ECON 8010
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PROBLEM SET #3 SOLUTIONS

1 MRS Functions

a. U(X,Y)=$X^{1/3}Y^{2/3}$ $\frac{\partial U}{\partial X} = \frac{1}{3}X^{-2/3}Y^{2/3}$
$\frac{\partial U}{\partial X} = \frac{2}{3}X^{1/3}Y - 1/3$
Thus,
$\text{MRS}_{Y,X} = \frac{U_X}{U_Y} = \frac{1}{2}X^{-1}Y$

b. U(X,Y)=$0.5X^{1/2}+0.5Y^{1/2}$
$\frac{\partial U}{\partial X} = \frac{1}{4}X^{-1/2}$
$\frac{\partial U}{\partial Y} = \frac{1}{4}Y^{-1/2}$
Thus,
$\text{MRS}_{Y,X} = \frac{U_X}{U_Y} = X^{-1/2}Y^{1/2}$

c. U(X,Y)=Y-X^{-1}
$\frac{\partial U}{\partial X} = X^{-2}$
$\frac{\partial U}{\partial Y} = 1$
Thus,
$\text{MRS}_{Y,X} = \frac{U_X}{U_Y} = X^{-2}$

Since none of the MRS expressions are equal and the utility functions are not monotonic transformations of each other, the preferences are unique.

2 Uncompensated Demand Functions

a. U(X,Y)=$X^{1/3}Y^{2/3}$
$\mathcal{L}(X, Y, \lambda) = X^{1/3}Y^{2/3} + \lambda(M - P_X X - P_Y Y)$
Tangency condition: $\text{MRS}_{Y,X} = \frac{P_X}{P_Y}$
$\frac{1}{2}X^{-1}Y = \frac{P_X}{P_Y} => Y = 2X\frac{P_X}{P_Y}$
$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 => M - P_X X - P_Y Y = 0 => M - P_X X - 2XP_X = 0$
$=> X^*(P_X, P_Y, M) = \frac{M}{3P_X}$
$=> Y^*(P_X, P_Y, M) = \frac{2M}{3P_Y}$

b. U(X,Y)=$0.5X^{1/2}+0.5Y^{1/2}$
$\mathcal{L}(X,Y,\lambda) = 0.5X^{1/2} + 0.5Y^{1/2} + \lambda(M - P_X X - P_Y Y)$
Tangency condition: $\text{MRS}_{Y,X} = \frac{P_X}{P_Y}$
$X^{-1/2}Y^{1/2} = \frac{P_X}{P_Y} => Y = (\frac{P_X}{P_Y})^2 X$
$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 => M - P_X X - P_Y Y = 0 => M - P_X X - \frac{P_X^2}{P_Y} X = 0$
$=> X^*(P_X, P_Y, M) = \frac{MP_Y}{P_X(P_X+P_Y)}$
$=> Y^*(P_X, P_Y, M) = \frac{MP_X}{P_Y(P_X+P_Y)}$

c. U(X,Y)=Y-X^{-1}
$\mathcal{L}(X,Y,\lambda) = Y - X^{-1} + \lambda(M - P_X X - P_Y Y)$
Tangency condition: $\text{MRS}_{Y,X} = \frac{P_X}{P_Y}$
$X^{-2} = \frac{P_X}{P_Y} => X = (\frac{P_Y}{P_X})^{1/2}$
$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 => M - P_X X - P_Y Y = 0 => M - P_X^{1/2} P_Y^{1/2} - P_Y Y = 0$
$=> Y^*(P_X, P_Y, M) = \frac{M - P_X^{1/2} P_Y^{1/2}}{P_Y}$
$X^*(P_X, P_Y, M) = (\frac{P_Y}{P_X})^{1/2}$

3 Indirect Utility Functions

a. U(X,Y)=$X^{1/3}Y^{2/3}$
$V(P_X, P_Y, M) = \frac{M}{3}(\frac{4}{P_X P_Y^2})^{1/3}$

b. U(X,Y)=$0.5X^{1/2}+0.5Y^{1/2}$
$V(P_X, P_Y, M) = \frac{1}{2}(\frac{M}{P_X P_Y (P_X+P_Y)})^{1/2}(P_X + P_Y)$

c. U(X,Y)=Y-X^{-1}
$V(P_X, P_Y, M) = \frac{M - 2P_X^{1/2} P_Y^{1/2}}{P_Y}$