ECON 8010 **TEST #1** **FALL 2009**

<u>Instructions</u>: All questions must be answered on this examination paper. No additional sheets of paper are permitted; use the backs of the pages if necessary. For every question, show all of your work in arriving at your answers. Point values for each question are in parentheses to the left of the question number as a guide for the allocation of your time.
<u>Time Limit</u>: <u>75 minutes</u>.

(20) 1. TRUE OR FALSE <u>and</u> EXPLAIN: Label each statement TRUE or FALSE, <u>and</u> rigorously explain your answer.

a. "The conditional labor-demand function $L^*(r,w,x)$ for a cost-minimizing firm is more wage-elastic than the labor-demand function $L^*(r,w,p)$ for a profit-maximizing firm when labor is a normal input."

b. "Consider an industry comprised of profit-maximizing firms that, collectively, face a price-elastic output demand, operate in perfectly competitive input markets, and produce a homogeneous product with a Cobb-Douglas technology. An increase in the price elasticity of demand for the output of that industry will increase the industry-wide wage elasticity of labor demand."

(10) 2. Let $x = f(K,L)$ be a production function that is homogeneous of degree one. Use Euler's Theorem on homogeneous functions to show that, for output levels where the average product of labor is decreasing, the marginal product of capital is positive.

(15) 3. Suppose that the firm's profit function is

$$\pi^*(p,w) = p^2/4w,$$

where p is the per-unit price of output x, and w is the per-unit price of labor L.

a. Derive the firm's output-supply function $x^*(p,w)$.

b. Derive the firm's labor-demand function $L^*(p,w)$.

c. What is the firm's production function $x = f(L)$?

(15) 4. Suppose that a firm produces output x from the production function $x = f(L)$, where $L = e(w)\cdot N$ is the labor input, e is the effort level or "efficiency" of each worker (which is a positive function of the hourly wage rate w), and N is the number of worker-hours hired by the firm. Product price $p = 1$ is fixed, and the firm chooses w and N to maximize profits.

a. Derive the first-order conditions for a profit maximum.

b. Use the first-order conditions from (a) to calculate the elasticity of worker effort e with respect to the wage rate w.

(30) 5. Suppose that a cost-minimizing firm's (minimum) total cost function is

$$C^*(r,w,x) = w[1 + x + \log_e(r/w)],$$

where r is the per-unit price of capital, w is the per-unit price of labor, and x is output.

a. Derive the firm's conditional (constant-output) demand functions for labor $L^*(r,w,x)$ and capital $K^*(r,w,x)$, and rigorously analyze the effects of an increase in w on $L^*(r,w,x)$ and on $K^*(r,w,x)$.

b. Derive the firm's (minimum) <u>marginal</u> cost function $MC^*(r,w,x)$, and rigorously analyze the effect of an increase in x on $MC^*(r,w,x)$.

(10) 6. MATCHING: Fill in the blank space with the number of the phrase that <u>best</u> matches the mathematical expression.

a.	___________	$f_{xy} = f_{yx}$	1. Euler's Theorem
b.	___________	$f_K \cdot K + f_L \cdot L = r \cdot f(K,L)$	2. Hotelling's Lemma
c.	___________	$\partial C^*/\partial w = L^*(r,w,x)$	3. Reciprocity Relation
d.	___________	$\partial \pi^*/\partial p = x^*(r,w,p)$	4. Shephard's Lemma
e.	___________	$\partial \lambda^*/\partial r = \partial K^*/\partial x$	5. Young's Theorem