ECON 8010 **TEST #1** **FALL 2009**

<u>Instructions</u>: All questions must be answered on this examination paper. No additional sheets of paper are permitted; use the backs of the pages if necessary. For every question, show all of your work in arriving at your answers. Point values for each question are in parentheses to the left of the question number as a guide for the allocation of your time.
<u>Time Limit</u>: <u>75 minutes</u>.

(20) 1. TRUE OR FALSE <u>and</u> EXPLAIN: Label each statement TRUE or FALSE, <u>and</u> rigorously explain your answer.

a. "The conditional labor-demand function $L^*(r,w,x)$ for a cost-minimizing firm is more wage-elastic than the labor-demand function L*(r,w,p) for a profit-maximizing firm when labor is a normal input."

False. From the Marshall-Hicks equation $\varepsilon_{LW} = -(1-S_L)\sigma_{KL} + S_L\eta_{xp}$, we know that the total effect of a change in w on $L^*(r,w,p)$ can be decomposed into a substitution elasticity and an output elasticity, both of which are negative. For conditional labor demand, there is no output effect, by definition, since output is held fixed at $\bar{x}$; there is only a substitution effect along an isoquant. Therefore, $L^*(r,w,p)$ is more wage elastic than $L^*(r,w,x)$.

b. "Consider an industry comprised of profit-maximizing firms that, collectively, face a price-elastic output demand, operate in perfectly competitive input markets, and produce a homogeneous product with a Cobb-Douglas technology. An increase in the price elasticity of demand for the output of that industry will increase the industry-wide wage elasticity of labor demand."

True. The Marshall-Hicks rules state that $\frac{\partial \varepsilon_{LW}}{\partial \eta_{xp}} = S_L > 0$. Therefore, as the price elasticity of demand becomes more elastic ($\partial \eta_{xp}$ decreases), the demand for labor becomes more elastic ($\partial \varepsilon_{LW}$ decreases).

(10) 2. Let x = f (K,L) be a production function that is homogeneous of degree one. Use Euler's Theorem on homogeneous functions to show that, for output levels where the average product of labor is decreasing, the marginal product of capital is positive.

If $x = f(K,L)$ is homogeneous of degree one, then Euler's Theorem on homogeneous functions states that:

$f(K,L) = f_K \cdot K + f_L \cdot L$ or (dividing through by L) $f(K,L)/L = f_K \cdot (K/L) + f_L$.

Define $AP_L = f(K,L)/L$ and $MP_L = f_L$.

Then, if AP_L is decreasing in L, $f(K,L)/L > f_L$ or $f(K,L)/L - f_L > 0$.

However, by Euler's Theorem $f(K,L)/L - f_L = f_K \cdot (K/L)$. Therefore, $f_K > 0$ assuming $(K/L) > 0$.

(15) 3. Suppose that the firm's profit function is

$$\pi^*(p,w) = p^2/4w,$$

where p is the per-unit price of output x, and w is the per-unit price of labor L.

a. Derive the firm's output-supply function $x^*(p,w)$.

$$\frac{\partial \pi^*}{\partial p} = x^*(p,w) = \frac{2p}{4w} = \frac{p}{2w} = x^*(p,w)$$

b. Derived the firm's labor-demand function $L^*(p,w)$.

$$\frac{\partial \pi^*}{\partial w} = -L^*(p,w) = \frac{-w^{-2}p^2}{4} = \frac{-p^2}{4w^2} \quad \Rightarrow \quad L^*(p,w) = \frac{p^2}{4w^2}$$

c. What is the firm's production function $x = f(L)$?

Substitute $x^*(p,w) = p/2w$ from part (a) into $L^*(p,w) = \frac{p^2}{4w^2}$ from part (b) to obtain $L^* = (x^*)$. Therefore, $x(L) = L^{1/2}$.

(15) 4. Suppose that a firm produces output x from the production function $x = f(L)$, where $L = e(w) \cdot N$ is the labor input, e is the effort level or "efficiency" of each worker (which is a positive function of the hourly wage rate w), and N is the number of worker-hours hired by the firm. Product price $p = 1$ is fixed, and the firm chooses w and N to maximize profits.

a. Derive the first-order conditions for a profit maximum.

The firm's profit equation is given by: $\pi = f[e(w).N] - w.N$
The first order conditions for a profit maximization are:

(i) $\frac{\partial \pi}{\partial w} = [f'(de/dw) - 1].N = 0 \Leftrightarrow f'(de/dw) = 1$

(ii) $\frac{\partial \pi}{\partial N} = f'e - w = 0 \Leftrightarrow f' = w/e$

b. Use the first-order conditions from part (a) to calculate the elasticity of worker effort e with respect to the wage rate w.

Substituting (ii) into (i), we have $\frac{w}{e}.\frac{de}{dw} = 1 = \varepsilon_{ew}$
So that the elasticity of worker effort with respect to the wage rate is unity.

(30) 5. Suppose that a cost-minimizing firm's (minimum) total cost function is

$$C^*(r,w,x) = w[1 + x + \log_e(r/w)],$$

where r is the per-unit price of capital, w is the per-unit price of labor, and x is output.

a. Derive the firm's conditional (constant-output) demand functions for labor $L^*(r,w,x)$ and capital $K^*(r,w,x)$, and rigorously analyze the effects of an increase in w on $L^*(r,w,x)$ and on $K^*(r,w,x)$.

$$L^*(r,w,x) = \frac{\partial C^*}{\partial w} = x + \log_e\left(\frac{r}{w}\right)$$

$$K^*(r,w,x) = \frac{\partial C^*}{\partial r} = \frac{w}{r}$$

$$\frac{\partial L^*}{\partial w} = -\frac{1}{w} < 0 \quad \text{and} \quad \frac{\partial K^*}{\partial w} = \frac{\partial^2 C^*}{\partial r \partial w} = \frac{1}{r} > 0$$

b. Derive the firm's (minimum) <u>marginal</u> cost function $MC^*(r,w,x)$, and rigorously analyze the effect of an increase in x on $MC^*(r,w,x)$.

$$\frac{\partial C^*}{\partial x} = MC^*(r, w, x) = w$$

$$\frac{\partial MC^*}{\partial x} = 0$$

(10) 6. MATCHING: Fill in the blank space with the number of the phrase that <u>best</u> matches the mathematical expression.

a.	_____5______	$f_{xy} = f_{yx}$	1. Euler's Theorem
b.	_____1______	$f_K \cdot K + f_L \cdot L = r \cdot f(K, L)$	2. Hotelling's Lemma
c.	_____4______	$\partial C^* / \partial w = L^*(r, w, x)$	3. Reciprocity Relation
d.	_____2______	$\partial \pi^* / \partial p = x^*(r, w, p)$	4. Shephard's Lemma
e.	_____3______	$\partial \lambda^* / \partial r = \partial K^* / \partial x$	5. Young's Theorem