ECON 8010 **PROBLEM SET #1** **FALL 2009**
SOLUTIONS

Total Points: 40

a. $X(K,L) = K^{1/4}L^{3/4}$

b. $X(K,L) = K^{1/2} + L^{1/2}$

1. B & H, p. 258, #2 (***10 points***)

a. $X(K,L) = K^{1/4}L^{3/4}$

$$MRTS = \frac{f_L}{f_K} = \frac{\frac{3}{4}K^{1/4}L^{-1/4}}{\frac{1}{4}K^{-3/4}L^{3/4}} = 3\frac{K}{L}$$

b. $X(K,L) = K^{1/2} + L^{1/2}$

$$MRTS = \frac{f_L}{f_K} = \frac{\frac{1}{2}L^{-1/2}}{\frac{1}{2}K^{-1/2}} = \left(\frac{K}{L}\right)^{1/2}$$

B & H, p. 258, #3 (***10 points***)

a. $\overline{X} = K^{1/4}L^{3/4} \quad \Rightarrow \quad K = \frac{\overline{X}^4}{L^3}$

$$MRTS = 3\frac{K}{L} = 3\frac{\overline{X}^4}{L^4}$$

$$\left.\frac{dMRTS}{dL}\right|_{dX=0} = -12\frac{\overline{X}^4}{L^5} < 0$$ (diminishing MRTS)

b. $\bar{X} = K^{1/2} + L^{1/2} \quad \Rightarrow \quad K = \left(\bar{X} - L^{1/2}\right)^2$

$$MRTS = \left(\frac{K}{L}\right)^{1/2} = \frac{\bar{X} - L^{1/2}}{L^{1/2}} = \frac{\bar{X}}{L^{1/2}} - 1$$

$$\left.\frac{dMRTS}{dL}\right|_{dX=0} = -\frac{1}{2}\bar{X}L^{-3/2} < 0$$ (diminishing MRTS)

2. B & H, p. 258, #6 (***10 points***)

a. $X(\alpha K, \alpha L) = (\alpha K)^{1/4}(\alpha L)^{3/4} = \alpha^1 K^{1/4} L^{3/4} = \alpha X(K, L)$

$X(K, L)$ shows constant returns to scale.

b. $X(\alpha K, \alpha L) = (\alpha K)^{1/2} + (\alpha L)^{1/2} = \alpha^{1/2}\left(K^{1/2} + L^{1/2}\right) = \alpha^{1/2} X(K, L)$

$X(K, L)$ shows decreasing returns to scale.

3. B & H, p. 300, #2 (***10 points***)

a. $\sigma_{K,L} = \frac{dk}{d\delta}\frac{\delta}{k}$ where $k = \frac{K}{L}$ and $\delta = \frac{w}{r} = MRTS = 3\frac{K}{L} = 3k$

$$k = \frac{1}{3}\delta \quad \Rightarrow \quad \frac{dk}{d\delta}\frac{\delta}{k} = \frac{1}{3}\frac{3k}{k} = 1$$

b. $\sigma_{K,L} = \dfrac{dk}{d\delta}\dfrac{\delta}{k}$ where $k = \dfrac{K}{L}$ and $\delta = \dfrac{w}{r} = MRTS = \left(\dfrac{K}{L}\right)^{1/2} = k^{1/2}$

$k = \delta^2 \quad \Rightarrow \quad \dfrac{dk}{d\delta}\dfrac{\delta}{k} = 2\delta\dfrac{\delta}{k} = \dfrac{2\delta^2}{\delta^2} = 2$