Growth and Inequality:
Dependence on the Time Path of Productivity Increases (and other Structural Changes)[†]

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Abstract

This paper examines the significance of the *time path* of a given productivity increase on growth and inequality. We show that whereas the time path impacts only the transitional path of aggregate quantities and has no effect on their ultimate steady-state levels, it has both transitional and permanent consequences for wealth and income distribution. As a result, the growth-inequality tradeoff generated by a given discrete increase in productivity contrasts sharply with that obtained when the same ultimate productivity increase is acquired gradually. This is true both in transition and across steady states. We show that a gradual productivity change can generate a Kuznets-type inverted U-shaped relationship between inequality and per-capita income. The distance from the technology frontier is also shown to have important implications for both the magnitude and persistence of inequality. Finally, our results suggest that economies with similar aggregate structural characteristics may have very different outcomes for income and wealth inequality, depending on the intrinsic nature of the productivity growth path.

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1. Introduction

The relationship between income inequality and economic growth has been extensively discussed since Kuznets' (1955) pioneering work first appeared over half a century ago. Since then, the question of whether these two key economic variables are positively or negatively related has been extensively debated, although no definitive conclusion has been reached. Empirical evidence has not resolved the issue. Early growth regressions by Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996), and others, yield a negative growth-inequality relationship.[1] But more recent studies obtain a positive, or at least more ambiguous, relationship; see for example, Li and Zou (1998), Forbes (2000), and Barro (2000).[2] From a theoretical perspective, this empirical controversy should not be surprising. Because an economy's growth rate and its income distribution are both endogenous equilibrium outcomes, the income inequality-growth relationship – whether positive or negative – will reflect the underlying set of forces to which both are simultaneously reacting. To understand these linkages it is necessary to examine the growth-inequality relationship using a consistently specified general equilibrium growth model.

In this paper we employ such a model to consider the impact of one of the major determinants of the growth-inequality relationship, namely an increase in productivity.[3] The key result we shall establish is that the effects a productivity increase of a given magnitude on wealth and income inequality depend crucially upon the time path along which the productivity increase accrues. This in turn has important consequences for the growth-inequality tradeoff, and further, may help explain why economies with similar aggregate structural characteristics may nevertheless have very different income and wealth distributions. While we focus on a productivity increase as being particularly salient, it will become evident that the argument in fact applies to any structural

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[1] The various explanations for this include: the political economy consequences of inequality (Alesina and Rodrik, 1994), the potential harm inequality may cause for investment in physical or human capital, (Galor and Zeira, 1993; Aghion and Bolton, 1997), and the unequal distribution of natural resources (Gylfason and Zoega, 2003).

[2] In particular, Forbes finds a positive relationship when the short-term impact is considered. Barro finds a negative relationship between inequality and growth for poorer countries, but a positive relationship for richer countries. Explanations for the positive relationship include: a positive relationship between inequality and higher tax rates to finance public education (Saint-Paul and Verdier, 1993), socio-economic stratification (Bénabou, 1996a), and the nature of technological progress (Galor and Tsiddon, 1997).

[3] This is one of the key factors influencing the growth-inequality relationship identified by Solimano (1998); see also Piketty (2006).

change that occurs over time. Hence the issue we are addressing is quite general, and therefore highly significant for understanding the dynamics of the growth-inequality tradeoff.

In a completely general setup, in which the equilibrium growth rate and income distribution are mutually dependent, their joint determination and the analysis of their relationship becomes intractable; see e.g. Sorger (2000). This paper, on the other hand, is related to a growing body of research that exploits the fact that if the underlying utility function is homogeneous in its relevant arguments, the aggregate economy can be summarized by a representative agent, as a result of which aggregate behavior becomes independent of the economy's distributional characteristics. Rather, the distributions (e.g. of wealth and income) track the evolution of the aggregate economy; see e.g. Caselli and Ventura (2000), García-Peñalosa and Turnovsky (2006, 2007, 2008), Kraay and Raddatz (2007), Carroll and Young (2009) and Barnett et al. (2009). While awareness of this aggregation property dates back to Gorman (1953), by rendering the analysis so tractable, it assumes particular importance in studying the growth-inequality relationship. Moreover, the class of utility functions for which the aggregation simplifies in this way includes the constant elasticity utility function that dominates contemporary growth theory.

Inequality is necessarily associated with heterogeneity across agents. Recently, differential initial endowments of capital across economic agents have received a lot of attention as an underlying source of heterogeneity.[4] A crucial mechanism generating the endogenous distribution of income is the relationship between agents' relative capital stock and their relative allocation of time to leisure. In the long run, this relationship is positive, as wealthier agents who have a lower marginal utility of wealth increase their consumption of all goods, including leisure.[5] In the short

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[4] By identifying agents' heterogeneity with their initial physical asset endowments, we are embedding distributional issues within a more traditional growth-theoretic framework. Indeed, the role of the return to capital, which is essential in that literature, has largely been ignored in the recent discussions of income inequality, which has emphasized other aspects such as human capital and growth; see e.g. Galor and Zeira (1993), Bénabou (1996b), and Viaene and Zilcha (2003), among others. The argument that the return to capital is essential to understanding distributional differences has, however, been addressed by Atkinson (2003), and is supported by recent empirical evidence for the OECD (see Checchi and García-Peñalosa (2008).

[5] This long-run negative relationship between wealth and labor supply is supported by empirical evidence, obtained from a variety of sources. For example, Holtz-Eakin, Joulfaian, and Rosen (1993) find that large inheritances decrease labor participation. Using data from the stock market boom of the 1990s, Cheng and French (2000) and Coronado and Perozek (2003) find a substantial negative effect of wealth on labor supply and retirement. Algan, Chéron, Hairault, and Langot (2003) use French data and find a significant wealth effect on the extensive margin of labor supply.

run, however, this relationship is conditioned by the time path a given productivity change is expected to follow, and the consumption-smoothing motives it generates for rich and poor agents.

A key feature of this labor allocation-relative wealth mechanism is that it introduces *hysteresis* into the dynamic adjustment characterizing the relative holdings of capital. This occurs because the impact of any structural change on the long-run evolution of wealth inequality, and subsequently on income inequality, depends critically upon the initial response of leisure (labor supply) to the underlying shock. This initial response, in turn, depends upon the time path that the structural change is expected (known) to follow. That is, not only the dynamic evolution but also the long-run distributions of wealth and income inequality become *path dependent*. Thus, a central insight of this paper is that the effects of a productivity increase of a *given* magnitude on the *long-run distributions* of both wealth and income are crucially dependent upon the time path that the productivity increase is assumed to follow. This is in sharp contrast to the dynamics of the aggregate economy. In this case, the time path of a productivity increase affects only the transitional path of the aggregate economy and has no impact on its steady state.

To illustrate the role of path dependence we compare the consequences of two alternative specifications of the productivity increase. The first is the conventional one, where the full productivity change occurs instantaneously as an unanticipated permanent *discrete* increase in the level of productivity. The second is where the same overall increase in the level of productivity occurs, but is acquired *gradually* over a known time path, and is therefore anticipated after the first instant. These two specifications of the productivity increase have exactly opposite consequences for the initial responses of leisure. In the first case leisure initially declines, while in the latter case it initially increases, leading to profoundly different distributional consequences.

The path dependence of the distributions of wealth and income inequality helps provide some key insights regarding the ambiguous empirical relationship between growth and inequality. Given the analytical complexity of the theoretical framework, our results are derived using numerical simulations. The main findings are summarized below:

1. Whereas a discrete productivity increase always leads to a monotonic decline in wealth inequality, its gradual introduction leads to a non-monotonic adjustment, with an initial

increase followed by a gradual decline after some period of time. In the long run, this is likely to lead to *more*, rather than less, wealth inequality unless the flexibility of production is extremely high. Furthermore, whereas a discrete productivity increase leads to an initial increase in income inequality, followed by a monotonic decline to below its initial level, its gradual introduction generates essentially the opposite time profile. In short, the gradual introduction of a technological increase is likely to completely reverse the growth-inequality tradeoffs obtained when the productivity increase occurs discretely.

2. The model permits a diversity of distributional equilibria for structurally similar countries: we show that countries with similar structural conditions and aggregate levels of development may end up with very different levels of inequality, depending on the time path of productivity changes. This result is consistent with the experiences of countries in East Asia and Latin America, who have similar levels of per-capita income but very different levels of income inequality.[6]
3. A striking aspect of the transitional behavior of this model is that a gradual productivity increase can generate the Kuznets-type inverted-U relationship between inequality and per-capita income. In contrast, a discrete change in productivity, as discussed above, can only generate an inverse monotonic relationship. The fact that the much-debated Kuznets relationship can be generated in the context of a simple one-sector Ramsey model is a salient feature of this paper, providing a simple theoretical justification for a controversial empirical relationship; see Ray (1998).
4. Finally, the distance of a country from its technological frontier is shown to have important implications for both inequality and its persistence.

The standard procedure of assuming that productivity increases occur fully on impact, rather than gradually, while convenient analytically, is arguably less realistic than positing some form of

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[6] The diversity of distributional equilibria prevents Deininger and Squire (1996, 1998) from finding a statistically significant relationship between the *level* of income and inequality in over 75 per cent of their cross-country sample even after controlling for initial differences in inequality. Of course, there may be other reasons that economies structurally similar in the aggregate may have different distributional characteristics, the most obvious being differential fiscal policies; see e.g. García-Peñalosa and Turnovsky (2007) where this issue is discussed in the context of a Romer-type endogenous growth model.

continuing adjustment. Several reasons support this view. First, to the extent that productivity increases reflect government investment in infrastructure and general productive capacity, the notion that they occur gradually over time, rather than instantaneously, seems more plausible. Budget restrictions inevitably force governments to spread their investments over time and develop them as multi-year projects. The US interstate highway system, initiated in the 1950's, is a good example; the recent public infrastructure policies of China and India also reflect this notion. Second, productivity increases generally reflect the assimilation of new productive techniques that may require learning for complete adaptation, and this too takes time. An example of this is the general purpose technologies (GPT) such as steam, railroads, lasers and, more recently information technology; see Aghion and Howitt (2009, ch 9). Third, from the perspective of a developing economy, one can interpret the productivity increase as representing a "productivity gap' which again is likely to take years to eliminate. Finally, foreign aid to developing countries, especially when "tied" to investment projects as schools, roads, hospitals, etc. is likely to be granted over time, and is therefore yet another good example of a gradual productivity change.

The rest of the paper proceeds as follows. Section 2 sets out the analytical model and the evolution of the aggregate dynamics, while Section 3 discusses the distributional dynamics of wealth and income. Since the focus is on the transitional dynamics, which are too complex to solve analytically, we employ numerical simulations. These are reported in Section 4. Section 5 discusses implications of our analysis for the empirical literature on growth and inequality, focusing specifically on the Kuznets curve, and Section 6 presents concluding remarks. While many of the earlier papers in this area conduct their dynamic analysis using standard linearization techniques, we shall use the shooting methods of Atolia and Buffie (2010a, 2010b) to obtain an exact solution to the dynamics. Nevertheless, the linearization procedure, albeit an approximation, is useful in guiding our intuition, particularly in the role of the initial response in leisure.[7] Therefore, the linearized solutions for both the aggregate economy and the distributions are set out in the Appendix.

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[7] Elsewhere, we have investigated the accuracy of conventional linearization procedures in characterizing the dynamics of a standard aggregate Ramsey model; see Atolia, Chatterjee, and Turnovsky (2009). In fact, for a model having the structure here, linearization can accommodate quite large structural changes without committing unacceptably large errors, at least for moderate values of the elasticity of substitution (less than unity). For large values (around 1.25) the errors become more significant.

2. Analytical Framework

We consider a decentralized economy having a single representative firm and heterogeneous households. The source of heterogeneity among consumers is the initial distribution of capital endowments. For simplicity, we assume a completely laissez-faire economy which operates in the absence of a government or social planner.

2.1 Technology and Factor Payments

Aggregate output is produced by a single representative firm according to a standard neoclassical production function[8]

$$Y = A(t)F\left(K(t), L(t)\right) \qquad F_L > 0, F_K > 0, F_{LL} < 0, F_{KK} < 0, F_{LK} > 0 \qquad (1)$$

where, K, L, and Y denote the per-capita stock of capital, labor supply, and output. In addition, $A(t)$ represents the level of productivity, which is exogenous to the firm's decisions.

The key feature of our analysis is that the level of productivity is assumed to increase gradually from its initial level, A_0, to a higher long-run level, $\tilde{A}$, both of which are known to the firm. This is specified by the (known) deterministic growth path

$$A(t) = \tilde{A} + \left(A_0 - \tilde{A}\right)e^{-\theta t}, \ \theta \geq 0 \qquad (2)$$

or equivalently

$$\dot{A}(t) = \theta\left[\tilde{A} - A(t)\right] \qquad (2')$$

The parameter θ thus defines the time path followed by the increase in productivity. The conventional approach to modeling productivity increases is to assume that they occur instantaneously. This is obtained (or at least approximated) as a special case by letting $\theta \to \infty$ in (2), so that the new productivity level is achieved virtually instantaneously. However, the more general specification introduced in (2) is important for two reasons. First, as we will demonstrate

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[8] Both factors of production have positive, but diminishing, marginal physical products and the production function exhibits constant returns to scale, with $F_{KL} > 0$ being a consequence of the latter assumption.

subsequently, there is a sharp contrast between how θ affects the behavior of *aggregates* and *distributions*. As one would expect, it affects the transitional path of the aggregate economy, but not the aggregate steady state. In contrast, it has profound impacts on *both* the time paths and the steady-state levels of both wealth and income inequality.

From a practical standpoint, the specification in (2) is quite general and can be applied to a variety of structural shocks, such as government policies on infrastructure, General Purpose Technologies, and foreign aid programs which are absorbed by the economy gradually, rather than instantaneously. As a specific example, we can think of the productivity enhancement described in (2) as a consequence of government policy, reflecting, for example, its investment in public infrastructure. It is then natural to view $\tilde{A}$ as some pre-specified long-run target, which because of budgetary limitations and other bureaucratic impediments can be attained only gradually over time. Consequently, the long-run growth-inequality tradeoff faced by the economy in response to a structural change will depend crucially upon the speed with which the change occurs.[9]

The wage rate, *w*, and the return to capital, *r*, are determined by the marginal physical products of labor and capital:

$$w(t) \equiv w(K,L) = AF_L(K,L) \tag{3a}$$

$$r(t) \equiv r(K,L) = AF_K(K,L) \tag{3b}$$

where we have dropped the time notation from the variables. Note that both the wage rate and the return on capital reflect the current level of productivity, $A(t)$.

2.2 Households

At time 0, the economy is populated by N_0 households, represented as a continuum between 0 and N_0, and each indexed by *i*. Population grows uniformly across households at an exponential rate, *n*, so that at time *t*, household *i* has grown to e^{nt} and the total population of the economy is $N(t) \equiv N_0 e^{nt}$. Households are identical in all respects except for their initial endowments of capital,

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[9] It is straightforward to generalize (2) to the case where the new level of productivity is reached in finite time, *T*. This is specified by the productivity growth function: $A(t) - \tilde{A} = (A_0 - \tilde{A})[e^{-\theta t} - e^{-\theta T}](1 - e^{-\theta T})^{-1} \quad t \le T; \quad A(t) - \tilde{A} = 0 \quad t \ge T$.

$K_{i,0}$, so that the average initial stock of capital in the economy is

$$K_0(t) = \frac{1}{N_0}\int_0^{N_0} K_{i,0}di$$

From a distributional perspective, we are interested in household *i*'s relative share of the total capital stock in the economy, $k_i(t) \equiv K_i(t)/K(t)$. At time *t*, with the growing population and accumulation of capital, the average per-capita amount of capital is

$$K(t) = \frac{1}{N_0 e^{nt}}\int_0^{N_0} K_i(t)e^{nt}di = \frac{1}{N_0}\int_0^{N_0} K_i(t)di\,.$$

and the share of capital owned by household *i* is

$$k_i(t) \equiv \frac{K_i(t)e^{nt}}{\frac{1}{N_0}\int_0^{N_0} K_i(t)e^{nt}di} = \frac{K_i(t)}{\frac{1}{N_0}\int_0^{N_0} K_i(t)di} = \frac{K_i(t)}{K(t)}$$

At all points of time, the mean of the distribution is normalized to unity, while the the initial (given) standard deviation of relative capital is $\sigma_{k,0}$.

We now consider household *i*, which, like all others, is endowed with a unit of time that it can allocate to either leisure, l_i, or work, $L_i \equiv 1 - l_i$. The household chooses its rates of consumption, C_i, and leisure to maximize lifetime utility represented by the iso-elastic function:

$$\max \int_0^\infty \frac{1}{\gamma}\left(C_i(t)l_i^\eta\right)^\gamma e^{-\beta t}dt, \quad \text{with } -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1 \tag{4}$$

where $1/(1-\gamma)$ equals the intertemporal elasticity of substitution. The preponderance of empirical evidence suggests that this is relatively small, certainly well below unity, so that we shall restrict $\gamma < 0$.[10] This maximization is subject to the household's initial endowment of capital, $K_{i,0}$, together with its capital accumulation constraint

$$\dot{K}_i(t) = (r(t) - n)K_i(t) + w(t)(1 - l_i(t)) - C_i(t) \tag{5}$$

The first-order conditions are standard and give rise to the following key relationships:

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[10] See, for example, the discussion of the empirical evidence summarized and reconciled by Guvenen (2006).

$$\eta \frac{C_i}{l_i} = w(t) \tag{6}$$

$$(\gamma - 1)\frac{\dot{C}_i}{C_i} + \eta\gamma \frac{\dot{l}_i}{l_i} = \frac{\dot{\lambda}_i}{\lambda_i} = \beta + n - r(t) \tag{7}$$

where λ_i is agent *i*'s shadow value of capital. Equation (6) equates the marginal rate of substitution between consumption and leisure to the price of leisure, while (7) is the Euler equation modified to take into account the fact that leisure changes over time. Using (6), we may write the individual's accumulation equation, (5), in the form

$$\frac{\dot{K}_i}{K_i} = r(t) - n + \frac{w(t)}{K_i}\left(1 - l_i \frac{1+\eta}{\eta}\right) \tag{8}$$

Combining (8) with the corresponding conditions for the aggregate economy, we can derive the macroeconomic equilibrium and the dynamics of the aggregate economy. Having determined these, we shall then obtain the dynamics of the distribution of capital and income.

2.3 Macroeconomic Equilibrium

In general, we define economy-wide averages as

$$Z(t) \equiv \frac{1}{N_0 e^{nt}} \int_0^{N_0} Z_i(t) e^{nt} di = \frac{1}{N_0} \int_0^{N_0} Z_i(t) di$$

Summing over all households, equilibrium in the capital and labor markets is described by

$$K = \frac{1}{N_0} \int_0^{N_0} K_i(t) di \tag{9a}$$

$$L = 1 - l = \frac{1}{N_0} \int_0^{N_0} (1 - l_i(t)) di \tag{9b}$$

Note that in equations (3a) and (3b), we have expressed the wage and the return to capital, *w*, *r*, as functions of average capital, *K*, and employment, L. From (9b), we can equivalently write them as functions of aggregate leisure time, $(1-l)$, namely, $w = w(K, l)$ and $r = r(K, l)$.

The key element facilitating the aggregation is that, because of homogeneity and perfect factor markets, all agents choose the same growth rate for the shadow value of capital, as seen in (7). As a result of this, one can show[11]

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C}; \quad \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \qquad \text{for all } i \tag{10a, 10b}$$

That is, all agents will choose the same growth rate for consumption and leisure, implying further that average consumption, *C*, and leisure, *l*, will also grow at the same common growth rates.

Using (6)-(8), and summing over all households, the macro-dynamic equilibrium is described by the following equations, expressed in terms of average capital, leisure (labor supply) and productivity:

$$\dot{K} = A(t)F(K,L) - \frac{A(t)F_L(K,L)l}{\eta} - nK \tag{11a}$$

$$\dot{l} = \frac{1}{G(K,l)}\left\{A(t)F_K(K,L) - \beta - n - (1-\gamma)\frac{F_{KL}K,L)}{F_L}\left[A(t)F(K,L) - \frac{A(t)F_L(K,L)l}{\eta} - nK\right]\right\} \tag{11b}$$

$$\dot{A}(t) = \theta\left(\tilde{A} - A(t)\right) \tag{11c}$$

where

$$G(K,l) \equiv \frac{1-\gamma(1+\eta)}{l} - (1-\gamma)\frac{F_{LL}}{F_L} > 0, \qquad l + L = 1$$

With this aggregation, the complete dynamics of the economy can be represented by the core dynamic system consisting of l_i, l, k_i, k, A, the evolution of which is described by (8), (10b), and (11a)-(11c). There are two key points to note about the macroeconomic equilibrium of the economy. First, the aggregate dynamics are entirely *independent* of any distributional characteristics. This is a consequence of the homogeneity of the underlying utility function and, as previously acknowledged, has been known since Gorman (1953). Second, the dynamics of both capital and leisure depend on the current level of productivity, $A(t)$. At any time, this in turn depends upon the anticipated long-run change, $(\tilde{A} - A_0)$, together with its growth rate along the transitional path, θ.

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[11] To derive (10) we take the time derivative of (6) and combine with (7); see Turnovsky and García-Peñalosa (2008) for a similar example.

In the simulations conducted in Section 4, we solve this aggregate dynamic system using the shooting algorithm of Atolia and Buffie (2010b). While shooting algorithms yield highly accurate solutions, being purely computational procedures they do not provide any economic insights. For this reason, the solution to the linearized dynamic system is presented in the Appendix. This exposition and equation (A.7) in particular, highlights the role played by the initial response of leisure in the aggregate dynamics, as it internalizes the information regarding the time profile of the productivity increase.

The steady-state equilibrium is attained when $\dot{K} = \dot{l} = \dot{A} = 0$, and is described by the following equations

$$\tilde{A}F(\tilde{K},\tilde{L}) - n\tilde{K} = \frac{F_L(\tilde{K},\tilde{L})\tilde{l}}{\eta} \tag{12a}$$

$$\tilde{A}F_K(\tilde{K},\tilde{L}) = \beta + n \tag{12b}$$

$$\tilde{L} + \tilde{l} = 1 \tag{12c}$$

The steady-state conditions (12a)-(12c) determine $\tilde{K}$, $\tilde{l}$, and $\tilde{L}$, given the (known) steady-state level of productivity $\tilde{A}$.[12] Since the steady state is independent of θ, the long-run effects of an increase in productivity are *independent* of the time path by which it is achieved. Thus, a productivity increase of a given magnitude, whether it occurs instantaneously as a discrete jump, or is attained only gradually, will lead to identical steady-state changes for the aggregate economy. However, the transitional responses may be significantly different (as illustrated later in the numerical experiments). In contrast, as we will see below, this has fundamental consequences for wealth and income inequality, both in transition and across steady states.

One further point: from (12a) and (12b), together with the homogeneity of the production function, we immediately infer

$$\tilde{l} > \frac{\eta}{1+\eta} \tag{13}$$

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[12] These equations are standard. Eq. (12a) describes the goods market-clearing condition, (12b) describes the modified golden rule condition, while (12c) is the labor market clearing condition.

This inequality yields a lower bound on the steady-state allocation to leisure that is consistent with a feasible equilibrium. As we shall see below, this condition is critical in characterizing the distributional dynamics.

3. Distributional Dynamics

To derive the distribution dynamics, we need to solve for the remaining variables, namely the relative stock of capital and leisure (k_i and l_i), in the core-dynamic system. An important difference in the distributional dynamics from the aggregate dynamics described in the previous section is that it generates *hysteresis*, i.e., the dependence of long-run outcomes on initial conditions. This is a direct consequence of (10a) and (10b): the proportionality of individual and aggregate consumption and leisure along the transition path.[13] Note that, having already solved for the path of *l*, the path of individual leisure, l_i, is determined except for the initial jump at $t = 0$. This initial jump, therefore, determines the labor supply for a household in the final steady state, which also determines its steady-state relative income and capital stock. As we will show below, the initial response of the labor-leisure choice, in turn, depends critically on the time path of the underlying shock (discrete versus continuous).

3.1 Distribution of Capital (Wealth)

Wealth inequality is characterized in terms of household *i*'s capital stock relative to the average, namely by the evolution of $k_i(t) \equiv K_i(t)/K(t)$. Combining (8) and the analogous aggregate relationship, leads to the following dynamic equation for the relative capital stock:

$$\dot{k}_i(t) = \frac{AF_L(K,l)}{K}\left[1 - \nu_i l\left(1+\frac{1}{\eta}\right) - \left\{1 - l\left(1+\frac{1}{\eta}\right)\right\}k_i\right] \tag{14}$$

where K, l, A evolve in accordance with (11a)-(11c) and the initial relative capital $k_{i,0}$ is given from

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[13] Hysteresis arises because of the relationships $\dot{l}_i/l_i = \dot{l}/l$ and $\dot{C}_i/C_i = \dot{C}/C$, as a result of which l_i and C_i are proportional to *l* and *C*, respectively. In the continuous-time specification, this introduces a zero root into the individual-level dynamics. Hysteresis has been shown to occur in other macrodynamic models. The earliest example is the Blanchard and Summers (1986) analysis of the European unemployment of the 1980s, while Sen and Turnovsky (1990) show how it arises in the context of a small open economy facing a perfect international capital market.

the initial endowment. Since $\dot{l}_i/l_i = \dot{l}/l$ we may write $l_i = \nu_i l$, where $\left(1/N_0\right)\int_0^{N_0}\nu_i di = 1$ and ν_i (relative leisure) is constant for each *i*, and to be determined. Setting $\dot{k}_i = 0$, and recalling (13), this leads to the following positive long-run relationship between relative leisure and relative capital:

$$\tilde{l}_i - \tilde{l} = \left(\tilde{l} - \frac{\eta}{1+\eta}\right)\left(\tilde{k}_i - 1\right) \text{ for each } i \tag{15}$$

While our simulations employ shooting algorithms to solve (14) for the time path of the relative stock of capital, in conjunction with the aggregate dynamics specified in (11a)-(11c), the intuition underlying the dynamic structure can be better understood by characterizing a linear approximation. To do this, we linearize (14) around the steady state defined in (12) and (15). In the Appendix we show that the resulting bounded solution for the relative stock of capital is:

$$k_i(t) - 1 = \delta(t)(\tilde{k}_i - 1) \tag{16a}$$

where

$$\delta(t) \equiv \left[1 + \frac{\tilde{A}F_L}{\tilde{K}}\int_t^\infty\left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\beta(\tau - t)}d\tau\right] \tag{16b}$$

Setting $t = 0$ in (16b), we can solve for agent *i*'s steady-state relative capital stock:

$$k_{i,0} - 1 = \delta(0)(\tilde{k}_i - 1) = \left(1 + \frac{\tilde{A}F_L}{\tilde{K}}\int_0^\infty\left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\beta\tau}d\tau\right)(\tilde{k}_i - 1) \tag{17}$$

where $k_{i,0}$ is given from the initial distribution of relative capital endowments.

Equations (16) and (17) characterize the evolution of relative capital. First, given the time path of the aggregate economy, in particular $l(\tau)$, and the distribution of initial capital endowments, (17) determines the steady-state distribution of capital, $(\tilde{k}_i - 1)$. Once this is known, (16a) and (16b) then describe the time path of relative capital, which can be expressed in the convenient form[14]

$$k_i(t) - \tilde{k}_i = \left[\frac{\delta(t) - 1}{\delta(0) - 1}\right]\left(k_{i,0} - \tilde{k}_i\right) \tag{18}$$

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[14] Note also that the constant $\nu_i = l_i/l$ can be determined from (15), and is given by $\nu_i = 1 + \left(1 - (1/\tilde{l})(\eta/(1+\eta))\right)(\tilde{k}_i - 1)$.

Because of the linearity of (16), (17) and (18), we can immediately transform these expressions into corresponding relationships for the standard deviation of the distribution of capital over agents, which serves as a convenient measure of wealth inequality. Specifically, corresponding to these three equations we obtain

$$\sigma_k(t) = \delta(t)\tilde{\sigma}_k \tag{16'}$$

$$\sigma_{k,0} = \delta(0)\tilde{\sigma}_k \tag{17'}$$

$$\sigma_k(t) - \tilde{\sigma}_k = \left(\frac{\delta(t)-1}{\delta(0)-1}\right)(\sigma_{k,0} - \tilde{\sigma}_k) \tag{18'}$$

The crucial difference between this analysis and previous work lies in the evolution of the productivity shock *A*(*t*), which is reflected in the time path of $\delta(t)$. In the case where the complete productivity increase occurs instantaneously, $l(\tau) - \tilde{l} = \left(l(0) - \tilde{l}\right)e^{\mu\tau}$ and $\delta(t)$, $\delta(0)$ simplify to

$$\delta(t) = 1 + \left(\frac{1}{\beta - \mu}\right)\frac{\tilde{A}F_L(\tilde{K},\tilde{L})}{\tilde{K}}\left(1 - \frac{l(t)}{\tilde{l}}\right);\ \delta(0) = 1 + \left(\frac{1}{\beta - \mu}\right)\frac{\tilde{A}F_L(\tilde{K},\tilde{L})}{\tilde{K}}\left(1 - \frac{l(0)}{\tilde{l}}\right)$$

Note that only the current allocation of time to leisure relative to its steady-state allocation is relevant in determining current wealth inequality relative to its long run level. When the productivity increase occurs gradually over time, the entire time profile of $A(t)$, as reflected in $l(t)$, also needs to be taken into account; see equation (A.5) in the Appendix.

In general, the term $\delta(t)$ in (16b) highlights the role played by the time path of leisure in determining the long-run change in wealth inequality. For example, if during the transition $l(\tau) < \tilde{l}$, so that leisure approaches its long-run steady state from below, then $\delta(t) < 1$ and wealth inequality will decline over time; see (18’). As our simulations show, this is the case for a discrete productivity increase, where leisure increases (following an initial drop) and wealth inequality declines monotonically over time. On the other hand, a gradual productivity increase leads to an initial increase in leisure, taking it initially above its new (lower) steady-state level. But since the transitional path is U-shaped, eventually approaching $\tilde{l}$ from below, whether inequality rises or falls over time depends upon the extent to which $l(\tau) > \tilde{l}$ during the early phase of the adjustment.

The other point to observe is that the closer $l(\tau)$ is to its steady state, $\tilde{l}$, the smaller is the subsequent adjustment in $l(t)$, and hence the smaller is the overall change in the distribution of wealth. This is because if the economy and therefore all individuals fully adjust their respective leisure times instantaneously, they will all accumulate wealth at the same rate, causing the wealth distribution to remain unchanged.

3.2 Distribution of Income

Defining household *i*'s income as $Y_i(t) = r(t)K_i(t) + w(t)\left(1 - l_i(t)\right)$, and average economy-wide income as $Y(t) = r(t)K(t) + w(t)\left(1 - l(t)\right)$, we define relative income by $y_i(t) = Y_i(t)/Y(t)$. This leads to the following equation of motion for relative income:[15]

$$y_i(t) - 1 = \varphi(t)\left[k_i(t) - 1\right] \tag{19}$$

where

$$\varphi(t) = 1 - \left(1 - s(t)\right)\left[1 + \frac{l(t)}{1 - l(t)}\left(1 - \frac{1}{\tilde{l}}\frac{\eta}{1+\eta}\right)\frac{1}{\delta(t)}\right] \tag{20}$$

s(*t*) represents the share of capital in total output. Again, because of the linearity of (19) in $(k_i(t) - 1)$, we can express the relationship between relative income and relative capital in terms of corresponding standard deviations of their respective distributions, namely

$$\sigma_y(t) = \varphi(t)\sigma_k(t) \tag{19'}$$

4. Numerical Analysis

The model set out in Sections 2 and 3 will be solved and analyzed numerically, using the following functional forms and parameterization:[16]

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[15] See Turnovsky and García-Peñalosa (2008) for details regarding the derivation of the equations of motion for relative income and capital.

[16] These parameters are generally standard in the literature and noncontroversial. For an extensive discussion of the calibration of the Ramsey model, see Cooley (1995).

Utility function:	$\gamma = -1.5,\ \beta=0.04,\ \eta = 1.75$
Production function:	$Y = A(\alpha K^{-\rho} + (1-\alpha)L^{-\rho})^{-1/\rho}$
	$A_0 = 1,\ \alpha = 0.4$
Elasticity of substitution:	$\sigma = 0.75,\ 1,\ 1.25$
Productivity level:	$A_o = 1,\ \tilde{A} = 1.5$
Productivity growth:	$\theta = 0.1$
Population growth:	$n = 0.015$

Preferences remain specified by a constant elasticity utility function with an intertemporal elasticity of substitution of 0.4, while the elasticity of leisure in utility is 1.75. The production function is of the Constant Elasticity of Substitution (CES) form, where we allow the elasticity of substitution to vary between $\sigma = 0.75$ (low substitution), $\sigma = 1$ (Cobb-Douglas), and $\sigma = 1.25$ (high substitution).

We adopt the following strategy. We consider the aggregate and distributional consequences of a 50% increase in productivity (A increases from its benchmark value of $A_0 = 1$ to $\tilde{A} = 1.5$), which we allow to take effect in two alternate ways: (i) an immediate one-time unanticipated jump in productivity from 1 to 1.5. This represents a *discrete* increase in *A*, and corresponds qualitatively to much of the previous literature, and (ii) the same increase in *A* ($A_0 = 1$ to $\tilde{A} = 1.5$) taking place *gradually* over time, where $A(t)$ adjusts at the (known) rate $\theta = 10$ % per period (year). In the latter case, the higher productivity level is achieved asymptotically. As a result, the instant it starts to increase, the subsequent levels of productivity are fully anticipated along the transition path. For each of these scenarios, we numerically characterize the economy's aggregate and distributional dynamics for the three specified values of the elasticity of substitution in production.

As already mentioned, we use a forward shooting algorithm proposed recently by Atolia and Buffie (2010b) to solve for systems having unit (zero) roots. This algorithm has the convenience of enabling one to solve the entire system together to obtain the global nonlinear saddle path.[17]

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[17] Alternatively, because of the block-recursive structure one can solve the problem sequentially, first solving for the aggregate dynamics using a reverse-shooting procedure in Atolia and Buffie (2010a) and then feeding this into the "individual-level" dynamics.

4.1 An Increase in Productivity: Discrete versus Continuous Adjustment

4.1.1. Aggregate Dynamics

Fig. 1 depicts the transition paths for the aggregate variables, K and l, corresponding to the two specifications of the productivity increase and the three values of the elasticity of substitution in production, σ. Although the long-run responses of leisure (labor supply), capital, and output are identical in both cases [consistent with eq. (12)], their short-run responses and transitional paths are dramatically different. Irrespective of the elasticity of substitution, we see that a discrete productivity increase causes leisure to *decline* instantaneously (labor supply to increase), after which it immediately reverses and increases monotonically to its new steady-state level, which lies above or below its original pre-shock level, depending upon whether $\sigma \gtrless 1$. By contrast, a continuous (gradual) productivity increase causes an immediate *increase* in leisure, which is then immediately reversed, overshooting its long-run equilibrium during the subsequent decline to steady state. Similar differences are displayed in the initial phases of the transitional path for capital. While a discrete productivity increase leads to a gradual monotonic accumulation of capital to the new steady-state, a continuous increase actually leads to a short-run *decumulation* (for about 10 years), before capital accumulation begins. This gives the time-path of the capital stock a U-shaped trajectory, the depth of which increases as σ declines. These differences in the adjustments of leisure (labor supply) and capital translate directly into differences in the dynamic adjustment of output (not shown). A discrete productivity increase causes output to increase instantaneously before increasing in transition, while a continuous shock causes output to fall on impact. For the reasons discussed in Section 3.1 [particularly eq. (17)], and as we shall illustrate soon, these transitional differences in the adjustment of leisure, in particular, have a critical impact on the economy's distributional dynamics.

Why does the dynamic response of the aggregate economy differ so dramatically for the two types of productivity change? The answer lies in the information being revealed to the agent on impact of the shock, relative to the time path of the higher productive capacity associated with the long-run realization of the shock. For a one-time discrete increase in A, the enhanced long-run productivity is fully realized instantaneously by the agent, and immediately raises the marginal product of both labor and capital. Consequently, labor supply immediately increases on impact of the shock, and the enhanced productivity of capital generates immediate incentives for capital accumulation, and the stock of capital begins to rise. Output increases instantaneously, and can accommodate the increase in consumption associated with the higher level of permanent income resulting from the productivity increase.

In contrast, if the productivity increase impinges on the economy only as a gradual process, the enhanced productive capacity necessary to support the increase in consumption will take effect only over time. In the short run, the long-run change in the level of productivity is fully anticipated by the agent, thereby increasing permanent income, and raising aggregate current consumption. But the immediate increase in productive capacity is reflected only as an increase in its growth rate, $\dot{A}(0) = \theta(\tilde{A} - A_0)$. Thus, since the instantaneous *level* of productivity remains unchanged, current output cannot rise, and the increase in consumption resulting from the anticipation of higher future income is achieved through reduced investment and a decline of the capital stock. In fact, the increase in short-run consumption and lower productivity (relative to the long-run) causes the agent to increase leisure, which causes output to also decline on impact of the shock.

4.1.2 Distributional Dynamics: Diversity of Transitory and Long-run Outcomes

Fig. 2 illustrates the dynamic responses of the distributions of capital (wealth) and income to the two specifications of the productivity increase, corresponding to the three values of the elasticity of substitution, $\sigma = 0.75, 1, 1.25$. The most striking feature of these distributional time paths is that not only do the two specifications of productivity increases have contrasting effects on the short-run distributions of capital and income, but contrary to the aggregate economy in fig. 1, the long-run effects are also dramatically different. In other words, while the aggregate economy reaches identical steady-states, irrespective of whether the productivity change occurs discretely or gradually, the distributions do not. This reflects the fact that the long-run distributions of wealth and income are *path dependent*, depending critically on the underlying process through which the steady-state equilibrium is attained (i.e., whether the productivity increase occurs discretely or gradually).

The contrasts between the dynamic adjustments in wealth and income distributions in response to the two types of productivity increase are sharpest for low values of σ. Focusing first on $\sigma = 0.75$ and 1 (in fig. 2), we see that a discrete productivity increase generates a gradual monotonic decline in wealth inequality over time. Income inequality increases instantaneously in the short run, before declining monotonically to an equilibrium value that is below its pre-shock level. By contrast, a gradual increase in productivity of the same magnitude increases wealth inequality in transition. However, the time path of wealth inequality is non-monotonic and follows an inverted U-shaped trajectory. On the other hand, income inequality falls instantaneously on impact of the shock, before it rises in transition to a higher equilibrium relative to its pre-shock benchmark. Like wealth inequality, the transitional adjustment of income inequality also follows a non-monotonic inverted U-shaped trajectory, peaking at around 15 years. For high values of the elasticity of substitution, such as $\sigma = 1.25$ (the third panel of fig. 2), the differences in the responses are less pronounced. The instantaneous and transitional responses of wealth and income inequality remain as above. But now in the long-run, both wealth and income inequality decline for the two types of productivity increases, though the continuous productivity shock leads to higher levels of long-run wealth and income inequality relative to the discrete shock.

The long-run outcome with a gradual productivity increase depends on the rate of productivity growth θ. Fig. 3 shows the dynamics of wealth and income inequality for alternative values of θ for $\sigma = 0.75$. In particular, the following patterns can be detected:

(i) The slower the rate at which a given increase in productivity is achieved (longer is the "catch-up" time), the greater are the long-run increases in wealth and income inequality.

(ii) Slower productivity growth increases the *persistence* of both wealth and income inequality.

The experiments illustrated in fig. 3 indicate that countries that experience a faster "catch-up" process will also experience smaller increases in inequality, which in turn are also less persistent over time, compared to countries that experience slower catch-up speeds. These results are consistent with the fact that Asian economies that have been developing at faster rates than Latin

American economies also have substantially less wealth and income inequality.[18,] We should note that we have performed these experiments for much larger values of θ and do indeed find that as θ becomes large, the time paths of wealth and income distributions converge to those associated with the discrete productivity increase.

More generally, we can conclude that a diverse range of distributional outcomes are possible along transitional paths as well as in the long run, depending on the time path followed by the underlying productivity shock. In particular, two countries starting with the same initial distribution of wealth, and having the same per-capita income today may have very different degrees of wealth and income inequality, if they have experienced the same overall productivity increase but at different rates. Thus, the diversity of growth experiences of different countries may be reflected in the cross-section diversity of wealth and income inequality.

As stressed above, the critical element in determining the evolution of the distributions is the time path followed by average leisure, $l(t)$ and its implications for the rates of factor return. To assist in understanding the intuition underlying these diverse time paths, it is convenient to recall (15) and (17), which we can rewrite as

$$l_i - \tilde{l} = \frac{1}{\delta(0)}\left(\tilde{l} - \frac{\eta}{1+\eta}\right)\left(k_{i,0} - 1\right)$$

$$\tilde{k}_i - 1 = \frac{1}{\delta(0)}\left(k_{i,0} - 1\right)$$

$$\delta(0) \equiv \left[1 + \frac{\tilde{A}F_L}{\tilde{K}}\int_0^{\infty}\left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\beta\tau}d\tau\right]$$

A *discrete* increase in productivity raises the return to both labor and capital. On impact, average leisure, $l(0)$, falls as agents substitute toward labor supply. This decrease in average leisure increases $\delta(0)$, implying an overall monotonic reduction in wealth inequality over time. As a consequence of this anticipated decline in long-run wealth inequality, the amount of leisure time chosen by agents with above (below) average wealth declines (increases). That is, wealthier people initially increase their work time, while poorer people work less, and income inequality increases.

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[18] The Gini coefficients for income are typically 10 or more points higher for Latin American countries than they are for Asian countries.

Over time, as average leisure increases, the relative income of agents having above-average wealth declines, and income inequality declines accordingly.

In contrast, a *continuous* increase in productivity leads to an initial increase in $l(0)$, taking it above $\tilde{l}$. This increase in the initial average leisure decreases $\delta(0)$, implying an increase in wealth inequality in the long-run; see (17). This initial increase in leisure is, however, immediately reversed and falls below $\tilde{l}$ during the subsequent transition. The net effect of this on the evolution of wealth inequality depends on whether the positive amounts of $l(\tau)-\tilde{l}$ during the early phase dominate the negative amounts in the latter phase. This depends upon the elasticity of substitution. If $\sigma \leq 1,$ we see that the early excesses dominate $\left(\delta(0) < 1\right)$, implying a non-monotonic inverted-U time path for the distribution of wealth, as it approaches its higher steady-state level.

As a consequence of the increase in long-run wealth inequality, the amount of leisure time chosen by people with above (below) average wealth increases (decreases), causing income inequality to decline. Over time, as average leisure decreases, the relative income of agents having above average wealth increases, and income inequality increases accordingly, although it reflects the inverted-U time path of wealth inequality. In the case of a high elasticity of substitution, $\sigma = 1.25$, most of the time $l(\tau) < \tilde{l}$, so that the downward pressure on wealth inequality dominates and except for a brief period at the start of the transition, wealth inequality declines over time. As a consequence of this, income inequality also declines, albeit slightly over time.

5. The Growth-Inequality Relationship

As noted in the introduction, the relationship between growth and inequality has been extensively discussed. This paper belongs to a growing strand of research that contends that these processes are endogenous outcomes in the course of economic development. The critical issue then concerns the underlying mechanisms or shocks that affect the joint evolution of growth and inequality. A comprehensive survey by Solimano (1998) identifies several factors, such as the national savings rate, investment in physical or human capital, productivity growth, education, capital markets, and public policy that can significantly influence the growth-inequality relationship. Viewed in this context, can we address the consequences of productivity growth (technological change) for the growth-inequality relationship, both in transition and in the steady-state?

In considering this issue, we focus on two aspects: (i) what is the relationship between the evolution of per-capita income and income inequality in the face of a productivity increase? In particular, what are the consequences (if any) of the nature of productivity change for the well-known empirical Kuznets' curve? (ii) does the distance from the technology frontier have implications for inequality and its persistence? In other words, do countries that require a lot of "catching-up" with regard to technology also generate more inequality in the process?

5.1. Inequality and Per-capita Income: The Kuznets Curve Revisited

The celebrated Kuznets curve yields an inverted U-shaped relationship between income inequality and a country's level of development, say the level of per-capita income. The idea is that in the initial stages of development, the accumulation of physical capital is important and, therefore, capital-rich agents gain disproportionately relative to the capital-poor. Consequently, at low levels of per-capita income, income inequality rises. After a certain level of income or development is attained, physical capital becomes less important for development (possibly due to the emergence of human capital and knowledge), and income inequality declines relative to the increase in per-capita income.[19] Clearly, such a relationship cannot be generated by an unanticipated *discrete* increase in productivity. This is because after the initial jumps in output and income inequality, the subsequent increases in income are associated with a monotonic decline in income inequality.

On the other hand, our more flexible specification where the level of productivity increases gradually is more promising. As we have already noted in figs. 2 and 3, after an initial decline on impact, income inequality does indeed follow an inverted U-shaped trajectory over time. Therefore, fig. 4 plots wealth and income inequality relative to (normalized) per-capita income following the realization of a gradual change in productivity, for the three values of the elasticity of substitution in production (the time range considered in fig. 4 starts from the period after the instantaneous adjustments are complete). As is clearly evident, elements of a Kuznets curve do emerge for both

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[19] Galor and Moav (2004) develop an elegant theory of growth in which human capital accumulation replaces physical capital accumulation as the prime engine of growth in the process of development. They argue that their theory offers a unified explanation for the effect of income inequality on the process of economic growth.

wealth and income inequality along the transition path, after the initial adjustments have been completed. Thus, our flexible, more general specification of productivity change not only allows a diversity of distributional outcomes depending on nature of growth experiences, but simultaneously generates a Kuznets type relationship during transition.

It is important to stress that the inverted U-shaped relationship between inequality and per-capita income generated by our model relies on a mechanism that is very different from the traditional explanations of the Kuznets curve. While the sectoral composition of capital, labor, and knowledge is the traditionally understood mechanism behind the Kuznets curve, we focus on the role of consumption-smoothing in response to a gradual, but anticipated, change in productivity, and its effects on investment and the choice of labor and leisure. Recently, Piketty (2006) has discussed the role of "waves" of technological change (such as general purpose technologies) that can generate waves of an inverted U-shaped relationship between inequality and income over time. Our findings fit nicely into that story.

How robust is the Kuznets curve and does the rate of "catch-up" matter? Fig. 5 reports some sensitivity analysis for the Kuznets curve discussed above with respect to the speed with which the productivity change is introduced. Specifically, we consider the following experiment: for a given increase in the *level* of productivity (50% increase in A from 1 to 1.5), we consider three alternative *rates* of change, i.e., $\theta = 0.05$, 0.1, and 0.2. These three cases characterize scenarios where a given "catch-up" in productivity occurs slowly ($\theta = 0.05$), at the benchmark rate ($\theta = 0.1$), or quickly ($\theta = 0.2$). Wealth and income inequality are plotted relative to normalized per-capita income. The corresponding time paths are drawn in fig. 5 for the case of an elasticity of substitution in production equal to 0.75. As we can see, the essence of the Kuznets curve that is generated by a continuous productivity change is robust with respect to the speed of productivity growth in transition.

5.2. Productivity Gaps and Inequality

Fig. 6 depicts the sensitivity of the distributional dynamics for different levels of productivity "catch-up," for any given rate of productivity growth. Specifically, for a given productivity growth rate ($\theta = 0.1$), we consider three magnitudes of "catch-up," i.e., $(\tilde{A} - A_0)/A_0 = 0.25$, 0.5, and 1. In other words, these characterize cases where the difference between the initial and steady-state levels of productivity are small (25%), at the benchmark (50%), or large (100%). The following patterns can immediately be identified:

(i) The larger the initial gap in productivity, the greater is the long-run rise in wealth and income inequality.

(ii) The higher the initial productivity gap, the larger is the short-run increase (following the instantaneous responses) in wealth and income inequality, relative to their initial (pre-shock) levels.

These results indicate that the further away a country is from its long-run technology or productivity frontier, the larger will be the inequality generated in converging to the frontier. Countries that are closer to their long-run technology frontier will have less persistent inequality than countries that are further away.

6. Conclusions

The relationship between growth and inequality is one of the most fundamental (and elusive) ones in development economics. We employ a general-equilibrium heterogeneous-agent growth model with certain well-known aggregation properties that generates hysteresis in the dynamics of wealth and income inequality, but not in the aggregate dynamics. This is manifested in dynamic adjustments of distributional variables being dependent on the initial response of leisure (labor supply), following a structural change. Since with forward-looking agents, the initial response is dependent upon future anticipations of these structural changes, this implies further that their long-run effects on the distributions of wealth and income are path-dependent.

This paper, therefore, examines the consequences of the time path of a productivity change on the distributions of wealth and income. As a benchmark, we have considered an increase in productivity of a given magnitude, and compared the distributional implications when (i) it is introduced gradually, with (ii) the more conventional situation where it all occurs instantaneously. The main conclusion is that the time path along which a productivity increase of a given magnitude is introduced has dramatic consequences for both wealth and inequality. In general we find that the gradual introduction of a given productivity increase has adverse distributional consequences, and certainly much more adverse than when they are introduced instantaneously.

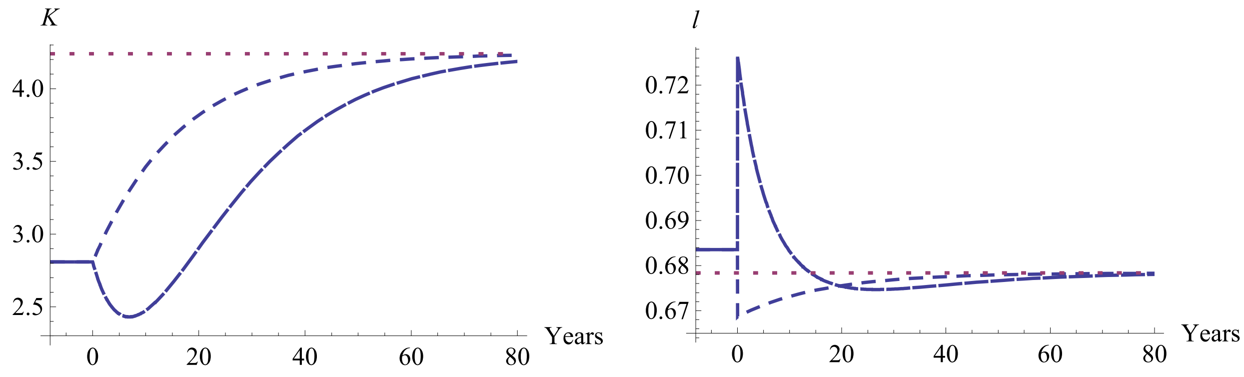
This has important consequences for policy. It suggests that if the government introduces some productivity-enhancing policy, such as investment in infrastructure, with the objective of stimulating economic growth, it should do so rapidly. While delay and gradual implementation will have no adverse permanent effects on the aggregate performance of the economy, they will generate expectational effects on the labor-leisure choice that will lead to a worsening of wealth and income inequality in the long-run.

The paper also has some interesting implications for the empirical relationship between inequality and growth. We show that a gradual productivity change can indeed generate a Kuznets-type inverted U-shaped relationship between inequality and per-capita income, with a diverse set of possible long-run outcomes for inequality across structurally similar countries. This is also shown to be a very robust finding. Further, we show that a country's distance from the technology frontier has important implications for inequality and its persistence. As Ray (1998, chapter 7) points out, one problem with the generally inconclusive empirical literature on the Kuznets curve is the lack of an underlying theory that can generate a testable specification of this relationship. By articulating an explicit mechanism through which per-capita income and inequality can be linked – namely the differences in the dynamic responses of the labor-leisure choice between the rich and the poor in conjunction with the time profile of structural changes – and in the context of the simple one-sector neoclassical growth model, our results provide a step in that direction, one that may provide a useful basis for future empirical work.

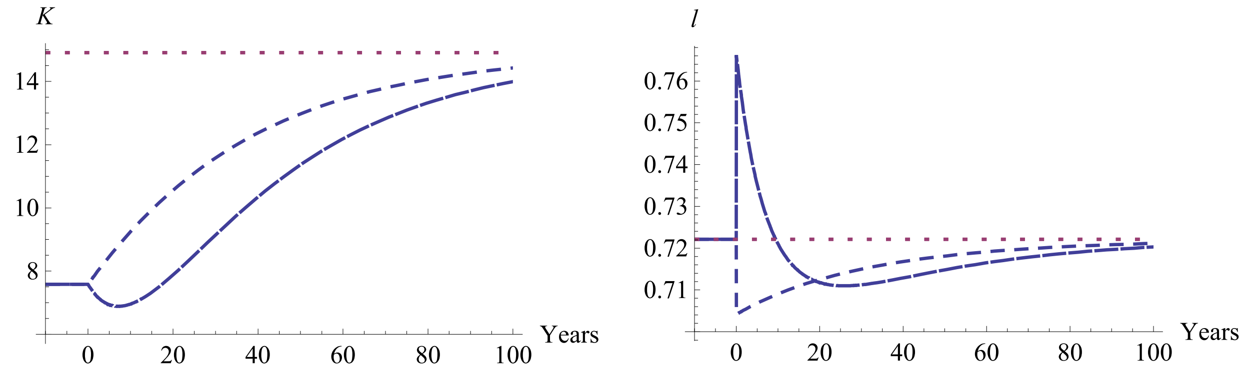
Finally, although we choose to focus on the time path followed by a productivity increase, the issue is in fact a generic one, applying to any form of structural change. As long as the time path for wealth inequality depends upon the initial response of leisure, its subsequent time path and in turn that of income inequality will depend on the path followed by the structural change.

Figure 1 : Aggregate Dynamics

σ = .75



σ = 1.00



σ = 1.25

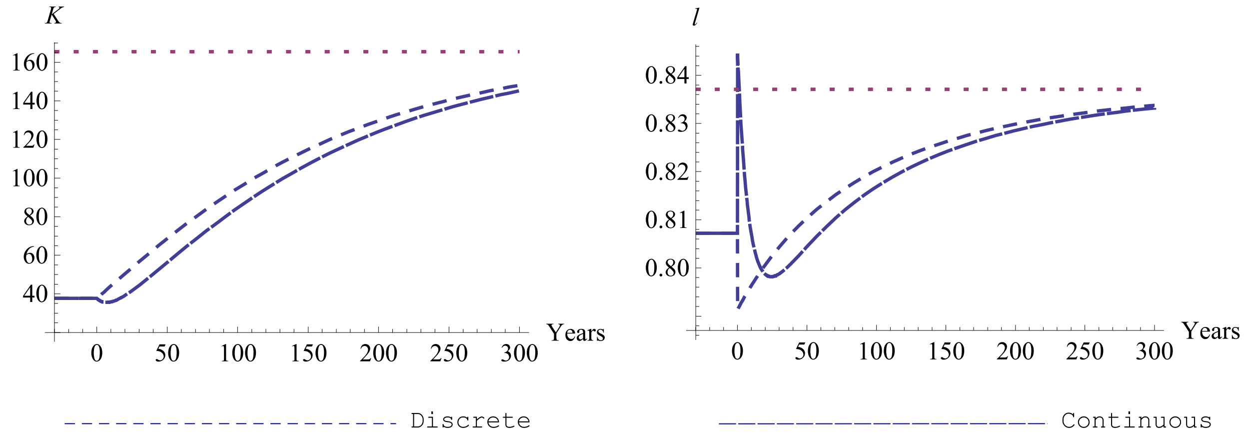


Figure 2 : Distributional Dynamics

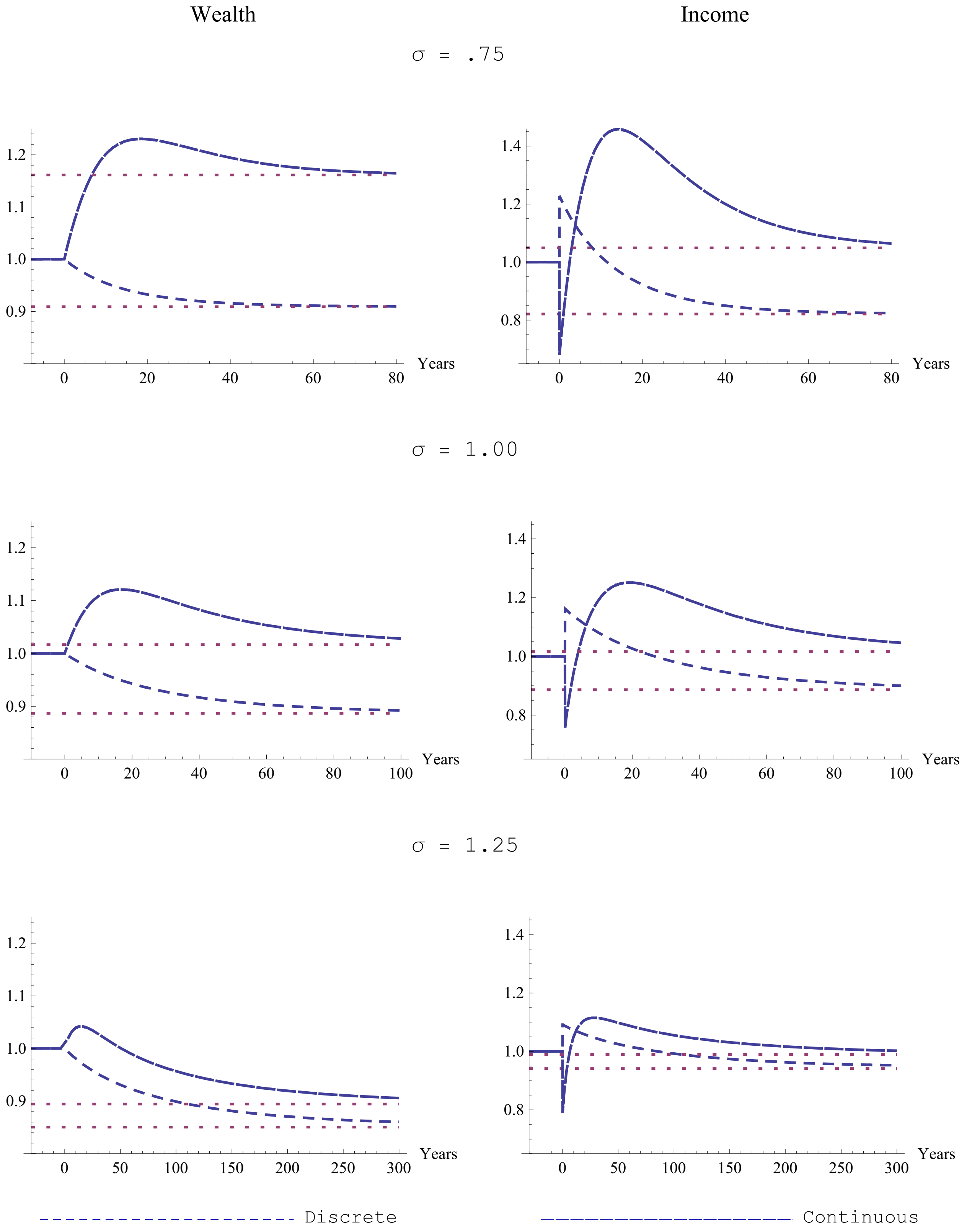
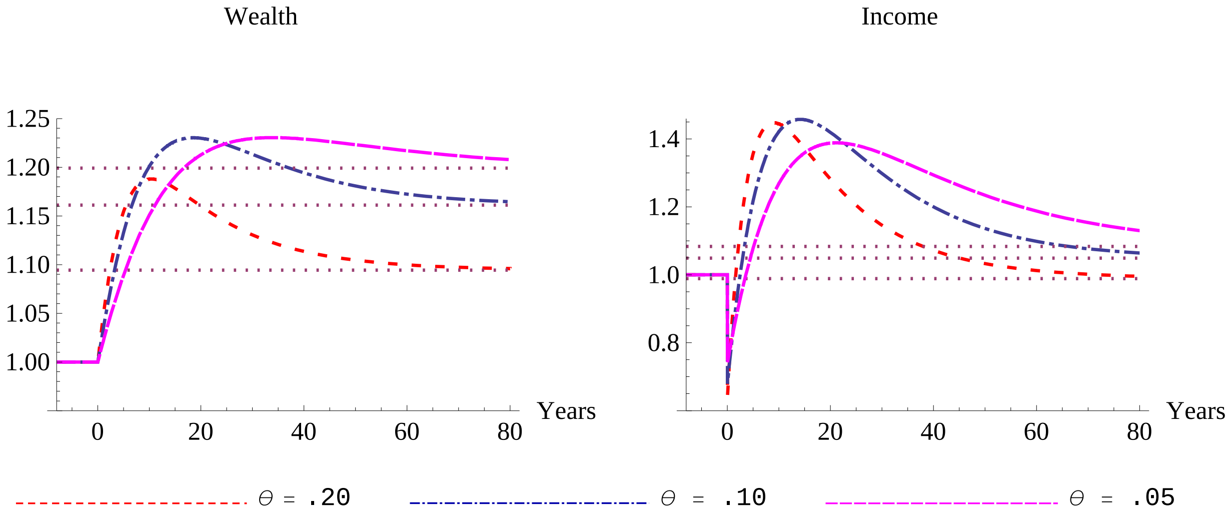
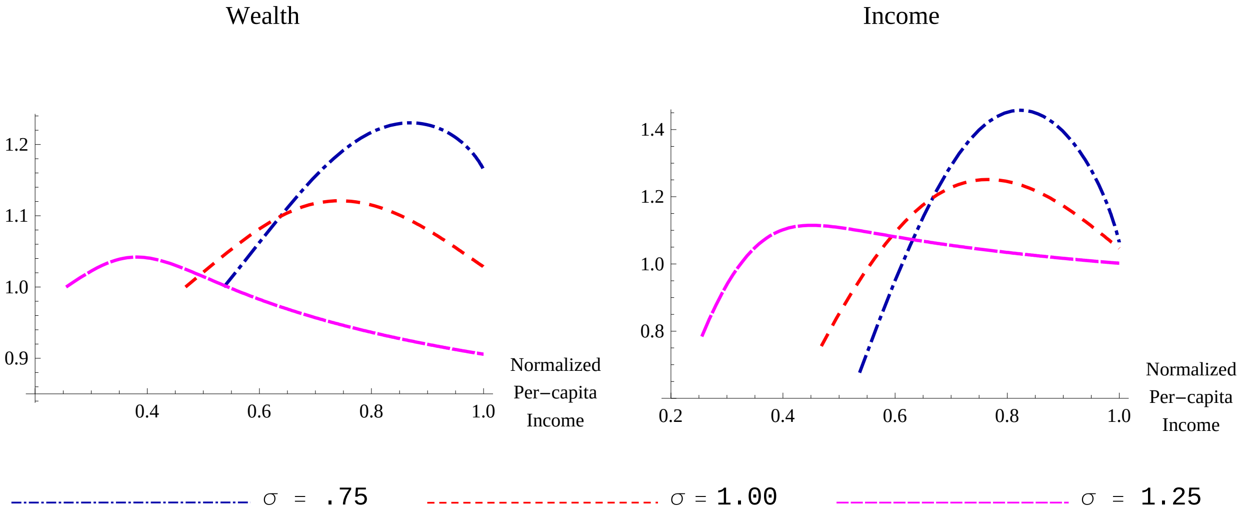


Figure 3 : Robustness of Distributional Dynamics


Wealth

Income

Figure 4 : Kuznets' Curve


Wealth

Income

Figure 5 : Robustness of Kuznets' Curve

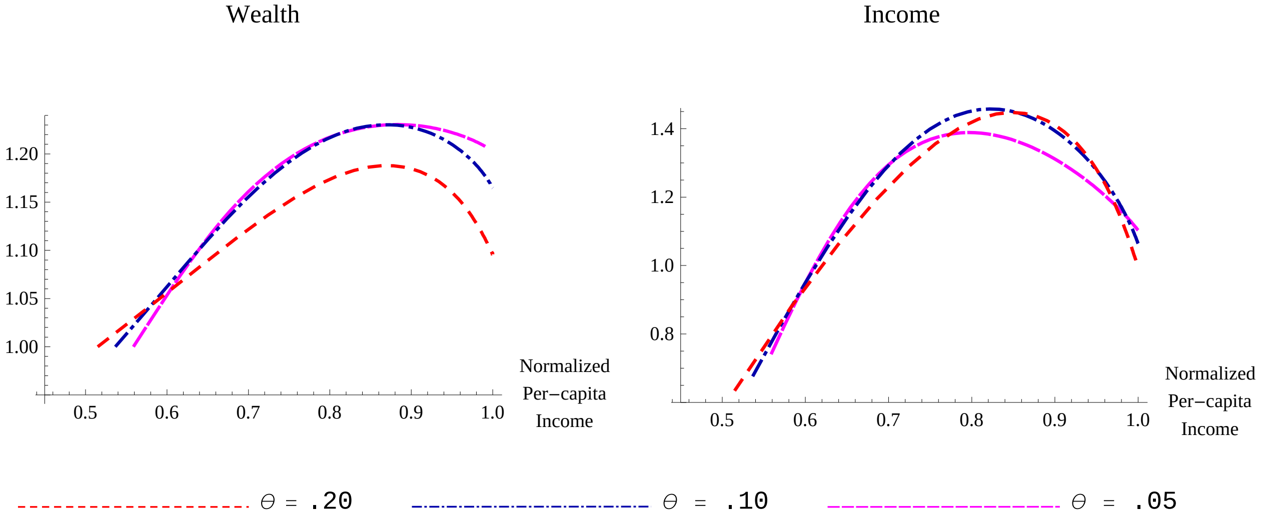
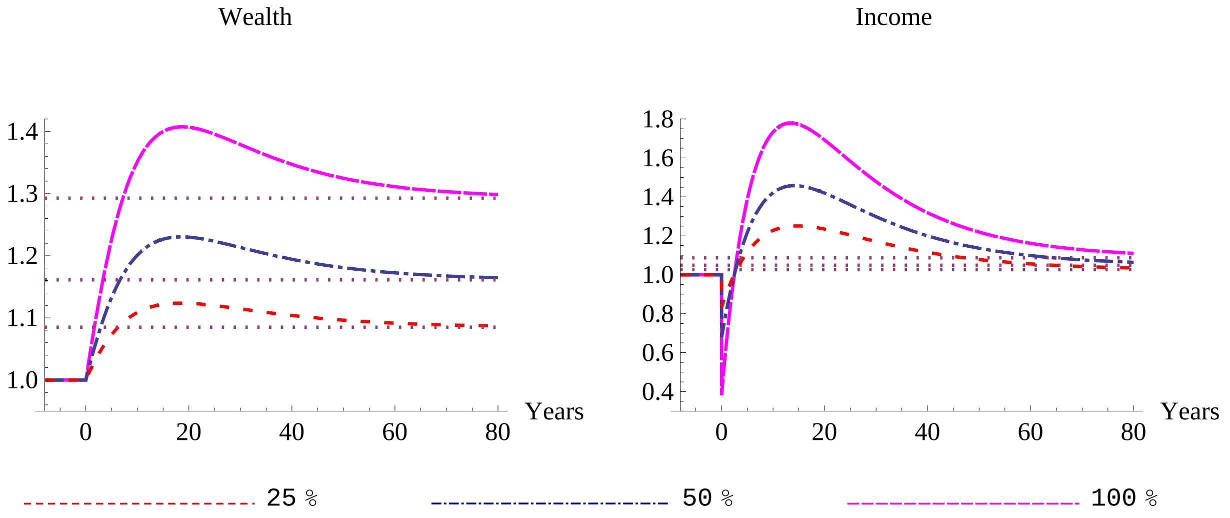


Figure 6 : Productivity Gaps, Catch up, and Distributional Dynamics



Appendix

Linearized Solution for the Dynamic System for a Gradual Increase in Productivity

A.1 Dynamics of aggregate economy

The dynamic system for the aggregate economy is described by (7a)-(7c):

$$\dot{K} = A(t)F(K,L) - \frac{A(t)F_L(K,L)l}{\eta} - nK \tag{A.1a}$$

$$\dot{l} = \frac{1}{G(K,l)}\left\{A(t)F_K(K,L) - \beta - n - (1-\gamma)\frac{F_{KL}K,L)}{F_L}\left[A(t)F(K,L) - \frac{A(t)F_L(K,L)l}{\eta} - nK\right]\right\} \tag{A.1b}$$

$$\dot{A}(t) = \theta\left(\tilde{A} - A(t)\right) \tag{A.1c}$$

where

$$G(K,l) \equiv \frac{1-\gamma(1+\eta)}{l} - (1-\gamma)\frac{F_{LL}}{F_L} > 0, \qquad l + L = 1$$

Linearizing (A1.a) and (A1.b) around the steady state equilibrium (8a)-(8c) and $\tilde{A}$, yields

$$\begin{pmatrix}\dot{K}\\ \dot{l}\\ \dot{A}\end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ 0 & 0 & -\theta\end{pmatrix}\begin{pmatrix}K-\tilde{K}\\ l-\tilde{l}\\ A-\tilde{A}\end{pmatrix} \tag{A.2}$$

where,

$$a_{11} = \tilde{A}\left[F_K - \frac{F_{KL}l}{\eta}\right] - n\,; \;\; a_{12} = -\tilde{A}F_L\left[1+\frac{1}{\eta}\right] + \tilde{A}F_{LL}\frac{\tilde{l}}{\eta}\,; \;\; a_{13} = \frac{n\tilde{K}}{\tilde{A}}$$

$$a_{21} = \frac{1}{G}\left[\tilde{A}F_{KK} - (1-\gamma)\frac{F_{KL}}{F_L}a_{11}\right]; \;\; a_{22} = \frac{1}{G}\left[-\tilde{A}F_{KL} - (1-\gamma)\frac{F_{KL}}{F_L}a_{12}\right]; \;\; a_{23} = \frac{1}{G}\left[-F_K - (1-\gamma)\frac{F_{KL}}{F_L}a_{13}\right]$$

This is a third order system with two stable eigenvalues; (i) $\mu,$ where $\mu < 0$ is the negative root to $\mu^2 - (a_{11}+a_{22}) - (a_{11}a_{22} - a_{12}a_{21}) = 0$ and (ii) $-\theta$. With $A(t)$ evolving exogenously in accordance with (A.1c), (A.2) can also be viewed as being a second-order non-homogeneous differential equation system in K, l, where the forcing term is time-varying. Using standard solution procedures, the general form of the stable solution is

$$K(t) = \tilde{K} + Ce^{\mu t} + \varphi_1 (A_0 - \tilde{A})e^{-\theta t} \tag{A.3a}$$

$$l(t) = \tilde{l} + \left(\frac{\mu - a_{11}}{a_{12}} \right) \left[K(t) - \tilde{K} \right] + \varphi_2 \left(A_0 - \tilde{A} \right) e^{-\theta t} \tag{A.3b}$$

where C is arbitrary and

$$\varphi_1 \equiv \left[\frac{a_{12}a_{23} - a_{13}(a_{22} + \theta)}{\Delta} \right] \tag{A.4a}$$

$$\varphi_2 \equiv \left[\frac{a_{21}a_{13} - a_{23}(a_{11} + \theta)}{\Delta} \right] \tag{A.4b}$$

and $\Delta \equiv (a_{11} + \theta)(a_{22} + \theta) - a_{12}a_{21}$. As long as the two stable eigenvalues are distinct – a very weak restriction – $\Delta \neq 0$.[20] Imposing the initial condition, $K(0) = K_0$ by setting $t = 0$ in (A.10a) yields

$$C = (K_0 - \tilde{K}) - \varphi_1 (A_0 - \tilde{A})$$

so that starting from $K(t) = K_0, A(t) = A_0$ the time paths for $K(t)$ and $l(t)$ are

$$K(t) = \tilde{K} + [(K_0 - \tilde{K}) - \varphi_1 (A_0 - \tilde{A})]e^{\mu t} + \varphi_1 (A_0 - \tilde{A})e^{-\theta t} \tag{A.5a}$$

$$l(t) = \tilde{l} + \left(\frac{\mu - a_{11}}{a_{12}} \right) \left[K(t) - \tilde{K} \right] + \varphi_2 \left(A_0 - \tilde{A} \right) e^{-\theta t} \tag{A.5b}$$

The solution (A.5a)-(A.5b) represents a general form, which covers all ranges of θ, including the conventional case where the full change in productivity occurs as a discrete change at time zero. This is obtained by letting $\theta \to \infty$ in which case $\varphi_1, \varphi_2 \to 0$, and the solution reduces to the following relationships:

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t} \tag{A.5a'}$$

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[20] The case where $\theta = -\mu$ can also be easily solved.

$$l(t) = \tilde{l} + \left(\frac{\mu - a_{11}}{a_{12}}\right)\left[K(t) - \tilde{K}\right] \tag{A.5b'}$$

In the long-run, as $t \to \infty$, $K(t) \to \tilde{K},\ l(t) \to \tilde{l}$ independently of the path as defined by θ.

To see the role of the initial adjustment in leisure, consider (A.5b) at time $t = 0$, namely

$$l(0) = \tilde{l} + \left(\frac{\mu - a_{11}}{a_{12}}\right)\left[K_0 - \tilde{K}\right] + \varphi_2\left(A_0 - \tilde{A}\right) \tag{A.6}$$

Thus the initial response of leisure to a productivity increase at that time is:

$$\frac{dl(0)}{dA} = \frac{d\tilde{l}}{dA} - \left(\frac{\mu - a_{11}}{a_{12}}\right)\frac{d\tilde{K}}{dA} - \varphi_2 \tag{A.7}$$

where the role of the time path is contained in φ_2.

(i) If the productivity increase occurs as a discrete jump at time zero, $\varphi_2 \to 0$, and the instantaneous response of leisure is the standard expression

$$\frac{dl(0)}{dA} = \frac{d\tilde{l}}{dA} - \left(\frac{\mu - a_{11}}{a_{12}}\right)\frac{d\tilde{K}}{dA} \tag{A.8}$$

which implies an initial decline in $l(0)$.

(ii) If the increase in productivity A occurs only gradually, at the rate θ, in accordance with (1c), we find that $\varphi_2 < 0$, implying an initial increase in $l(0)$, as suggested by our simulations.

A.2 Dynamics of the Relative Capital Stock

To obtain the dynamics of individual capital we linearize equation (14) around the steady-state $\tilde{K}, \tilde{l}, \tilde{k}_i, \tilde{l}_i$. This is given by

$$\dot{k}_i(t) = \frac{\tilde{A}F_L}{\tilde{K}}\left[\left(1 + \frac{1}{\eta}\right)(\tilde{k}_i - v_i)(l(t) - \tilde{l}) + \left[\left(\tilde{l}\left(1 + \frac{1}{\eta}\right) - 1\right)\left(k_i(t) - \tilde{k}_i\right)\right]\right] \tag{A.9}$$

Combining the steady-state conditions (12a) and (12b) to obtain

$$\frac{F_L(\tilde{K}, \tilde{L})}{\tilde{K}}\left[\tilde{l}\left(1 + \frac{1}{\eta}\right) - 1\right] = \beta$$

and rewriting equation (15) as

$$v_i = \frac{(1-\tilde{k}_i)}{\tilde{l}\left(1+\frac{1}{\eta}\right)} + \tilde{k}_i$$

enables us to express (A.9) in the more compact form

$$\dot{k}_i(t) = \beta\left(k_i(t) - \tilde{k}_i\right) + \frac{\tilde{A}F_L}{\tilde{K}}\frac{\left(\tilde{k}_i - 1\right)}{\tilde{l}}\left(l(t) - \tilde{l}\right) \qquad \text{(A.9')}$$

The stable solution to this equation is

$$k_i(t) - 1 = \left(\tilde{k}_i - 1\right)\left[1 + \frac{\tilde{A}F_L}{\tilde{K}}\int_t^\infty \left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\beta(\tau - t)}d\tau\right] \qquad \text{(A.10)}$$

Setting $t = 0$ in (A.10) and noting that $k_{i.0}$ is given, we obtain

$$k_{i,0} - 1 = \left(\tilde{k}_i - 1\right)\left[1 + \frac{\tilde{A}F_L}{\tilde{K}}\int_0^\infty \left(1 - \frac{l(\tau)}{\tilde{l}}\right)e^{-\beta\tau}d\tau\right] \qquad \text{(A.11)}$$

Thus, having determined $\tilde{K}, \tilde{L}$, and the time path for $l(t)$ from (A.5), equation (A.11) determines $\tilde{k}_i$, and knowing $\tilde{k}_i$, (A.10) in turn determines the entire time path for $k_i(t)$.

In the case of the discrete productivity shock, when the aggregate economy follows (A.5'), we find $l(\tau) - \tilde{l} = (l(0) - \tilde{l})e^{\mu\tau}$ and (A.10), (A.11) reduce to

$$k_i(t) - 1 = (\tilde{k}_i - 1)\left[1 + \left(\frac{1}{\beta - \mu}\right)\frac{\tilde{A}F_L(\tilde{K}, \tilde{L})}{\tilde{K}}\left(1 - \frac{l(t)}{\tilde{l}}\right)\right] \qquad \text{(A.10')}$$

$$k_{i,0} - 1 = (\tilde{k}_i - 1)\left[1 + \left(\frac{1}{\beta - \mu}\right)\frac{\tilde{A}F_L(\tilde{K}, \tilde{L})}{\tilde{K}}\left(1 - \frac{l(0)}{\tilde{l}}\right)\right] \qquad \text{(A.11')}$$

References

Aghion, P. and P. Bolton, 1997, "A trickle-down theory of growth and development with debt overhang," *Review of Economic Studies* 64, 151-72.

Aghion, P. and P. Howitt, 2009, *The Economics of Growth*. The MIT Press: Cambridge, MA.

Alesina, A. and D. Rodrik, 1994, "Distributive politics and economic growth," *Quarterly Journal of Economics* 109, 465-490.

Algan, Y., A. Cheron, J-O. Hairault, and F. Langot, 2003, Wealth effect on labor market transitions, *Review of Economic Dynamics* 6, 156-178.

Atkinson, A.B., 2003, "Income inequality in OECD countries: Data and explanations," *CESifo Economic Studies* 49, 479-513.

Atolia, M. and E.F. Buffie, 2010a, "Reverse shooting made easy: Automating the search for the global nonlinear saddle path," *Computational Economics* (forthcoming).

Atolia, M. and E.F. Buffie, 2010b, "Solving the unit root problem in models with an exogenous world market interest rate," *Macroeconomic Dynamics* (forthcoming).

Atolia, M., S. Chatterjee, and S.J. Turnovsky, 2009, "How misleading is linearization? Evaluating the dynamics of the neoclassical growth model," *Journal of Economic Dynamics and Control* (forthcoming).

Barnett, R., J. Bhattacharya, and H. Bunzel, 2009, "Choosing to keep up with the Joneses and income inequality." *Economic Theory,* forthcoming.

Barro, R.J., 2000, "Inequality and growth in a panel of countries," *Journal of Economic Growth* 5, 5-32.

Bénabou, R., 1996a, "Heterogeneity, stratification, and growth: Macroeconomic implications of the community structure and school finance," *American Economic Review* 86, 584-609.

Bénabou, R., 1996b, "Equity and efficiency in human capital investment: The local connection," *Review of Economic Studies* 63, 237-264.

Blanchard, O.J. and L.H. Summers, 1986, "Hysteresis and the European unemployment problem," *NBER Macroeconomics Annual* 1, 15-78.

Carroll, D. and E. Young, 2009, "The stationary wealth distribution under progressive taxation."

Review of Economic Dynamics 12, 469-478.

Caselli, F., and J. Ventura, 2000, "A representative consumer theory of distribution," *American Economic Review* 90, 909-926.

Checchi, D. and C. García-Peñalosa, 2008, "Labour market institutions and the personal distribution of income in the OECD," *Economica*, (forthcoming).

Cheng, I-H. and E. French, 2000, "The effect of the run-up in the stock market on labor supply, *Federal Reserve Bank of Chicago Economic Perspectives* 24, 48-65.

Cooley, T.F., 1995, *Frontiers of Business Cycle Research,* Princeton University Press, Princeton, N. J.

Coronado, J.L. and M. Perozek, 2003, "Wealth effects and the consumption of leisure: Retirement decisions during the stock market boom of the 1990s," *Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series*, 2003-20.

Deininger, K., and L. Squire, 1998, "New way of looking at old issues: inequality and growth," *Journal of Development Economics 57*, 259-287.

Deininger, K., and L. Squire, 1996, "A new data set measuring income inequality," *World Bank Economic Review 10*, 565-591.

Forbes, K., 2000, "A reassessment of the relationship between inequality and growth," *American Economic Review* 90, 869-887.

Galor, O. and O. Moav, 2004, "From physical to human capital accumulation: Inequality and the process of development," *Review of Economic Studies* 71, 1001-1026.

Galor, O. and D. Tsiddon, 1997, "Technological progress, mobility, and economic growth," *American Economic Review* 87, 363-382.

Galor, O. and J. Zeira, 1993, "Income distribution and macroeconomics," *Review of Economic Studies* 60, 35-52.

García-Peñalosa, C. and S.J. Turnovsky, 2006, "Growth and income inequality: A canonical model," *Economic Theory* 28, 25-49.

García-Peñalosa, C. and S.J. Turnovsky, 2007, "Growth, income inequality and fiscal policy: What are the relevant tradeoffs?" *Journal of Money, Credit and Banking* 39, 369-394.

Gorman, W., 1953, “Community preference fields,” *Econometrica* 21, 63-80.

Guvenen, F., 2006, “Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective,” *Journal of Monetary Economics* 53, 1451-1472.

Gylfason, T. and G. Zoega, 2003, “Inequality and growth: Do natural resources matter?” in *Inequality and Growth: Theory and Policy Implications*, T.S. Eicher and S.J. Turnovsky (eds.), MIT Press, Cambridge MA.

Holtz-Eakin, D., D. Joulfaian, and H.S. Rosen, 1993, “The Carnegie conjecture: Some empirical evidence,” *Quarterly Journal of Economics* 108, 413-435.

Kraay, A. and C. Raddatz, 2007, “Poverty traps, aid, and growth.” *Journal of Development Economics* 82, 315-347.

Kuznets, S., 1955, “Economic growth and income inequality,” *American Economic Review* 45, 1-28.

Li, H.Y. and H.F. Zou, 1998, “Income inequality is not harmful to growth: Theory and evidence” *Review of Development Economics* 2, 318-334.

Perotti, R., 1996, “Growth, income distribution, and democracy: What the data say,” *Journal of Economic Growth* 1, 149-187.

Persson, T. and G. Tabellini, 1994, “Is inequality harmful for growth?” *American Economic Review* 84, 600-621.

Piketty, T., 2006, “The Kuznets curve: Yesterday and tomorrow,” in Banerjee, A.V., R Bénabou, and D. Mookherjee (eds.), *Understanding Poverty*, Cambridge University Press.

Ray, D. (1998), *Development Economics*. Princeton University Press, Princeton, NJ.

Saint-Paul, G. and T. Verdier, 1993, “Education, democracy, and growth,” *Journal of Development Economics* 42, 399-407.

Sen, P. and S.J. Turnovsky, 1990, “Investment tax credit in an open economy.” *Journal of Public Economics* 42, 277-299.

Solimano, A., 1998, “The end of the hard choices? Revisiting the relationship between income distribution and growth,” in A. Solimano (ed.), *Social Inequality: Values, Growth, and the State*, University of Michigan Press.

Sorger, G., 2000, “Income and wealth distribution in a simple growth model,” *Economic Theory* 16, 23-42.

Turnovsky, S.J. and C. García-Peñalosa, 2008, “Distributional dynamics in a neoclassical growth model: The role of elastic labor supply,” *Journal of Economic Dynamics and Control* 32, 1399-1431.

Viane, J.-M. and I. Zilcha, 2003, “Human capital formation, Income inequality, and growth,” in *Inequality and Growth: Theory and Policy Implications*, T.S. Eicher and S.J. Turnovsky (eds.), MIT Press, Cambridge MA.