ECON 4750/6750 **TEST #2** **FALL 2009**

SOLUTIONS

Instructions: All questions are to be answered on this examination paper; no extraneous sheets of paper are permitted. Point values for each question are in parentheses to the left of the corresponding numerals and should govern your allocation of time. You may use your textbook, problem sets, and class notes, but you may not collaborate with others. Show all of your work. Time limit: 75 minutes.

(20) 1. The most commonly used method for obtaining sample estimates of the population parameters of the linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \mu_i \qquad i = 1, \ldots, N$$

is ordinary least squares (OLS), which chooses estimates of β_0 and β_1 so as to minimize the sum of the squared deviations of the Y's from their predicted values; that is, OLS minimizes $\Sigma\ \hat{\mu}_i^2 = \Sigma\ (Y_i - \hat{Y}_i)^2$.

Circle the letter(s) corresponding to the equations, below, which are necessarily the result of using OLS to estimate this model:

a. $\Sigma\ \hat{\mu}_i = 0$

b. $\Sigma\ \hat{\mu}_i^2 = 0$

c. $\Sigma\ \hat{\mu}_i X_i = 0$

d. $\hat{\beta}_1 = \overline{Y}/\overline{X}$

(20) 2. Data on price and absorbency were collected for each of eleven brands of paper towels by a well-known consumer-information magazine. Using these data, the following bivariate regression model was estimated:

$$\hat{P} = \underset{(0.113)}{0.530} + \underset{(0.038)}{0.308}A$$

$$R^2 = 0.90$$

where $\hat{P}$ is the estimated price per roll in dollars, A is an index of (water) absorbency, and R^2 is the coefficient of determination. The estimated standard errors of the coefficient estimates are in parentheses.

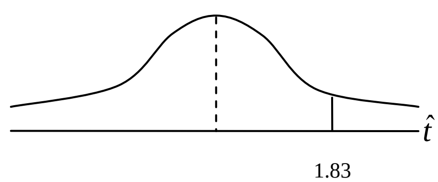
a. Conduct a test of the null hypothesis that there is no relationship between absorbency and price ($\beta_1 = 0$) against the one-sided alternative $\beta_1 > 0$, using an 0.05 level of significance.

H_0: B_1=0
H_a: B_1>0

Under H_0, the test statistic $\hat{t} = \dfrac{\hat{\beta}_1 - \beta_1^{H_0}}{s_{\hat{\beta}_1}} = \dfrac{0.308 - 0}{0.038} = 8.11$

has a t distribution with N-2=9 d.f.

Since, $\hat{t} = 8.11 > t^*_{9;0.05} = 1.83$, **we reject H_0.**



b. Test the null hypothesis of "no gain from regression" ($R^2 = 0$) against the one-sided alternative $R^2 > 0$, using an 0.05 level of significance.

H_0: R^2=0
H_a: R^2>0

Under the null H0, the ratio $\dfrac{N-K}{K-1} \cdot \dfrac{R^2}{1-R^2} = \dfrac{9}{1} \cdot \dfrac{0.9}{0.1} = 81$

has an F distribution with K-1=1 and N-K=9 degrees of freedom.
Since $\hat{F} = 81 > F^*_{1;9} = 5.12$, we reject the null hypothesis.

(15) 3. The relationship between price (Y), in dollars, and size (X), in square feet, for N = 213 single-family residential transactions in Athens, GA in 2003 is given by the sample regression function

$$\hat{Y} = -853.4 + 92.01\,X$$
$$(5061.2) \quad (5.606)$$

where estimated standard errors are in parentheses under the corresponding coefficient estimates.

a. A real-estate agent claims that single-family homes in Athens sell for \$100 per square foot, on average. Conduct a two-tailed test of the validity of this claim, using an .05 level of significance.

H_0: $B_1=100$
H_a: $B_1 \neq 100$

Under H_0, the test statistic $\hat{t} = \dfrac{\hat{\beta}_1 - \beta_1^{H_0}}{s_{\hat{\beta}_1}} = \dfrac{92.01 - 100}{5.606} = -1.43$

has a $t_{211;\,\alpha}$ distribution. Set $\alpha = 0.05$.

Then $t^* = \pm 1.96 \;<\; \hat{t} = -1.43$ so **we cannot reject H_0.**

b. What is the model's prediction of the selling price for a home with 3,000 square feet?

$$(\hat{Y} \mid x = 3000) = -853.4 + 92.01(3{,}000) = -853.4 + 276{,}030 = 275{,}176.60$$

(20) 4. The population regression function for the relationship between the number of pounds (Q) of cara cara oranges sold at the westside Publix supermarket each day over a twelve-day period and the price per pound (P) of these oranges is

$$Q_t = \beta_0 + \beta_1 P_t + \mu_t$$

where β_0 and β_1 are population parameters to be estimated, and μ_t is the population error term. Using the data on Q and P, we find that

$$\overline{P} = 70,\ \overline{Q} = 100,\ \Sigma(P - \overline{P})(Q - \overline{Q}) = -3550$$

$$\Sigma(P - \overline{P})^2 = 2250, \text{ and } \Sigma(Q - \overline{Q})^2 = 6300.$$

a. Calculate the least-squares estimates of β_0 and β_1.

$$\hat{\beta}_1 = \frac{\sum (P - \overline{P})(Q - \overline{Q})}{\sum (P - \overline{P})^2} = \frac{-3550}{2250} = -1.578$$

$$\hat{\beta}_0 = \overline{Q} - \hat{\beta}_1 \overline{P} = (100) - (-1.578)(70) = 210.46$$

b. Calculate the estimated (point) price elasticity of demand for cara cara oranges, evaluated at the sample means $\overline{Q}$ = 100 and $\overline{P}$ = 70.

$$\varepsilon_{QP} = \frac{\overline{P}}{\overline{Q}} = \frac{dQ}{dP} = \frac{70}{100}.(-1.578) = -1.105$$

i.e a 10% increase in the price of cara cara oranges will reduce the quantity demanded by approximately 11%.

c. Calculate the R^2 for the sample regression function.

$$R^2 = \frac{\hat{\beta}_1^{\,2} \sum (P-\overline{P})^2}{\sum (Q-\overline{Q})^2} = \frac{(-1.578)^2 (2250)}{6300} = 0.889$$

(15) 5. The relationship between the number of visitors per month (V) to Luray Caverns in Virginia and the admission price (P) was estimated to be

$$\hat{V}_t = 15.763 - 0.252P_t \qquad t = 1, \ldots, T = 84$$
$$(0.517) \quad (0.041)$$

where V is measured in thousands of persons, P is measured in dollars, estimated standard errors are in parentheses under the coefficient estimates, and T is the number of monthly observations.

a. Current admission price is \$8.00 per visitor. How many visitors per month are predicted if no admission price were charged?

$$\hat{V} = 15.763 - 0.252(0) = 15.763$$

15,763 visitors per month are predicted.

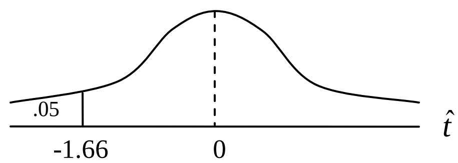
b. If, on the other hand, the admission price were increased to \$10.00 per visitor, how many visitors each month are predicted?

$$\hat{V} = 15.763 - 0.252(10) = 13.243$$

13,243 visitors per month are predicted.

c. Test the null hypothesis that $\beta_1 = 0$ against the one-sided alternative $\beta_1 < 0$, at the 0.05 significance level.

H_0: $B_1=0$
H_a: $B_1<0$



Under H_0, the test statistic $\hat{t} = \dfrac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \dfrac{-0.252}{0.041} = -6.146$ has a $t_{82;\,\alpha}$ distribution with T-2=82 d.f.
At $\alpha = 0.05$, $t^* \cong -1.66 > -6.146$ so we **reject H_0**.

(10) 6. Suppose we have the following paired observations on Y and X,

Y	X
3	0
6	1
10	1
5	0

and we specify the bivariate regression model

$$Y_i = \beta_0 + \beta_1 X_i + \mu_i \qquad i = 1, \ldots, N = 4$$

Denote by $\overline{Y}_0$ the sample mean of Y for the 2 observations for which X = 0, and let $\overline{Y}_1$ be the sample mean of Y for the 2 observations for which X = 1. Calculate sample estimates of β_0 and β_1 as functions of $\overline{Y}_0$ and $\overline{Y}_1$.

$$E(Y|X=0) = \beta_0 + \beta_1(0) = \beta_0$$

$$E(Y|X=1) = \beta_0 + \beta_1(1) = \beta_0 + \beta_1$$

$\hat{Y}_0$ is a sample estimator of $E(Y|X=0) = \beta_0$.

$\hat{Y}_1$ is a sample estimator of $E(Y|X=1) = \beta_0 + \beta_1$.

Therefore, $\hat{\beta}_0 = \overline{Y}_0 \;\; \Rightarrow \; \hat{\beta}_0 = \frac{1}{2}\sum_{X=0} Y_i = \frac{3+5}{2} = 4 = \hat{\beta}_0$

and $\hat{\beta}_1 = \overline{Y}_1 - \hat{\beta}_0 = \overline{Y}_1 - \overline{Y}_0 \;\; \Rightarrow \; \hat{\beta}_1 = \frac{1}{2}\sum_{X=1} Y_i - 4 = \frac{6+10}{2} - 4 = 8 - 4 = 4 = \hat{\beta}_1$