

Instructions: All questions must be answered on this examination paper. No additional sheets of paper are permitted; use the backs of the pages if necessary. For every question, show all of your work in arriving at your answers. Time limit: 75 minutes.

- (15) 1. TRUE or FALSE and explain. Label each of the statements TRUE or FALSE, and rigorously justify your answer.
- a. _____ "If X and Y are Edgeworth-Pareto complements and they are each subject to diminishing marginal utility, then the utility function $U(X,Y)$ is quasi-concave.
- b. _____ "If two goods, X and Y, are net substitutes and X is a normal good, then X and Y must be gross substitutes.
- c. _____ "If the utility function $U(X,Y)$ is additively separable, then the cross-price effects on uncompensated (ordinary) demand are zero."

(15) 2. RESTRICTIONS: Calculate the missing parameter values from the restrictions imposed by the theory of consumer behavior.

a. Suppose that a consumer has the following uncompensated demand function:

$$X^*(P_x, P_y, M) = aP_x^{-0.4}P_y^bM^{0.7}$$

where P_x and P_y are the per-unit prices of goods X and Y, respectively, M is money income, and a and b are fixed parameters. The value of the parameter b is _____. EXPLAIN your answer:

b. Suppose that the consumer has the following expenditure function:

$$M^*(P_x, P_y, U) = P_yU + 2P_x^{1/2}P_y^c$$

where P_x and P_y are per-unit prices of goods X and Y, respectively, U is utility, and c is a fixed parameter. The value of the parameter c is _____. EXPLAIN your answer.

c. If a consumer spends 1/3 of her income on good X and 2/3 of her income on good Y, and the income elasticity of demand for good X is 0.6, then the income elasticity of demand for good Y is _____. EXPLAIN your answer.

(15) 3. Consider a consumer who chooses a bundle of goods (X,Y) at per-unit prices $P_X = 2$ and $P_Y = 4$. Denote the own-substitution and cross-substitution effects S_{XX} and S_{YY} , and S_{XY} and S_{YX} , respectively. Suppose that $S_{XX} = -1$. Determine the values of S_{YY} , S_{XY} , and S_{YX} .

- (15) 4. Suppose that an individual maximizes the utility function

$$U(X,Y) = Y + X^{1/2}$$

subject to the budget constraint $M = P_x X + P_y Y$.

- a. Derive the uncompensated demand functions for X and Y .

- b. Is either X or Y an inferior good? Justify your answer rigorously.

- (15) 5. Suppose that a consumer's preferences can be represented by the indirect utility function

$$V(P_X, P_Y, M) = \log_e(P_Y/P_X) + (M/P_Y) - 1$$

where P_X and P_Y are the per-unit prices of goods X and Y , respectively, M is money income, and $P_Y > P_X$ and $M > P_Y$.

- a. Use Roy's Identity to derive the uncompensated demand functions for X and Y .

b. What is the direct utility function, $U(X,Y)$? Show all of your work.

(15) 6. An alternative representation of the preferences given in question 5 is the expenditure function, obtained by substituting U for V in the indirect utility function and then solving for M .

a. Write down this expenditure function.

b. Derive the compensated demand functions for X and Y .

(10) 7. The distance function $D(X,Y,U)$ is a dual to the expenditure function $M(P_x,P_y,U)$, where X and Y are quantities of goods, U is utility, P_x is the per-unit price of X , and P_y is the per-unit price of Y . The distance function associated with the utility function $U(X,Y) = X \cdot Y$ is

$$D(X,Y,U) = [(X \cdot Y)/U]^{1/2}$$

Using this distance function, derive the inverse, compensated demand functions for X and Y , $P_x^*(X,Y,U)$ and $P_y^*(X,Y,U)$.