginal benefit to marginal cost. The left-hand side represents the benefit of increasing the rate of capital utilization on the margin. However, this benefit must be traded off with its opportunity cost, in terms of the higher depreciation it entails, which is given by the right-hand side. Thus, the agent will set the rate of capital utilization at the point where the marginal production benefit equals the depreciation cost. Eq. (6c) equates the marginal return on consumption to the marginal product of capital. Eq. (6d) is the aggregate resource constraint, and (6e) is the familiar transversality condition.

[12]The marginal depreciation cost of capital utilization is $\delta'(u) = \phi\,\bar{d}u^{\phi-1}$.

3.3. The optimal choice of the capital utilization rate

Our aim in this section is to show that as long as the rate of depreciation is sensitive to capital utilization, the agent always finds it optimal to utilize capital 'less than fully', i.e., $0<u<1$, a result that will be crucial to the rest of our analysis. We focus on the equilibrium condition (6b), which determines the optimal rate of capital utilization. For the sake of exposition, we normalize $N = E = 1$ and set $n = g = 0$ for this section. The left-hand side of (6b) denotes the marginal benefit of utilizing capital, for any given level of K, and for all $u \in (0,1)$. Let us denote this by

$$g(u) = A + \sigma_K (uK)^{\sigma_K - 1}, \tag{6f}$$

where $g'(u)<0, g''(u)>0$, $g(0) = \infty$, $g(1) = A + \sigma_K K^{\sigma_K - 1}$, $g'(0) = \infty$, and $g'(1)<0$.

The right-hand side of (6b) denotes the marginal depreciation cost of utilizing capital, for all $u \in (0,1)$. Normalizing $\bar{d} = 1/\phi$, denote this by

$$h(u) = u^{\phi - 1}, \tag{6g}$$

where $h'(u)>0$, $h''(u)>0$ (if $\phi>2$), $h(0) = 0$, $h(1) = 1$, $h'(0) = 0$, and $h'(1)>0$.

Proposition. *There exists an optimal capital utilization rate* $u = u^* \in (0,1)$ *such that* $g(u^*) = h(u^*)$.

Proof. Given that both $g(.)$ and $h(.)$ are continuous and twice-differentiable functions with slopes of opposite sign, the proof will be complete by establishing that $g(0)>h(0)$ and $g(1)<h(1) = 1$. From (6f) and (6g), it can easily be established that $g(0)>h(0)$, since $g(0) = \infty$ and $h(0) = 0$. To show that $g(1)<h(1) = 1$, we parameterize $g(u)$ for the non-scale and endogenous growth models. For the non-scale growth model, $A = 0$. Since (6b) is an equilibrium condition, we note that for $u = 1$, $\sigma_K K^{\sigma_K - 1} = \beta + 1/\phi$ in steady state. Using these conditions, we see that in equilibrium, $g(1)<1$, under the mild condition that $\beta<(\phi - 1)/\phi$. In the endogenous growth model, with $A>0$ and sustained positive growth, $K \to \infty$. Therefore, $g(1) = A$. Imposing the mild restriction that $A<1$, we have $g(1)<1$. Therefore, there exists an optimal $u = u^* \in (0,1)$, such that $g(u^*) = h(u^*)$. □

The above result is graphically illustrated in Fig. 1, which plots the marginal benefit and costs of capital utilization as functions of the rate of utilization, u, for any given stock of capital. Marginal benefit, or $g(u)$, is shown by the downward-sloping locus MB. The marginal depreciation cost of utilization, $h(u)$, is depicted by the upward-sloping locus MC. The equilibrium condition (6b) ensures that the MB and MC curves intersect to yield an interior equilibrium for $u \in (0,1)$. Therefore, as long as the rate of depreciation is sensitive to the rate of capital utilization, the agent will find it optimal not to utilize capital fully.

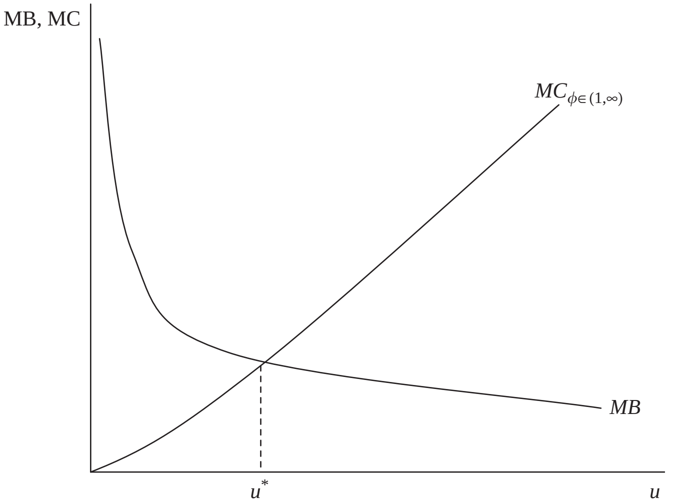


Fig. 1. The optimal choice of the capital utilization rate.

4. Capital utilization in the non-scale and neoclassical Ramsey growth models

The optimality conditions (6a)–(6b) reduce to those of the non-scale model when $A = 0$. Further, the neoclassical Ramsey model emerges as a special case when we set $\varepsilon = 0$, so that $\sigma_K = \alpha$. Therefore, the possibility of long-run growth arises only in the presence of an exogenous factor like population growth or technological progress. We begin by expressing the equilibrium conditions in terms of the following 'scale-adjusted' stationary variables:

$$k = \frac{K}{(EN)^{\sigma_N/1-\sigma_K}}, \quad c = \frac{C}{(EN)^{\sigma_N/1-\sigma_K}}.$$

Note that, for the neoclassical Ramsey model, $\sigma_N = 1 - \sigma_K$, so that the above variables have standard per-capita interpretations. Using the above definitions, we can solve for the optimal rate of capital utilization from (6b) as

$$u(k) = (\phi\bar{d})^{1/(\sigma_K-\phi)}(\sigma_K k^{\sigma_K-1})^{1/(\phi-\sigma_K)}. \tag{7}$$

From (7) it is evident that the rate of utilization and the marginal product of capital are positively related. Therefore, when the marginal product of the aggregate capital stock is high, the agent will utilize it more intensively than when the marginal product is low. Substituting (7) into (3), we get the reduced form depreciation function:

$$\delta(k) = \bar{d}[u(k)]^{\phi}. \tag{8}$$

Therefore, both the rate of capital utilization and the rate of depreciation are increasing functions of the marginal product of capital and decreasing functions of the aggregate capital stock. Substituting (7) into (2c) yields the reduced form production function:

$$f(k) = B^{\sigma_K} k^{\eta}, \tag{9}$$

where $B = (\sigma_K/\phi\bar{d})^{1/\phi-\sigma_K}$ and $\eta = \sigma_K(\phi-1)/\phi-\sigma_K$.

η represents the reduced form output elasticity of the *utilized* capital stock. It depends on both the output elasticity of capital, σ_K, and the elasticity of depreciation with respect to utilization, ϕ. Note that η is less than σ_K, the output elasticity of the *aggregate* capital stock.[13] In the limit, as $\phi \to \infty$, η approaches σ_K.

The steady-state levels of the scale-adjusted consumption and the utilized capital stock are given by[14]

$$\tilde{k} = \left[\left(\frac{\phi}{\sigma_K B^{\sigma_K}(\phi-1)}\right)\left(\beta-(1-\gamma)\left(\frac{(1-\sigma_K-\sigma_N)n-\sigma_N g}{1-\sigma_K}\right)\right)\right]^{1/(\eta-1)}, \tag{10a}$$

$$\tilde{c} = \left[\frac{\beta-(1-\gamma)\{(1-\sigma_K-\sigma_N)n-\sigma_N g\}/(1-\sigma_K)}{\eta} - \left(\frac{\sigma_N}{1-\sigma_K}\right)(n+g)\right]\tilde{k}. \tag{10b}$$

Using (10) in (7) and (8) in we can derive the steady-state capital utilization and depreciation rates:

$$\tilde{u} = \left[\frac{\beta-(1-\gamma)\{(1-\sigma_K-\sigma_N)n-\sigma_N g\}/(1-\sigma_K)}{\bar{d}(\phi-1)}\right]^{1/\phi}, \tag{11a}$$

$$\tilde{\delta} = \frac{\beta-(1-\gamma)\{(1-\sigma_K-\sigma_N)n-\sigma_N g\}/(1-\sigma_K)}{\phi-1}. \tag{11b}$$

For the neoclassical model, we set $\sigma_K = 1-\sigma_N$. The above expressions then reduce to

$$\tilde{u} = \left(\frac{\beta+(1-\gamma)g}{\bar{d}(\phi-1)}\right)^{1/\phi}; \quad \tilde{\delta} = \frac{\beta+(1-\gamma)g}{(\phi-1)}. \tag{11c}$$

Note that the steady-state depreciation rate is a decreasing function of ϕ. The relationship between the equilibrium capital utilization rate and ϕ, however, merits some comment. When ϕ is near its lower bound ($\phi \to 1$), $\tilde{u}$ is very large and approaches infinity. However, as ϕ increases above $1+\bar{d}/\beta$, $\tilde{u}$ falls below 1 and declines for low values of ϕ. As ϕ becomes large, $\tilde{u}$ rises and, in the limit, as $\phi \to \infty$, $\tilde{u} \to 1$.[15] Also note that the equilibrium capital utilization and depreciation rates in the non-scale model depend on both the population growth rate and the rate of technological progress. However, in the neoclassical Ramsey model, they only

[13] This is consistent with the recent empirical findings of Dowrick and Rogers (2002), who estimate η to be in the range of 0.17–0.23 in a panel of 51 countries for the 1970–1990 period, while the estimates of α, at least for the U.S., is in the range of 0.3–0.4.

[14] The formal derivations are provided in Appendix A.

[15] Further discussion and illustrations are provided in Appendix A.

depend on the rate of technological progress, but are independent of the population growth rate.

It can easily be demonstrated that the steady-state equilibrium is a saddle-point with one negative and one positive root. The negative root, say $\tilde{\mu}$, which by definition is the speed of convergence to the steady-state equilibrium, is given by

$$\tilde{\mu} = \tilde{\mu}(\beta, \phi, \gamma, \eta, g, n). \tag{12}$$

The convergence rate $\tilde{\mu}$ not only depends on the preference, production, population growth, and technological progress parameters of the economy, but also on the output-elasticity of the *utilized* capital stock (η) and consequently on the elasticity of depreciation with respect to capital utilization (ϕ), a feature absent from the traditional growth-convergence literature. It can easily be demonstrated that the convergence rate is a decreasing function of ϕ.

The rate of growth of the scale-adjusted capital stock along the transition path to the steady-state equilibrium is a one-dimensional locus given by

$$\begin{aligned}\frac{\dot{k}}{k} &\equiv \psi_k \\ &= \frac{1}{(1-\gamma)}\left[\sigma_K B^{\sigma_K} k^{\eta-1} - \bar{d}B^{\phi}k^{\eta-1} + (1-\gamma)\left\{\frac{(1-\sigma_K-\sigma_N)n - \sigma_N g}{1-\sigma_K}\right\} - \beta\right]\end{aligned} \tag{13}$$

From (13) we see that the growth rate depends not only on the dynamic adjustment path of the marginal product of utilized capital (the first term in the parenthesis on the right-hand side), but also on the endogenous depreciation rate (the second term), which is no longer constant, as in standard growth models, but is a convex function of the underlying capital stock. Diminishing returns to capital ensure that in the steady state the net marginal product of capital converges to the equilibrium depreciation rate, so that the steady-state growth rate of the scale-adjusted capital stock is zero, i.e., $\tilde{\psi}_k = 0$. The steady-state growth rate of the aggregate capital stock is then given by

$$\frac{\dot{K}}{K} = \left(\frac{\sigma_N}{1-\sigma_K}\right)(n+g).$$

The transitional dynamics of the rate of capital utilization can be obtained from (7)

$$\psi_u = \frac{\dot{u}}{u} = -\left(\frac{1-\sigma_K}{\phi-\sigma_K}\right)\psi_k. \tag{14}$$

Therefore, in transition, the *growth* rate of capital utilization is inversely related to that of the underlying capital stock. This is consistent with the empirical findings of Nadiri and Rosen (1969) and Beaulieu and Mattey (1998). Imbs (1999) finds that, on average, the capital utilization rate in the U.S. displays a mild downward trend of 0.03 percent per quarter. This result can be rationalized in the following manner. In the initial stages of development, when the capital stock is low, the growth rate of its utilization is at a high level, due to the high marginal product. As capital grows towards its steady-state level, its marginal product declines, and consequently the growth rate of its utilization declines over time towards its steady-state level.

4.1. Implications of the capital utilization decision

Henceforth, we will refer to the model incorporating the capital utilization decision as the 'capital utilization model', and the model that assumes constant depreciation and full (or fixed) capital utilization as the 'standard model'. We adopt a two-pronged strategy for our analysis. First, we would like to compare the speed of convergence and the magnitude of the steady-state equilibrium in the two models in order to isolate the long-run effect of the capital utilization decision. Second, we would like to capture solely the effect of agents' internalizing the tradeoff between higher capital utilization and higher depreciation along the transition path, a feature that is absent from the standard growth model.

What implications does the capital utilization decision have on the steady-state equilibrium? To answer this question, consider the standard neoclassical growth model with a *constant* depreciation rate $\bar{\delta}$. Since the depreciation rate is constant, the marginal cost of utilization is zero, and the capital utilization decision is independent of the rate of depreciation. Therefore, the agent will fully utilize capital at each instant, i.e., $\bar{u} = 1$, for all t. However, for the moment, let us relax the condition that $\bar{u} = 1$and just assume that the rate of utilization is *constant* in the standard model and is equal to $u = \bar{u}$, for all t with $0 < \bar{u} \leqslant 1$. In order to make the comparison between the two models meaningful, we will first control for the steady-state depreciation rate. Therefore, in the standard model, we set $\bar{\delta} = \tilde{\delta}$, the steady-state depreciation rate in the capital utilization model, for all t. The standard growth model with a constant depreciation rate $\bar{\delta} = \tilde{\delta}$, and a fixed capital utilization rate $\bar{u}$, yields a steady-state scale-adjusted capital stock given by

$$\bar{k} = \left(\frac{\beta + \tilde{\delta} - (1-\gamma)\{(1-\sigma_K-\sigma_N)n - \sigma_N g\}/(1-\sigma_K)}{\sigma_K \bar{u}^{\sigma_K}} \right)^{1/\sigma_K - 1}. \tag{15}$$

Comparing (10) and (15) and using (11), we see that

$$\frac{\tilde{k}}{\bar{k}} = \left(\frac{\tilde{u}}{\bar{u}} \right)^{\sigma_K/(1-\sigma_K)}. \tag{16}$$

Therefore, as long as $\tilde{u} \neq \bar{u}, \tilde{k} \neq \bar{k}$. In the standard model, the implicit assumption is that the rate of capital utilization is one, i.e., $\bar{u} = 1$, for all t. Under this condition, (16) implies that $\tilde{k} < \bar{k}$. Similar arguments apply for consumption and output. The higher transitional depreciation costs due to the agent's optimal choice of the time path of capital utilization lead to lower new investment, and as a consequence, a lower steady-state per-capita capital stock, consumption, and output than those implied by the standard growth model where the rate of capital utilization is fixed at unity. Therefore, by ignoring the optimal choice of capital utilization, the standard one-sector growth model may be overstating the magnitude of the steady-state equilibrium. The numerical simulations we conduct in Section 6 will further illustrate the magnitude of this overstatement.

What effect does the agents' internalization of the costs and benefits of capital utilization have on the speed of convergence to the steady-state equilibrium? For the sake of simplicity of exposition, consider the case where $n = g = 0$ (we will, however, relax this restriction while conducting the numerical experiments). Then, the speed of convergence in the capital utilization model is given by

$$\widetilde{\mu} = \frac{\beta - [\beta^2 + (4(\phi - 1)(1 - \eta)(\beta + \widetilde{\delta})/(\phi(1 - \gamma)((\beta + \tilde{\delta}/\sigma_K) - \tilde{\delta}]^{1/2}}{2}. \tag{17a}$$

The speed of convergence implied by the standard model with a *constant* utilization rate ($u = \bar{u} \leqslant 1, \quad \forall t$) is

$$\bar{\mu} = \frac{\beta - [\beta^2 + (4(1 - \sigma_K)(\beta + \tilde{\delta})/(1 - \gamma))((\beta + \tilde{\delta}/\sigma_K) - \tilde{\delta})]^{1/2}}{2}. \tag{17b}$$

Note that in (17b), the speed of convergence in the standard model, $\bar{\mu}$, is independent of the fixed rate of capital utilization, $\bar{u}$. Therefore, as long as the rate of capital utilization is constant, its value will have no bearing on the speed of convergence. As a result, (17b) would apply both to the standard model with $\bar{u} < 1$ and the one with $\bar{u} = 1$, for all t. On the other hand, from (17a) we see that the speed of convergence in the capital utilization model, $\tilde{\mu}$, depends on η, the output elasticity of *utilized* capital, as well as ϕ, the elasticity of depreciation with respect to capital utilization. Comparing (17a) and (17b), we see that as long as $1 < \phi < \infty$, and consequently $\eta < \sigma_K$, we must have $|\tilde{\mu}| < |\bar{\mu}|$. Therefore, the optimal choice of capital utilization slows down the rate at which an economy approaches its steady-state equilibrium.

The intuition behind the above result can be explained as follows. A direct consequence of incorporating the decision to utilize capital on the margin is an endogenous depreciation rate whose time path depends on that of capital. Specifically, from (8) we see that the depreciation rate is a decreasing and convex function of the capital stock. As the capital stock approaches its steady-state level, the capital utilization and depreciation rates gradually decline to their respective steady-state values, given by (11). Then, for any level of capital stock $k < \tilde{k}$, we must have $\delta(k) > \tilde{\delta} = \bar{\delta}$. This is shown in Fig. 2. The higher depreciation rate in transition requires a larger proportion of new investment to be devoted to maintaining the existing stock of capital. This reduces the amount of new investment every period, and as a consequence slows down the speed of convergence to the steady state. In contrast, a fixed depreciation rate and capital utilization rate during transition implies a zero marginal cost of utilization, and higher new investment, and consequently, a higher speed of convergence.

It is also evident from our analysis that the rate of capital utilization is a potential determinant of conditional convergence.[16] Even though international evidence of capital utilization is sparse, the studies discussed in Section 2 document large variations in the workweek of capital, both in manufacturing and non-manufacturing industries, across countries and continents. The framework we have developed shows that differences in the degree to which agents internalize the capital utilization decision along the transition path may lead to differences in the speed of 'catching

[16]The notion of conditional convergence refers to the possibility of structurally similar countries with similar initial conditions to converge to a common steady-state per-capita income.

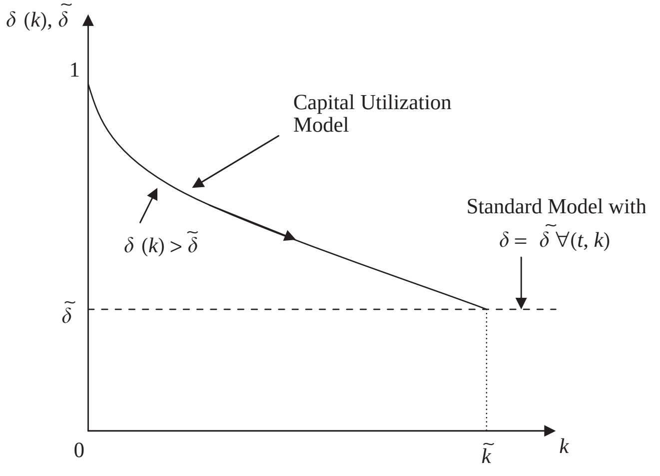


Fig. 2. Depreciation rates and capital utilization in the non-scale model.

up', even for countries that have similar initial and terminal conditions. To illustrate this possibility, we start by controlling for the steady-state equilibrium. From (16) we see that if we control for the steady-state capital utilization rates in the two models, i.e., if $\tilde{u} = \bar{u}$, then $\tilde{k} = \bar{k}$ and consequently, $\tilde{y} = \bar{y}$. We now have an example of two countries that have similar initial and terminal conditions and, therefore, a common per-capita level of income. As an extreme illustration, even though both countries have the same long-run capital utilization rates, we have one country where agents internalize the utilization decision along the transition path (the capital utilization model), and another where this decision is not internalized (the standard model). Then, by comparing (17a) and (17b), we can see that these two countries, starting from the *same* initial conditions, will approach their *common* steady-state per-capita income at *different* speeds of convergence. Stated differently, the greater is the extent of internalization of the capital utilization decision, longer it will take for a country to reach its steady-state equilibrium.

5. Capital utilization in the endogenous growth model

To obtain sustained long-run endogenous growth, we will now revert to the case where $A > 0$ and $\varepsilon = 0(\sigma_K = \alpha)$. This implies that the marginal product of capital is bounded from below in the long run, and provided that, net of depreciation costs, it exceeds the rate of time preference, the model generates sustained positive long-run growth. Since our focus is on endogenous growth, we will normalize $N = E = 1$ and set $g = 0$. Since both the per-capita capital stock and consumption grow continuously in the long-run equilibrium, we need to express the steady-state equilibrium and the dynamics in terms of stationary variables.

Let $y = f(uK)/K$, the output-capital ratio, and $c = C/K$, the consumption-capital ratio, be the stationary variables in terms of which we shall express the equilibrium. The steady-state equilibrium for this model can be expressed as

$$\tilde{c} = \frac{1}{(1-\gamma)}\left[\beta - \frac{\gamma(\phi-1)}{\phi}A\tilde{u}\right] - n, \tag{18a}$$

$$\tilde{u} = \left(\frac{\tilde{y}}{\phi\bar{d}}\right)^{1/\phi} = \left(\frac{A}{\phi\bar{d}}\right)^{1/(\phi-1)}, \tag{18b}$$

$$\tilde{y} = A\tilde{u} = \left(\frac{A^{\phi}}{\phi\bar{d}}\right)^{1/(\phi-1)}, \tag{18c}$$

$$\tilde{\delta} = \frac{1}{\phi}\tilde{y}. \tag{18d}$$

Therefore, in the steady-state equilibrium, the output-capital ratio and the rate of capital utilization converge to constant levels, determined by the long-run marginal product of capital (A) and the elasticity of depreciation with respect to capital utilization (ϕ). The steady-state balanced growth rate is given by

$$\tilde{\psi} = \frac{1}{(1-\gamma)}\left[\left(\frac{\phi-1}{\phi}\right)A\tilde{u} - \beta\right] = \frac{1}{(1-\gamma)}[A\tilde{u} - \tilde{\delta} - \beta]. \tag{19}$$

As in endogenous growth models, the equilibrium growth rate is independent of the rate of population growth, n.[17] It can be verified that the steady-state equilibrium is a saddle-point with one positive and one negative eigenvalue. The negative eigenvalue, which measures the speed of convergence, can be expressed as

$$\tilde{\mu} = -\frac{(1-\alpha)}{(1-\gamma)}\left[\left(\frac{\phi-1}{\phi}\right)A\tilde{u} - \beta\right]. \tag{20}$$

Note that, in contrast to the non-scale and neoclassical growth models, the speed of convergence in the endogenous growth model is a function of the steady-state rate of capital utilization and is also independent of the population growth rate, n. Further, it is an increasing function of the elasticity parameter, ϕ. This is also in contrast to the neoclassical model, where the relationship was exactly the opposite. This is because of the existence of long run constant returns to scale in utilized capital in the endogenous growth model. From (18b)–(18d), we see that an increase in ϕ, in addition to decreasing the depreciation rate, also increases the long run marginal and average product of capital, thereby increasing the steady-state rate of utilization and flow of new investment. This tends to increase the speed of convergence.

[17] It must, however, be noted that this result depends on the specification of per period utility. In the framework we have adopted, individual agents maximize total utility as per the 'Benthamite welfare criteria', resulting in an equilibrium growth rate that is independent of population growth. An alternative specification of utility is the 'Millian welfare criteria', which maximizes the average utility of generations. Palivos and Yip (1993) show that the equilibrium growth rate in the endogenous growth model under the 'Millian' welfare criteria depends negatively on the population growth rate.

5.1. Implications of the capital utilization decision

Since the steady-state equilibrium levels and the speed of convergence in the endogenous growth model are proportional to the steady-state rate of capital utilization, we will compare the capital utilization model with the standard model with full capital utilization, i.e., $\bar{u} = 1$, for all t. This comparison is dictated by the nature of the production function assumed which, for the endogenous growth model, reduces to the linear 'AK' model in the long run.

As in Section 4, we compare the two alternative models by controlling for the long-run depreciation rate. In the standard model with a constant depreciation rate $\bar{\delta} = \tilde{\delta}$, for all t, the long-run growth rate is given by $\bar{\psi} = 1/(1-\gamma)[A - \tilde{\delta} - \beta]$. Comparing this with (20), we see that

$$\tilde{\psi} - \bar{\psi} = \frac{A}{(1-\gamma)}(\tilde{u} - 1) < 0.$$

We can derive similar expressions for the equilibrium output-capital ratio, consumption-capital ratio, and the speed of convergence in the two models:

$$\tilde{y} - \bar{y} = A(\tilde{u} - 1) < 0; \quad \tilde{c} - \bar{c} = -\frac{\gamma A}{(1-\gamma)}[(\tilde{u} - 1)] < 0 \quad \text{if} \quad \gamma < 0;$$

$$|\tilde{\mu}| - |\bar{\mu}| = \frac{(1-\alpha)A}{(1-\gamma)}(\tilde{u} - 1) < 0.$$

Therefore, as in the previous section, ignoring the capital utilization decision leads the standard endogenous growth model to overstate the magnitude of the steady-state equilibrium and the speed of convergence.

6. Capital utilization and convergence: a numerical analysis

We now proceed to a numerical illustration of the implications of endogenous capital utilization for one-sector growth models. Our starting point is to assign numerical values to the structural parameters of the economy that are consistent with corresponding empirical estimates. The rate of time preference β is set at 0.03 and γ, which is a measure of the intertemporal elasticity of substitution, is set at -1.5. On the other hand, the production parameters are parameterized so as to yield either the non-scale (or 'semi-endogenous'), neoclassical or the endogenous growth model. For example, $A = 0$ in the non-scale and neoclassical growth models, but is positive in the endogenous growth model. The output elasticity of individual capital, α, is set at 0.35, which is consistent with estimates of the share of capital in U.S. GDP. The externality coefficient, ε, is set at 0.2 for the non-scale model, while it is zero for the neoclassical and endogenous growth cases.[18] There have been a few

[18]There are no published estimates for ε, but several researchers have estimated the externality associated with other forms of physical capital, such as public capital and infrastructure. These estimates lie in the range of 0.1–0.3; see Gramlich (1994). Our choice of $\varepsilon = 0.2$ for the non-scale model lies in the middle of that estimated range.

attempts in the literature to estimate the elasticity parameter, ϕ. For example, Burnside and Eichenbaum (1996) estimate ϕ to be 1.56 for U.S. manufacturing, while Finn's (1995) corresponding estimate is 1.44. More recently, Dalgaard (2003) uses data for Denmark to find ϕ equal to about 1.7. Finally, Basu and Kimball (1997) note that the upper bound of the 95 percent confidence interval for ϕ is about 2. Given these estimates, one can conclude that an empirically plausible range for ϕ is between 1.4 and 2. For our experiments, we allow ϕ to vary both within this empirically estimated range, as well as to infinity, in order to get some asymptotic results. The scale parameter $\bar{d}$ is set at 0.3.[19] Population growth (n) is assumed to be 1.5 percent, while the rate of technological progress (g) is set at 2 percent, following Barro and Sala-i-Martin (2004).

The approach we will adopt is in line with the theoretical comparisons outlined in Sections 4 and 5. For any given ϕ, the capital utilization model gives the equilibrium capital utilization rate $\tilde{u}$, the equilibrium depreciation rate $\tilde{\delta}$, and the corresponding speed of convergence to this equilibrium, along with the other equilibrium quantities. We then calibrate the standard growth model using the equilibrium solutions obtained from the capital utilization model. The implied speeds of convergence and other relevant equilibrium quantities from the two models are then compared for variations in ϕ, the elasticity of depreciation with respect to capital utilization.

6.1. The non-scale 'semi-endogenous' growth model

Table 1 presents a numerical comparison of the speed of convergence between the standard and the capital utilization versions of the 'semi-endogenous' or non-scale growth model. The presence of exogenous population growth and technological progress typically magnifies the speed of convergence in these models, and one point of interest would be to examine the effect of capital utilization on the speed of convergence, both in the absence and presence of these exogenous factors. Therefore, Tables 1A–C reports speeds of convergence under three alternate scenarios: (i) both technological progress and population growth are positive, i.e., $n = 0.015$, and $g = 0.02$, (ii) population growth is positive, but technological progress is absent, i.e., $n = 0.015$, and $g = 0$, and (iii) both population growth and technological progress are absent, i.e., $n = g = 0$.

In performing the comparisons, we use two insights obtained from our theoretical analysis. First, from (17a) and (17b), we see that the speed of convergence is independent of the steady-state rate of capital utilization. Second, (16) shows that the steady-state equilibrium levels depend on the equilibrium capital utilization rate. Therefore, in order to make the comparison meaningful, we control for the fixed capital utilization and depreciation rates in the standard model. In other words, we

[19]There is no documented estimate for the scale parameter $\bar{d}$ in the depreciation function (3). Therefore, we calibrate this parameter by using estimates of U.S. depreciation and capital utilization rates in the depreciation function $\delta = \bar{d}u^{\phi}$. Estimates of capital depreciation (δ) in the U.S. vary from 12 to 14 percent; see Epstein and Denny (1980), Nadiri and Prucha (1996), and Fraumeni (1997). Therefore, we set $\delta = 0.13$. Using $\phi = 1.7$, which lies in the middle of its estimated range of 1.4–2, and $u = 0.58$ from Beaulieu and Mattey (1998), we can calculate $\bar{d} \approx 0.3$.

Table 1

Speed of convergence in the non-scale growth model $A = 0$, $\beta = 0.03$, $\gamma = -1.5$, $\sigma_K = 0.55$ ($\alpha = 0.35$, $\varepsilon = 0.2$), $\bar{d} = 0.3$

	$\phi = 1.40$ $\tilde{\delta} = .30$, $\tilde{u} = .95$ $\eta = 0.26$	$\phi = 1.45$ $\tilde{\delta} = .26$, $\tilde{u} = .92$ $\eta = 0.28$	$\phi = 1.50$ $\tilde{\delta} = .24$, $\tilde{u} = .86$ $\eta = 0.29$	$\phi = 1.56$ $\tilde{\delta} = .21$, $\tilde{u} = .80$ $\eta = 0.30$	$\phi = 1.60$ $\tilde{\delta} = .20$, $\tilde{u} = .77$ $\eta = 0.31$	$\phi = 1.70$ $\tilde{\delta} = .17$, $\tilde{u} = .72$ $\eta = 0.34$	$\phi = 2$ $\tilde{\delta} = .12$, $\tilde{u} = .63$ $\eta = 0.38$	$\phi \to \infty$ $\tilde{\delta} \to 0$, $\tilde{u} \to 1$ $\eta = 0.55$
A. *Population growth rate: $n = 0.015$; rate of technological progress*: $g = 0.02$								
Cap. utilization model $u = u(K(t))$, $\delta = \delta(K(t))$	0.0906	0.0855	0.0813	0.0771	0.0747	0.0698	0.0603	0.0345
Standard model $\bar{u} = \tilde{u}$, $\bar{\delta} = \tilde{\delta}$, $\forall t$	0.1441	0.1316	0.1217	0.1121	0.1068	0.0962	0.0773	0.0345
	$\phi = 1.40$ $\tilde{\delta} = .12, \tilde{u} = .51$ $\eta = 0.26$	$\phi = 1.45$ $\tilde{\delta} = .1, \tilde{u} = .48$ $\eta = 0.28$	$\phi = 1.50$ $\tilde{\delta} = .09, \tilde{u} = .46$ $\eta = 0.29$	$\phi = 1.56$ $\tilde{\delta} = .08, \tilde{u} = .44$ $\eta = 0.30$	$\phi = 1.60$ $\tilde{\delta} = .07, \tilde{u} = .43$ $\eta = 0.31$	$\phi = 1.70$ $\tilde{\delta} = .06, \tilde{u} = .41$ $\eta = 0.34$	$\phi = 2$ $\tilde{\delta} = .05, \tilde{u} = .39$ $\eta = 0.38$	$\phi \to \infty$ $\tilde{\delta} \to 0, \tilde{u} \to 1$ $\eta = 0.55$
B. *Population growth rate:$n = 0.015$; Rate of technological progress*: $g = 0$								
Cap. utilization model $u = u(K(t)), \delta = \delta(K(t))$	0.0359	0.0340	0.0323	0.0306	0.0297	0.0277	0.0239	0.0137
Standard model $\bar{u} = \tilde{u}, \bar{\delta} = \tilde{\delta}, \forall t$	0.0569	0.0520	0.0481	0.0443	0.0422	0.0381	0.0306	0.0137
	$\phi = 1.40$ $\tilde{\delta} = .08$, $\tilde{u} = .37$ $\eta = 0.26$	$\phi = 1.45$ $\tilde{\delta} = .07$, $\tilde{u} = .35$ $\eta = 0.28$	$\phi = 1.50$ $\tilde{\delta} = .06$, $\tilde{u} = .34$ $\eta = 0.29$	$\phi = 1.56$ $\tilde{\delta} = .05$, $\tilde{u} = .33$ $\eta = 0.30$	$\phi = 1.60$ $\delta = .05$, $\tilde{u} = .326$ $\eta = 0.31$	$\phi = 1.70$ $\delta = .04$, $\tilde{u} = .318$ $\eta = 0.34$	$\phi = 2$ $\delta = .03$, $\tilde{u} = .316$ $\eta = 0.38$	$\phi \to \infty$ $\tilde{\delta} \to 0$, $\tilde{u} \to 1$ $\eta = 0.55$
C. *Population growth rate*: $n = 0$; *rate of technological progress*: $g = 0$								
Cap. utilization model $u = u(K(t))$, $\delta = \delta(K(t))$	0.0204	0.0193	0.0183	0.0172	0.0168	0.0157	0.0135	0.008
Standard model $\bar{u} = \tilde{u}$, $\bar{\delta} = \tilde{\delta}$, $\forall t$	0.0342	0.0311	0.0286	0.0263	0.0250	0.0224	0.0179	0.008

Note: $\tilde{X}$ = Equilibrium quantity in the capital utilization model, $\bar{X}$ = Equilibrium quantity in the standard model.

calibrate the standard model with a *constant* capital utilization rate and depreciation rate, where these rates are equal to those obtained from the equilibrium solution to the capital utilization model. Then, the two models will have the same steady-state equilibrium and the only difference between them would be that in one model the choice of capital utilization is internalized along the transition to the steady state, while in the other it is not. Our numerical calculations will then reveal the effect of this internalization (or the lack of it) on the speed of convergence in the two models.

A general pattern that emerges from Tables 1A–C is that the presence of endogenous capital utilization leads to slower speeds of convergence in the non-scale model, irrespective of the presence or absence of exogenous technological progress and population growth. However, the presence of these exogenous factors does tend to increase the speed of convergence to the steady-state equilibrium. For example, in Table 1A, when $n = 0.015$, $g = 0.02$, and $\phi = 1.56$ (the Burnside–Eichenbaum estimate), the capital utilization model yields an equilibrium utilization rate of 80 percent and a depreciation rate of 21 percent. The speed of convergence to the stationary steady state is about 7.7 percent. On the other hand, the standard model with capital utilization and depreciation rates *fixed* at 80 percent and 21 percent, respectively, yields an even higher speed of convergence of 11.2 percent, to the *same* steady state.

Table 1B presents an interesting comparison between the two models, where $n = 0.015$, but $g = 0$. The calibration of the standard model in this case corresponds exactly to the one-sector non-scale model presented in Turnovsky (2000, p. 518). Therefore, one can analyze the predictive power of the capital utilization model relative to that of the standard model. For example, for $\phi = 1.6$, the capital utilization model yields an equilibrium utilization rate of 43 percent and a depreciation rate of 7 percent. The corresponding speed of convergence in this model is about 2.97 percent, in line with recent empirical estimates. In comparison, the calibrated standard model corresponding to Turnovsky (2000) yields a much higher speed of convergence of about 4.2 percent, which is outside the estimated empirical range of 2–3 percent obtained by Barro and Sala-i-Martin (1992, 2004), Mankiw et al. (1992), and Sala-i-Martin (1994, 1996). As ϕ becomes large, the gap between the standard model and the capital utilization model shrinks, and in the limit, the two rates converge.

Table 1C compares the speed of convergence between the two models in the absence of population growth and technological progress. In this case, the calculations for the convergence rate are the lowest, when compared to Tables 1A and B. Nevertheless, the capital utilization model still implies slower convergence speeds than the standard model.

6.2. The neoclassical Ramsey model

Table 2 conducts an experiment similar to Table 1 for the neoclassical Ramsey growth model, which is a special case of the more general non-scale model. For the neoclassical case, we set $\varepsilon = 0$, and thus $\sigma_K = \alpha = 0.35$. As in the non-scale model, the presence of endogenous capital utilization in the Ramsey model also yields

Table 2
Speed of convergence in the neoclassical Ramsey growth model $A = 0,\quad \beta = 0.03,\quad \gamma = -1.5,\quad \sigma_K = \alpha = 0.35,\quad (\varepsilon = 0),\quad \bar{d} = 0.3$

	$\phi = 1.40$ $\tilde{\delta} = .2,\quad \tilde{u} = .75$ $\eta = 0.13$	$\phi = 1.45$ $\tilde{\delta} = .18,\quad \tilde{u} = .70$ $\eta = 0.14$	$\phi = 1.50$ $\tilde{\delta} = .17,\quad \tilde{u} = .66$ $\eta = 0.15$	$\phi = 1.56$ $\delta = .16,\quad \tilde{u} = .62$ $\eta = 0.16$	$\phi = 1.60$ $\tilde{\delta} = .14,\quad \tilde{u} = .60$ $\eta = 0.17$	$\phi = 1.70$ $\tilde{\delta} = .11,\quad \tilde{u} = .57$ $\eta = 0.18$	$\phi = 2$ $\tilde{\delta} = .08,\quad \tilde{u} = .52$ $\eta = 0.21$	$\phi \to \infty$ $\tilde{\delta} \to 0,\quad \tilde{u} \to 1$ $\eta = 0.35$
C. *Population growth rate: $n = 0.015$; rate of technological progress: $g = 0.02$*								
Cap. utilization model $u = u(K(t)),\quad \delta = \delta(K(t))$	0.1047	0.0994	0.0951	0.0907	0.0881	0.0830	0.0731	0.0448
Standard model $\bar{u} = \tilde{u},\quad \bar{\delta} = \tilde{\delta},\quad \forall t$	0.1816	0.1662	0.1539	0.1421	0.1355	0.1224	0.0989	0.0448

	$\phi = 1.40$ $\tilde{\delta} = .08,\quad \tilde{u} = .37$ $\eta = 0.13$	$\phi = 1.45$ $\tilde{\delta} = .07,\quad \tilde{u} = .35$ $\eta = 0.14$	$\phi = 1.50$ $\tilde{\delta} = .06,\quad \tilde{u} = .34$ $\eta = 0.15$	$\phi = 1.56$ $\tilde{\delta} = .054,\quad \tilde{u} = .33$ $\eta = 0.16$	$\phi = 1.60$ $\tilde{\delta} = .05,\quad \tilde{u} = .32$ $\eta = 0.17$	$\phi = 1.70$ $\tilde{\delta} = .04,\quad \tilde{u} = .318$ $\eta = 0.18$	$\phi = 2$ $\tilde{\delta} = .03,\quad \tilde{u} = .31$ $\eta = 0.21$	$\phi \to \infty$ $\tilde{\delta} \to 0,\ \tilde{u} \to 1$ $\eta = 0.35$
D. *Population growth rate: $n = 0.015$; rate of technological progress: $g = 0$*								
Cap. utilization model $u = u(K(t)),\quad \delta = \delta(K(t))$	0.0398	0.0379	0.0362	0.0346	0.0336	0.0316	0.0279	0.0172
Standard model $\bar{u} = \tilde{u},\quad \bar{\delta} = \tilde{\delta},\quad \forall t$	0.0686	0.0628	0.0582	0.0538	0.0513	0.0464	0.0376	0.0172

	$\phi = 1.40$ $\tilde{\delta} = .08,\quad \tilde{u} = .37$ $\eta = 0.13$	$\phi = 1.45$ $\tilde{\delta} = .07,\quad \tilde{u} = .35$ $\eta = 0.14$	$\phi = 1.50$ $\tilde{\delta} = .06,\quad \tilde{u} = .34$ $\eta = 0.15$	$\phi = 1.56$ $\tilde{\delta} = .054,\quad \tilde{u} = .33$ $\eta = 0.16$	$\phi = 1.60$ $\tilde{\delta} = .05,\quad \tilde{u} = .32$ $\eta = 0.17$	$\phi = 1.70$ $\tilde{\delta} = .04,\quad \tilde{u} = .318$ $\eta = 0.18$	$\phi = 2$ $\tilde{\delta} = .03,\quad \tilde{u} = .31$ $\eta = 0.21$	$\phi \to \infty$ $\tilde{\delta} \to 0,\quad \tilde{u} \to 1$ $\eta = 0.35$
C. *Population growth rate: $n = 0$; rate of technological progress: $g = 0$*								
Cap. utilization model $u = u(K(t)),\quad \delta = \delta(K(t))$	0.0357	0.0338	0.0322	0.0307	0.0298	0.0279	0.0245	0.0149
Standard model $\bar{u} = \tilde{u},\quad \bar{\delta} = \tilde{\delta},\quad \forall t$	0.0648	0.0591	0.0546	0.0502	0.0478	0.0429	0.0343	0.0149

Note: $\tilde{X}$ = Equilibrium quantity in the capital utilization model, $\bar{X}$ = equilibrium quantity in the standard model.

slower speeds of convergence, even though they are magnified by positive population growth rate and technological progress. However, there are some differences between the behavior of the two models. First, irrespective of population growth and technological progress, the calculated speeds of convergence are higher for the Ramsey model than the non-scale model. On the other hand, the equilibrium capital utilization and depreciation rates are lower than the non-scale model. Second, from (11c), we see that both the equilibrium capital utilization and depreciation rates are independent of the population growth rate in the Ramsey model and therefore remain unchanged for variations in n.

Table 1C presents an interesting comparison between the capital utilization and standard versions of the Ramsey model. We have already seen in Table 1 that the absence of exogenous factors like population growth and technological progress yields the slowest speeds of convergence. The capital utilization model in this case yields speeds of convergence that are quite close to the empirical upper bound of 3 percent, while the standard model, even with $n = g = 0$, yields speeds of convergence that are uniformly above the empirically estimated range, for the entire range of ϕ. For example, when $\phi = 1.56$, the capital utilization model predicts a speed of convergence of about 3 percent, while the corresponding speed in the standard model is much higher, at 5 percent.

6.3. The capital utilization decision and the convergence path

The results of Tables 1 and 2 also illustrate the effect of the agents' internalization of the capital utilization decision on the convergence path to the steady-state equilibrium. Since we control for the initial and steady state conditions, both the capital utilization and standard models converge to a *common* per-capita (or scale adjusted) output in the long run. However, there are some fundamental differences in their transitional adjustment paths. As an illustration, consider the non-scale model in Table 1B where $\phi = 1.56$. Our numerical calculations reveal that the speed of convergence in the capital utilization model is approximately 3 percent, while in the standard model it is 4.4 percent. This implies that in the model where the capital utilization decision is internalized along the transition path, the half-life of convergence is about 23 years, while in the standard model where the capital utilization rate is fixed, the corresponding half-life is only about 16 years.[20] Therefore, two economies that have identical initial and terminal conditions can converge to a *common* steady-state equilibrium at *different* rates, depending upon the underlying differences in the extent to which capital utilization is internalized along the transition path. This example, though a little simplistic and extreme, illustrates that variations in the degree to which agents internalize the capital utilization decision may provide a better understanding of convergence paths and speeds across countries. These differences may arise due to institutional factors governing the

[20] The time t for which the output-capital ratio is halfway between its initial and final steady-state values is given by the condition $e^{-\mu t} = 0.5$, where μ is the absolute value of the speed of convergence.

choice of multiple shifts in production (as in Europe), capital-labor ratios across industries and shifts, the vintage of the underlying capital stock, and the workweek of labor.

6.4. The endogenous growth model

Table 3 presents numerical exercises for the endogenous growth model similar to Tables 1 and 2, and further reinforces the results obtained in the previous section. The version of the endogenous growth model we study is similar to that of Jones and Manuelli (1990) and can be derived as a special case of the non-scale model when $A > 0$ and $\varepsilon = 0$. In addition, since in the endogenous growth model the steady state is characterized by sustained growth, we set $n = g = 0$. Further, since the steady-state equilibrium in this model has the property that all steady-state quantities, including the speed of convergence, is proportional to the steady-state rate of capital utilization, we can only compare the standard model, which sets $\bar{u} = 1$, with the capital utilization model with $\tilde{u} < 1$, by controlling for the steady-state rate of depreciation. For this model, we set A equal to 0.4.[21] From (18), we see that in contrast to the neoclassical model, the speed of convergence and equilibrium growth rates in the endogenous growth model are independent of the population growth rate. The population growth rate only affects the equilibrium consumption-capital ratio. Table 3 presents the differences in the speed of convergence between the standard and capital utilization models, for variations in ϕ.

As in the previous models, we find that for any value of ϕ, the speed of convergence in the standard endogenous growth model is always greater than that obtained from the capital utilization model. For example in Table 3 when $\phi = 1.56$, the equilibrium rate of capital utilization is 76 percent and the corresponding rate of depreciation is 19 percent. The implied speed of convergence from this model is 2.04 percent, which is well within the empirically estimated range. On the other hand, the standard model with a fixed depreciation rate and the rate of utilization set to unity yields a speed of convergence of 4.58 percent, which is much higher than empirical estimates. Also, note that for the empirical range of ϕ considered (1.4–2), the speed of convergence in the standard model varies from above 3 percent to about 6.15 percent, well *above* the empirically plausible range. On the other hand, the capital utilization model yields speeds of convergence from about 1.85–2.69 percent, which falls well within the empirical range of 2–3 percent.

Comparing Tables 1–3, we see that there are some fundamental differences in the role capital utilization plays in the non-scale, neoclassical and endogenous growth models. First, for any given ϕ, the speed of convergence in the endogenous growth model is lower than the ones in the non-scale and neoclassical models, once the

[21]The parameter A has been calculated using a two-step calibration method. We have already obtained $\bar{d} = 0.3$ from the depreciation function (3). Moreover, the first-order condition for the choice of the capital utilization rate (u) in the long-run is $A = \bar{d}\phi u^{\phi-1}$. Using Beaulieu and Mattey's (1998) estimate of $u = 0.58$, and $\phi = 1.7$, we get $A \approx 0.4$. Therefore, in Table 3, both u and ϕ change in a way such that this first-order condition is maintained. I would like to thank an anonymous referee for suggesting this procedure.

Table 3
Speed of convergence in the endogenous growth model $A = 0.4, \quad \beta = 0.03, \quad \gamma = -1.5, \quad n = 0, \quad g = 0, \quad \alpha = 0.35, \quad \bar{d} = 0.3$

	$\phi = 1.40$	$\phi = 1.45$	$\phi = 1.5$	$\phi = 1.56$	$\phi = 1.6$	$\phi = 1.7$	$\phi = 2$	$\phi \to \infty$
	$\tilde{\delta} = .25, \quad \tilde{u} = .89$	$\delta = .23, \quad \tilde{u} = .83$	$\tilde{\delta} = .21, \quad \tilde{u} = .79$	$\tilde{\delta} = .19, \quad \tilde{u} = .76$	$\tilde{\delta} = .18, \quad \tilde{u} = .74$	$\tilde{\delta} = .17, \quad \tilde{u} = .71$	$\tilde{\delta} = .13, \quad \tilde{u} = .67$	$\tilde{\delta} \to 0, \quad \tilde{u} \to 1$
Cap. utilization model $u = u(K(t)), \quad \delta = \delta(K(t))$	0.0185	0.0189	0.0196	0.0204	0.0209	0.0225	0.0269	0.0962
Standard model $\bar{u} = 1, \quad \bar{\delta} = \tilde{\delta}, \quad \forall t$	0.0305	0.0367	0.0414	0.0458	0.0482	0.0530	0.0615	0.0962

Table 4
The steady-state equilibrium: a comparison of the standard model ($u = 1$) with the capital utilization model ($0 < u < 1$) relative differences in equilibrium levels

	$\phi = 1.40$	$\phi = 1.45$	$\phi = 1.5$	$\phi = 1.56$	$\phi = 1.60$	$\phi = 1.70$	$\phi = 2$	$\phi \to \infty$
E. *The non-scale growth model*								
Scale-adjusted output	0.0100	0.1016	0.1726	0.2373	0.2716	0.3357	0.4320	0
Speed of convergence	0.3711	0.3502	0.3317	0.3121	0.3004	0.2748	0.2194	0

	$\phi = 1.40$	$\phi = 1.45$	$\phi = 1.5$	$\phi = 1.56$	$\phi = 1.60$	$\phi = 1.70$	$\phi = 2$	$\phi \to \infty$
F. *The neoclassical Ramsey growth model*								
Per-capita output	0.1443	0.1766	0.2020	0.2259	0.2388	0.2634	0.2994	0
Speed of convergence	0.4234	0.4018	0.3825	0.3618	0.3494	0.3219	0.2613	0

	$\phi = 1.40$	$\phi = 1.45$	$\phi = 1.5$	$\phi = 1.56$	$\phi = 1.60$	$\phi = 1.70$	$\phi = 2$	$\phi \to \infty$
C. *The endogenous growth model*								
Output-capital ratio	0.1148	0.1701	0.2099	0.2445	0.2620	0.2932	0.3333	0
Consumption-output ratio	0.1808	0.2241	0.2420	0.2484	0.2480	0.2393	0.1977	0
Growth rate	0.3923	0.4823	0.5270	0.5548	0.5650	0.5758	0.5634	0
Speed of convergence	0.3923	0.4823	0.5270	0.5548	0.5650	0.5758	0.5634	0

Note: The relative differences have been calculated by computing $(1 - \tilde{X}/\bar{X})$, where $\tilde{X}$ is an equilibrium variable in the capital utilization model, and $\bar{X}$ is the corresponding variable in the standard model.

capital utilization decision is endogenized. Therefore, in this sense, the endogenous growth model performs better than both the non-scale and neoclassical models. Second, the relationship between ϕ and the speed of convergence is also exactly opposite in these models. While the speed of convergence declines with ϕ in the non-scale and neoclassical models, it increases in the endogenous growth model. Finally, we have already noted that the speed of convergence in the endogenous growth model is not affected by population growth or technological progress, while in the non-scale and neoclassical models it increases with the inclusion of these exogenous factors.

6.5. The steady-state equilibrium

Table 4 reports the relative differences in equilibrium levels between the standard model which implicitly assumes full utilization, i.e., $\bar{u} = 1$, , and the capital utilization model where the equilibrium rate of utilization need not be one, i.e., $\tilde{u} < 1$.[22] For

[22]Specifically, let $\tilde{X}$ be an equilibrium variable in the capital utilization model and $\bar{X}$ be the corresponding variable in the standard model. Then, the relative difference between the two equilibrium levels is given by $(1 - \tilde{X}/\bar{X})$.

example, when $\phi = 1.56$, the standard model (with $\bar{u} = 1$) *overstates* equilibrium output by about 24 percent and the speed of convergence by about 39 percent relative to the capital utilization model in the non-scale model (Table 4A). The corresponding overstatement in the Ramsey model is about 23 and 36 percent, respectively (Table 4B). In the endogenous growth model, the standard model overstates the output-capital ratio and the consumption-output ratio by 25 percent, and the long run growth rate and the speed of convergence by about 56 percent (Table 4C). However, in the limit, as $\phi \to \infty$, the steady-state magnitudes in the capital utilization and standard models tend to converge. Table 4 highlights the fact that the assumption of fixed depreciation and full capital utilization may lead to a significant overstatement of the steady-state equilibrium in one-sector growth models.

7. Conclusions

This paper introduces capital utilization as an optimal choice in a general class of one-sector models of economic growth. Our objective has been to bring theoretical predictions regarding the speed of convergence obtained from one-sector growth models in line with observed empirical estimates. We show that incorporating the capital utilization decision into a standard growth model may help to resolve the discrepancy between theory and facts, and provide a simpler alternative, based on optimal choice, to the adjustment cost and multiple capital good frameworks. Contrary to the assumptions of theoretical growth models, the sensitivity of depreciation to capital utilization leads to 'less than full' utilization of capital by the economic agent, a result consistent with empirical facts. The relationship between the rate of depreciation and capital utilization, embodied in the variable marginal benefits and costs of capital accumulation along the transition path, plays a crucial role in slowing down the speed of convergence to the steady-state equilibrium. We also show that by assuming a constant depreciation rate and full capital utilization, the standard growth models may be significantly overstating the magnitude of both the steady-state equilibrium and the convergence rate. Finally, our numerical analysis suggests that differences across countries in the extent to which agents internalize the capital utilization decision along the transition path may lead to differences in the speed of adjustment to the steady state, even for countries that have similar initial and terminal conditions.

By underscoring the importance of capital utilization for the dynamics of growth and convergence, our analysis not only attempts to bring theory closer to the facts, but also opens up potential avenues for future research in this area. The framework we have developed opens up the question of understanding and separating the long-run determinants of the choice of the workweek of capital from that of labor, examining how that choice is affected by public policy and, in turn, how it affects the growth-convergence path of an economy. Specifically, since the internalization of capital usage provides the firm with an extra margin to change output, it is important to examine the dynamic response of the firm to capital income taxes and subsidies, and compare the results with the prediction of standard models. This becomes even more important in decentralized economies where the accumulation of capital may be subject to externalities. An interesting avenue for further research would be to characterize optimal taxation policies in decentralized economies where capital utilization and depreciation are endogenously determined. In the context of the two-sector 'non-scale' growth model, Eicher and Turnovsky (1999a, b) demonstrate that the convergence rate is time varying. Therefore, another point of interest would be to see how capital utilization affects the *dynamic path* of the convergence rate. Finally, incorporating capital utilization in a two-sector endogenous growth model, as in Ortiguera and Santos (1997), would enable an examination of its explanatory power relative to the multiple capital goods and adjustment cost frameworks. These remain exciting and intriguing questions for future research in this area.

Appendix A

A.1. Capital utilization in the non-scale growth model

A.1.1. Equilibrium dynamics

The core dynamics for the non-scale, or 'semi-endogenous' growth model with endogenous capital utilization can be derived from (6a)–(6d) by setting $A = 0$. Given (7) and (8) and the definitions of the scale-adjusted stationary variables k and c, we can express the dynamics in the following manner:

$$\dot{k} = \frac{B^{\sigma_K}(\phi - \sigma_K)}{\phi} k^{\eta} - c - \left[\left(\frac{\sigma_N}{1-\sigma_K}\right)(n+g)\right]k, \tag{A1.1}$$

$$\dot{c} = \frac{c}{(1-\gamma)}\left[\sigma_K B^{\sigma_K}\frac{(\phi-1)}{\phi}k^{\eta-1} + (1-\gamma)\left\{\frac{(1-\sigma_K-\sigma_N)n - \sigma_N g}{1-\sigma_K}\right\} - \beta\right], \tag{A1.2}$$

where $B = (\sigma_K/\phi\bar{d})^{1/\phi-\sigma_K}$.

In steady-state equilibrium, we must have $\dot{k} = \dot{c} = 0$. Using this condition in (A1.1) and (A1.2) we can solve for the steady-state level of per-capita consumption and the utilized capital stock, given in (10a) and (10b). The corresponding expressions for the neoclassical Ramsey model can be obtained by setting $\sigma_K = \alpha$ and $\sigma_K + \sigma_N = 1$.

The linearized dynamics around the steady-state equilibrium (10a)–(10b) can be expressed in the following manner:

$$\begin{pmatrix} \dot{k} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} k - \tilde{k} \\ c - \tilde{c} \end{pmatrix}, \tag{A1.3}$$

where

$$a_{11} = \beta - \left(\frac{\sigma_N}{1-\sigma_K}\right)(n+g) - (1-\gamma)\left\{\frac{(1-\sigma_K-\sigma_N)n - \sigma_N g}{1-\sigma_K}\right\}; \quad a_{12} = -1,$$

$$a_{21} = \frac{1}{(1-\gamma)}\left[(\eta-1)\left\{\beta - (1-\gamma)\left(\frac{(1-\sigma_K-\sigma_N)n - \sigma_N g}{1-\sigma_K}\right)\right\}\right]\left(\frac{\tilde{c}}{\tilde{k}}\right); \quad a_{22} = 0.$$

The determinant of the coefficient matrix in (A1.3) can easily be shown to be negative. For example, in the neoclassical case when $n = g = 0$, the determinant is given by

$$-\frac{(1-\eta)(\phi-1)(\beta+\tilde{\delta})(\beta+\tilde{\delta}(1-\alpha))}{\alpha\phi(1-\gamma)} < 0.$$

The equilibrium is therefore a saddle point with one negative and one positive root, $\mu = (\mu_1, \mu_2)$ which are the solutions to

$$\mu^2 - \beta\mu - \frac{(1-\eta)(\phi-1)(\beta+\tilde{\delta})(\beta+\tilde{\delta}(1-\alpha))}{\alpha\phi(1-\gamma)} = 0. \qquad \text{(A1.4)}$$

The negative root corresponding to (A1.4) is the speed of convergence and is given by (17a).

A.1.2. The steady-state capital utilization rate

We will now characterize the behavior of the steady-state capital utilization rate ($\tilde{u}$) with respect to the elasticity of depreciation with respect to capital utilization (ϕ). For analytical simplicity, consider the neoclassical Ramsey model with $n = g = 0$. Recalling (11c), the steady-state capital utilization and depreciation rates are

$$\tilde{u} = \left(\frac{\beta}{\bar{d}(\phi-1)}\right)^{1/\phi}; \quad \tilde{\delta} = \frac{\beta}{(\phi-1)}. \qquad \text{(A1.5)}$$

First, note that

$$\lim_{\phi\to 1} \tilde{u} = \infty \quad \text{and} \quad \lim_{\phi\to\infty} \tilde{u} = 1.$$

Further, for $1 + \frac{\bar{d}}{\beta} \leqslant \phi < \infty$, we get $\tilde{u} \leqslant 1$.

Also,

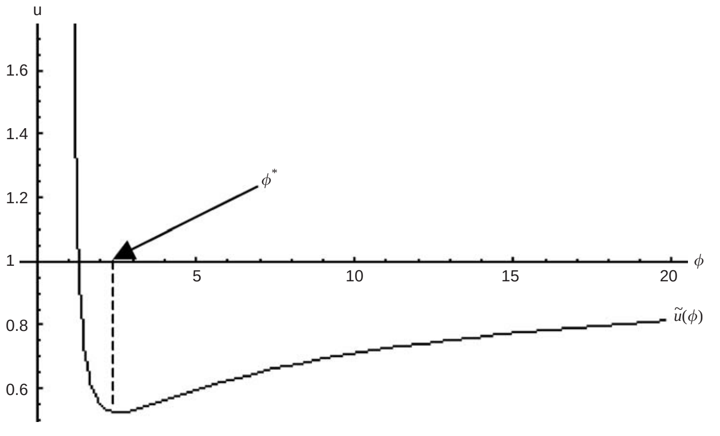
$$\frac{\partial \tilde{u}}{\partial \phi} = -\frac{(\beta/\bar{d}(\phi-1))^{1/\phi}[\phi + (\phi-1)\mathrm{Log}(\beta/\bar{d}(\phi-1))]}{\phi^2(\phi-1)} \lesseqgtr 0$$

according to whether $\phi \lesseqgtr \phi^*$, where ϕ^* is the point of inflexion. ϕ^* can be obtained from the solution of $(\beta/\bar{d}(\phi-1))^{1/\phi}[\phi + (\phi-1)\,\mathrm{Log}(\beta/\bar{d}(\phi-1))]/\phi^2(\phi-1) = 0$, given β and $\bar{d}$.

In other words, when $1<\phi<\phi^*$, $\frac{\partial \tilde{u}}{\partial \phi}<0$, i.e., $\tilde{u}$ declines with ϕ. On the other hand, when $\phi^*<\phi<\infty$, $\partial\tilde{u}/\partial\phi>0$, i.e., $\tilde{u}$ increases with ϕ. In the limit, we have

$$\operatorname*{Lim}_{\phi\to 1}\frac{\partial\tilde{u}}{\partial\phi}=-\infty \quad \text{and} \quad \operatorname*{Lim}_{\phi\to\infty}\frac{\partial\tilde{u}}{\partial\phi}=0.$$

The above results can be verified by the numerical calibrations reported in Tables 1–4. The figure below, which plots the relationship between $\tilde{u}$ and ϕ, for $\beta=0.03$ and $\bar{d}=0.3$, further illustrates these results.



A.2. Capital utilization in the endogenous growth model

A.2.1. Equilibrium dynamics

For the endogenous growth model described in Section 5, let $y=f(uK)/K$, the average product of utilized capital, and $c=C/K$, the consumption-capital ratio be the stationary variables in terms of which we shall express the equilibrium dynamics. We can then rewrite the first-order conditions (6b) and (6c) as

$$\alpha y+(1-\alpha)Au=\phi\,\bar{d}u^{\phi}, \tag{A2.1}$$

$$\beta-\frac{\dot{\lambda}}{\lambda}=\alpha y+(1-\alpha)Au-\bar{d}u^{\phi}. \tag{A2.2}$$

Eq. (A2.2) can be solved to yield the optimal rate of capital utilization as a function of the average product of capital:

$$u=u(y), \tag{A2.3}$$

where $u'(y)>0$ and $u''(y)<0$.

The core dynamics of the model can be expressed as

$$\dot{y}=-\frac{\phi(1-\alpha)(y-Au)}{(\phi-1)}\left[\frac{1}{\phi}\{(\phi-\alpha)y-(1-\alpha)Au\}-c-n\right], \tag{A2.4}$$

$$\frac{\dot{c}}{c}=\frac{1}{(1-\gamma)}\left[\left(\frac{\phi-1}{\phi}\right)\{\alpha y+(1-\alpha)Au\}-\beta\right]-\frac{1}{\phi}[(\phi-\alpha)y-(1-\alpha)Au]+c+n, \tag{A2.5}$$

$$\frac{\dot{u}}{u}=\frac{\alpha\dot{y}}{[\phi^2\,\bar{\mathrm{d}}u^{\phi}-(1-\alpha)Au]}. \tag{A2.6}$$

The steady state is attained when $\dot{y}=\dot{c}=\dot{u}=0$ and, using (A2.2), is described in (18a)–(18d) in Section 5. The linearized dynamics around this steady-state equilibrium is given by

$$\begin{pmatrix}\dot{y}\\ \dot{c}\end{pmatrix}=\begin{pmatrix}-(1-\alpha)\tilde{\psi} & 0\\ \dfrac{(\phi-1)\alpha\tilde{c}}{\phi(1-\gamma)(\phi+\alpha-1)} & \tilde{c}\end{pmatrix}\begin{pmatrix}y-\tilde{y}\\ c-\tilde{c}\end{pmatrix}. \tag{A2.7}$$

The determinant of the coefficient matrix of (A2.7) is $-(1-\alpha)\tilde{\psi}<0$, where $\tilde{\psi}$ is the steady-state growth rate. The equilibrium is therefore a saddle-point with one positive and one negative eigenvalue. The negative eigenvalue is a measure of the speed of convergence, and is given by (20).

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References

Abadir, K., Talmain, G., 2001. Depreciation rates and capital Stocks. The Manchester School 69, 42–51.
Ambler, S., Pacquet, A., 1994. Stochastic depreciation and the business cycle. International Economic Review 35, 101–116.
Anxo, D., Bosch, G., Bosworth, D., Cette, G., Sterner, T., Taddei, D., 1995. Work Patterns and Capital Utilization. Kluwer Academic Publications, Boston.
Barro, R.J., Sala-i-Martin, X., 1992. Convergence. Journal of Political Economy 100, 223–251.
Barro, R.J., Sala-i-Martin, X., 2004. Economic Growth. The MIT Press, Cambridge.
Basu, S., 1996. Procyclical productivity: increasing returns or cyclical utilization? The Quarterly Journal of Economics 111, 719–751.
Basu, S., Kimball, M., 1997. Cyclical productivity with unobserved input variation. NBER Working Paper, No. 5915.

Bautista, R.M., Hughes, H., Lim, D., Morawetz, D., Thoumi, F., 1981. Capital Utilization in Manufacturing. Oxford University Press, Washington, DC.

Beaulieu, J., Mattey, J., 1998. The workweek of capital and capital utilization in manufacturing. Journal of Productivity Analysis 10, 199–223.

Betancourt, R., Clague, C., 1981. Capital Utilization: A Theoretical and Empirical Analysis. Cambridge University Press, UK.

Burnside, C., Eichenbaum, M., 1996. Factor-hoarding and the propagation of business cycle shocks. American Economic Review 86, 1154–1174.

Burnside, C., Eichenbaum, M., Rebelo, S., 1996. Capital utilization and returns to scale. NBER Macroeconomics Annual 11, 67–123.

Calvo, G., 1975. Efficient and optimal utilization of capital services. American Economic Review 65, 181–186.

Christensen, L., Jorgenson, D., 1969. The measurement of US real capital input, 1929–1967. Review of Income and Wealth 15, 239–320.

Dalgaard, C., 2003. Idle capital and long-run productivity. Contributions to Macroeconomics 3, 1–42.

Dowrick, S., Rogers, M., 2002. Classical and technological convergence: beyond the Solow–Swan model. Oxford Economic Papers 54, 369–385.

Eicher, T., Turnovsky, S., 1999a. Non-scale models of economic growth. Economic Journal 109, 394–415.

Eicher, T., Turnovsky, S., 1999b. Convergence speeds and transitional dynamics in non-scale growth models. Journal of Economic Growth 4, 413–428.

Epstein, L., Denny, M., 1980. Endogenous capital utilization in a short-run production model: theory and empirical application. Journal of Econometrics 12, 189–207.

Finn, M., 1995. Variance properties of Solow's productivity residual and their cyclical implications. Journal of Economic Dynamics and Control 19, 1249–1281.

Foss, M., 1963. The utilization of capital equipment: postwar compared with pre-war. Survey of Current Business 43, 8–16.

Foss, M., 1981a. Long-run changes in the workweek of fixed capital. AEA Papers and Proceedings 71, 58–63.

Foss, M., 1981b. Changes in the Workweek of Fixed Capital: US Manufacturing, 1929–1976. American Enterprise Institute, Washington, DC.

Foss, M., 1995. Operating hours of US manufacturing plants, 1976–1988, and their significance for productivity change. In: Anxo, A., et al. (Eds.), Work Patterns and Capital Utilization. Kluwer Academic Publications, Boston.

Fraumeni, B., 1997. The measurement of depreciation in the US national income and product accounts. Survey of Current Business, July, pp. 7–23.

Gramlich, E., 1994. Infrastructure investment: a review essay. Journal of Economic Literature 32, 1176–1196.

Greenwood, J., Hercowitz, Z., Huffman, G., 1988. Investment, capacity utilization, and the real business cycle. American Economic Review 78, 402–417.

Gylfason, T., Zoega, G., 2001. Obsolescence. CEPR Discussion Paper, No. 2833.

Imbs, J., 1999. Technology, growth and the business cycle. Journal of Monetary Economics 44, 65–80.

Jones, C., 1995. R&D-based models of economic growth. Journal of Political Economy 103, 759–784.

Jones, L.E., Manuelli, R., 1990. A convex model of equilibrium growth: theory and implications. Journal of Political Economy 98, 1008–1038.

Keynes, J., 1936. The General Theory of Employment, Interest and Money. Macmillan, London.

Kim, Y., Watson, G., 1974. The optimal utilization of capital stock and the level of economic development. The Economic Journal 41, 377–386.

Kollintzas, T., Choi, J., 1985. A Linear Rational Expectations Equilibrium Model of Aggregate Investment with Endogenous Capital Utilization and Maintenance. Mimeo, State University of New York.

Kydland, F., Prescott, E., 1982. Time-to-build and aggregate fluctuations. Econometrica 50, 1345–1370.

Lucas, R., 1970. Capacity, overtime, and empirical production functions. American Economic Review 60, 23–27.

Lucas, R., 1988. On the mechanics of economic development. Journal of Monetary Economics 22, 3–42.
Mankiw, N., Romer, D., Weil, D., 1992. A contribution to the empirics of economic growth. Quarterly Journal of Economics 107, 407–438.
Marris, R., 1964. The Economics of Capital Utilization. Cambridge University Press, UK.
Mayshar, J., Halevy, Y., 1997. Shiftwork. Journal of Labor Economics 15, S198–S222.
Nadiri, M., Prucha, I., 1996. Estimation of the depreciation rate of physical and R&D capital in the US total manufacturing sector. Economic Inquiry 34, 43–56.
Nadiri, M., Rosen, S., 1969. Interrelated factor demand functions. American Economic Review 59, 457–471.
Oi, W., 1981. Slack capacity: productive or wasteful. American Economic Review, Papers and Proceedings 71, 64–69.
Orr, J., 1989. The average workweek of capital in manufacturing, 1952–84. Journal of the American Statistical Association 84, 88–94.
Ortiguera, S., Santos, M., 1997. On the speed of convergence in endogenous growth models. American Economic Review 87, 383–399.
Palivos, T., Yip, C., 1993. Optimal population size and endogenous growth. Economics Letters 41, 107–110.
Romer, P., 1986. Increasing returns and long-run growth. Journal of Political Economy 94, 1002–1037.
Rumbos, B., Auernheimer, L., 2001. Endogenous capital utilization in a neoclassical growth model. Atlantic Economic Journal 29, 121–134.
Sala-i-Martin, X., 1994. Cross-sectional regressions and the empirics of economic growth. European Economic Review 38, 739–747.
Sala-i-Martin, X., 1996. The classical approach to convergence analysis. Economic Journal 106, 1019–1036.
Shapiro, M., 1986. Capital utilization and capital accumulation: theory and evidence. Journal of Applied Econometrics 1, 211–234.
Smith, K., 1970. Risk and the optimal utilization of capital. Review of Economic Studies 37, 253–259.
Taubman, P., Gottschalk, P., 1971. The average workweek of capital in manufacturing. Journal of the American Statistical Association 66, 448–455.
Taubman, P., Wilkinson, M., 1970. User cost, capital utilization and investment theory. International Economic Review 11, 209–215.
Turnovsky, S., 2000. Methods of Macroeconomic Dynamics. The MIT Press, Cambridge.
Turnovsky, S., 2004. The transitional dynamics of fiscal policy: long-run capital accumulation and growth. Journal of Money, Credit and Banking 36, 883–910.
Turnovsky, S., 2003b. Old and new growth theories: a unifying structure. In: Salvador, N. (Ed.), Old and New Growth Theories: An Assessment. Edward Elgar, Cheltenham, UK, pp. 1–44.
Wen, Y., 1998. Capacity utilization under increasing returns to scale. Journal of Economic Theory 81, 7–36.