

# Patent Damages and Spatial Competition

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## Abstract

This paper analyzes price competition between a spatially differentiated product patentee and an imitator anticipating the possibility of future patent damages. We compare the performance of three damage regimes. The “reasonable royalty” regime, the only one that yields symmetric equilibrium pricing, maximizes static welfare and yields the highest incentives to innovate when patent enforcement is nearly certain. The “lost profits” regime, the only one that may deter infringement, yields the highest incentives to innovate when patent enforcement is less-than-certain and product value is sufficiently high. The “unjust enrichment” regime is weakest. Our results offer an efficiency argument for abandoning it. We also describe new insights into the “hypothetical negotiation” that the courts use to construct reasonable royalty damages.

**JEL Classification:** *K2, O3*

**Keywords:** Patent, litigation, damages, spatial, differentiated.

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## 1. Introduction

The patent grant is fundamentally spatial. The identity of the granting country determines a patent’s territorial boundaries and its claims determine its “product space” boundaries. Product innovations are particularly difficult to protect with trade secrets, so patent protection is important for reaping returns to innovation in such cases. Given these features, it is surprising that the patent literature has ignored how patent enforcement rates and the method of damage calculation affect spatial competition in the presence of product patents.

Using a model of entry and fixed-location Hotelling (1929) duopoly, we analyze the impact of damage regimes on market competition. We identify equilibrium prices, market shares, profits and welfare when a patentee and imitator compete anticipating probabilistic money damages under the three primary regimes that have been used by United States courts: “reasonable royalty,” where the court bases fixed fee and per-unit royalty damages on a hypothetical (pre-infringement) negotiation between patentee and imitator; “lost profits,” where damages restore the patentee to a hypothetical monopoly profit; and “unjust enrichment,” where the imitator must disgorge all profit.<sup>1</sup> We also use a patent-race model to compare incentives to innovate across damage regimes.

We model damage regimes to most closely resemble application of the law by US courts.<sup>2</sup> For the lost profit and unjust enrichment regimes, our formulae for determining damages are straightforward and follow Anton and Yao (2007). For the reasonable royalty regime, our approach includes an important innovation. Like Schankerman and Scotchmer (2001), we identify royalties that satisfy the requirements of a hypothetical negotiation. In contrast, however, we model threat points in the hypothetical negotiation to reflect the mutual assumption that an injunction would issue (and therefore a patentee-monopoly would ensue) if bargaining breaks down. This approach, which is standard practice in product patent cases in US courts, avoids the awkward “circularity” problem—where a court’s choice of “reasonable” damages determines equilibrium license fees and vice-versa—that Schankerman and

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<sup>1</sup>Typically, an injunction also issues when damages are awarded. Since our model has only one period of competition followed by litigation and damage awards, injunctions do not affect the comparison of damage regimes except through their impact on the hypothetical negotiation. In our conclusion, we discuss a multi-period extension of our model in which injunctions would make a bigger difference.

<sup>2</sup>Reitzig, Henkel and Heath (2002) discuss damage rules for other countries.

Scotchmer (2001) identify for the case of vertical licensing of research tools.

We show that an increase in the likelihood of patent enforcement causes equilibrium prices to rise under all damage regimes. Intuitively, the higher likelihood of damages softens price competition. For at least one firm (and possibly both), the best-response function shifts out because it prefers to increase price to give market share to the other firm. Because prices are strategic complements, both prices are higher in equilibrium. Duopoly prices may exceed monopoly prices and there may be multiple equilibria.<sup>3</sup>

Static welfare is maximized only under the reasonable royalty regime. Intuitively, reasonable royalty damages are unique in affecting the duopoly pricing incentives of the patentee and imitator *symmetrically*. For the imitator, the per-unit royalty multiplied by the likelihood of patent enforcement (the “expected royalty”) is like a marginal cost of market share. For the patentee, the expected royalty is like an opportunity cost of market share. As the expected royalty increases, both firms’ best-response functions shift out in the same way. With symmetric prices, all consumers buy from the closest firm, so welfare is maximized. With asymmetric prices, which always obtain under either the lost profits or unjust enrichment regimes, some consumers do not buy from the closest firm, so the market suffers welfare-reducing “excess” transportation costs.

Entry may be deterred only under the lost profits regime. Intuitively, lost profit damages are unique in that they increase one-for-one with the size of a hypothetical monopoly profit. As the product’s value increases, the price that a monopolist would charge increases (because a monopolist is free to hold consumers to reservation utility), but equilibrium duopoly prices do not. For sufficiently high product value, expected damages are high enough so that the imitator’s expected profit is negative if it enters.<sup>4</sup> Damages under the other regimes, being based on a hypothetical bargaining profit (reasonable royalty) and a duopoly profit (unjust enrichment), are never high enough to make the imitator’s expected profit negative.

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<sup>3</sup>Our main results on static welfare and entry are robust to cases of multiple equilibria. To clearly highlight the potential strengths of each regime, results on incentives to innovate are presented conditional on unique equilibria obtaining.

<sup>4</sup>Since our primary interpretation of the model is spatial competition, we think of transportation costs as being fixed and focus comparative static analysis on changes in product value. Alternatively, one could fix product value and focus on changes in degrees of product differentiation. With such a focus, our results would show that for sufficiently homogeneous products (i.e. sufficiently low transportation costs), the lost profits regime deters entry.

Incentives to invest in research and development to innovate first and become the patentee, instead of the imitator, are higher whenever both the patentee's expected profit and the difference in the patentee's and imitator's expected profits are higher. When patent enforcement is (nearly) certain, incentives are best under reasonable royalty. Intuitively, when enforcement is certain, (symmetric) equilibrium prices hold the consumer located halfway between the patentee and imitator to his reservation utility. In this case, total firm profits exceed both what a monopolist could achieve and what duopolists with asymmetric prices could achieve. Once damages are factored in, the patentee-duopoly profit and the difference in the patentee's and imitator's profit *both* exceed the patentee-monopoly profit. If patent enforcement is less-than-certain, however, then (regardless of the damage regime) the patentee-duopoly profit is lower than the patentee-monopoly profit if the product is sufficiently valuable. In this case incentives to innovate are highest under lost profits, because it is the only regime that may deter entry.

Unjust enrichment is the weakest regime. It neither maximizes static welfare nor generates strong incentives to innovate. Our findings suggest that the Patent Act of 1946 and the 1964 Supreme Court decision in *Aro Manufacturing Co. v. Convertible Top Replacement Co.* [377 US 476, 1964], which together ended the use of unjust enrichment damages, are supportable on economic grounds.

Prior to *Aro*, courts usually awarded unjust enrichment damages. In *Aro*, the Court found that when Congress amended the statute in 1946, it intended to proscribe such damages. The statutory language indicates that the preferred measure of compensation is lost profits. In *Panduit Corp. v. Stahlin Bros. Fibre Works, Inc.* [197 USPQ 726 (6 Cir), 1978], the court also announced a four-factor test a patentee must satisfy to recover lost profits.<sup>5</sup> When the *Panduit* test cannot be satisfied, many courts award a reasonable royalty on the basis of a hypothetical (pre-infringement) arm's-length negotiation between the patentee and the imitator. In *Georgia-Pacific Corporation v. U.S. Plywood-Champion Papers Inc.* [166 USPQ 235 (S.D.N.Y), 1970], the court established a widely-followed fifteen-factor test for using a hypothetical negotiation framework to determine the royalty rate.

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<sup>5</sup>A patentee must prove: (1) demand for the patented product, (2) absence of acceptable non-infringing substitutes, (3) his manufacturing and marketing capability to exploit the demand, and (4) the amount of the profit he would have made.

We focus most of our analysis on market competition absent a license, for two reasons. First, because of fear of possible antitrust consequences, inexperience with licensing or other frictions, firms often fail to bargain to patent licensing deals prior to producing. The intersection of patent law and antitrust law is not clearly defined, because the monopoly rights of patentees conflict with the goals of antitrust law. For example, the US Supreme Court held as legal contracts combining patent licenses with narrowly-tailored price fixing arrangements in *U.S. v. General Electric* [272 US 476 (1926)], but declared price fixing illegal per se in *U.S. v. Trenton Potteries Co* [273 US 392 (1927)]. This conflict has led to numerous exceptions to the *General Electric* holding and, in 1948 and 1965, to near reversals of it.<sup>6</sup>

Second, if firms do bargain over a license, then the profits from market competition absent a license form the threat points in bargaining. Total firm profits and static welfare under bargaining will not vary across damage regimes, but the firms' individual profits will vary with the threat points. Hence, the analysis of threat points remains important, particularly when considering incentives to innovate.

While several papers compare the performance of patent damage regimes (Schankerman and Scotchmer 2001, 2005; Reitzig et. al 2002; Choi 2006; Anton and Yao 2007), our paper is the first to show the welfare consequences of damages' symmetric or asymmetric effects on firms' behavior. Our spatial framework is also unique in permitting comprehensive study of cases where total duopoly profits may exceed the monopoly profit. This feature, a key component of many of the results described above, also has important implications for the hypothetical bargain. Since licensing a second firm increases total profit and profit depends crucially on the chosen royalty rate, some royalty rates are precluded on efficiency grounds.

The work of Anton and Yao (2007), which considers process patents and focuses on Cournot competition absent a license, relates closely to ours because they model lost profit and unjust enrichment damages in a similar way and directly consider patent races. In their setting, the lost profits regime does not deter entry. Process patents can be infringed without diminishing the patentee's profit, because the imitator can always choose the quantity consistent with no infringement. This "passive" infringement is an equilibrium for high levels

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<sup>6</sup>Maurer and Scotchmer (2006, p. 478) argue that doctrinal confusion often deters patentees from licensing their inventions. See also Hovenkamp, Janis and Lemley (2004) for a detailed discussion of the legal history.

of court enforcement of patents. Consequently, losing a patent race is less costly and firms have lower incentives to innovate. With product patents in a spatial setting, by contrast, passive infringement is often not even possible. Moreover, the lost profits regime may deter infringement altogether, which enhances the patentee’s profit and incentives to innovate.

Schankerman and Scotchmer (2001) consider the case of research tools vertically licensed by one non-producing party to another producing party. They approach damage calculations in a philosophically distinct way, basing lost profit damages on the profit a patentee would (hypothetically) get from an equilibrium bargain rather than the profit it would (hypothetically) get as a producing monopolist. They identify a circularity in the determination of “reasonable” damages using a hypothetical negotiation—firms’ beliefs about the court’s chosen level of damages determines the fee for the license in the equilibrium bargain and vice-versa. Their approach to damages is consistent with the existing (vague) statutory language governing the reasonable royalty regime and is applicable, in practice, to the case of research tools. It is not typically applied in cases where the patentee is an active market participant, however. Our results suggest that, when possible, implementation of the hypothetical negotiation (as we model it) may have emerged as standard practice because it yields a convenient way to overcome the awkward circularity problem.

Reitzig et al. (2002) and Choi (2006) study product patents in a Cournot duopoly model and also find that infringement is not deterred under lost profits. Reitzig et al. (2002) show that a type of passive infringement may also obtain when the patentee is capacity-constrained. In this case unjust enrichment damages may improve incentives to innovate relative to lost profits. Choi finds that, absent such constraints, the lost profits regime generates greater profit for the patentee and greater R&D incentives.<sup>7</sup>

Two other papers study patent licensing under spatial competition. Poddar and Sinha (2004) consider “inside” licensing (i.e. the licensor is a producer) of a process patent, while Caballero-Sanz et al. (2002) study “outside” licensing. Neither paper considers uncertain enforcement or damage regimes.<sup>8</sup> Eswaran and Gallini (1996) study spatial competition

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<sup>7</sup>Using a different approach, Ayres and Klemperer (1999) argue that making patent enforcement probabilistic and delaying damage awards is superior to injunctive relief. See also Blair and Cotter (1998).

<sup>8</sup>Their main results compare the various ways of selling a license in a non-cooperative framework. The literature on optimal patent licensing has considered fixed fees, per-unit royalties, auctions, etc., for “outside”

between a patentee and imitator, but do not analyze damage regimes.

Section 2 introduces the model and analyzes some benchmark cases under zero liability. Section 3 analyzes market competition in absence of a license for all damage regimes. Section 4 introduces bargaining and analyzes the hypothetical negotiation. Section 5 considers incentives to innovate in a patent-race model. Section 6 concludes.

## 2. The Model

We begin with a standard Hotelling linear city model with fixed firm locations. Consumers of unit mass have identical *reservation value*  $V$  for the good and are distributed uniformly along a line of unit length. They bear *transportation cost*  $t$  per unit of distance they travel to a seller. The patentee sells at location 0. There are two possible market structures: single-price monopoly or duopoly including an imitator who sells at location 1. We denote the patentee's price as  $P_H$  and the imitator's price as  $P_I$ . The firms produce with the same constant-marginal-cost technology and marginal costs are normalized to zero. To focus on whether damages are sufficient to preclude entry, we also normalize entry costs to zero. If the imitator does not enter and compete, it earns a profit of 0. All parties are risk neutral and their preferences do not display wealth effects.

Consider first the monopoly case. A consumer located at point  $x$  on the line will buy from the patentee-monopolist if  $P_H^M \leq V - tx$  (the superscript  $M$  denotes "monopolist"). The optimal price depends on the value of the product, relative to transportation costs:

$$P_H^{M*} = \begin{cases} V - t & \text{if } V > 2t \\ \frac{V}{2} & \text{if } V \leq 2t. \end{cases} \quad (1)$$

For sufficiently high  $V$ , the monopolist serves the entire market and holds the consumer at  $x = 1$  to his reservation utility. Revenue equals  $V - t$ . Otherwise, the monopolist prices so that marginal revenue for the  $x = \frac{V}{2t}$  buyer equals zero and serves only consumers with  $x \leq \frac{V}{2t}$ . In this case, revenue equals  $\frac{V^2}{4t}$ . Since imitation does not occur, the patentee's revenue is its entire profit and is independent of the patent damage regime.

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licensing of process patents (Kamien and Tauman 1986) and product patents (Katz and Shapiro 1986), as well as for "inside" licensing (Marjit 1990). See Kamien (1992) for a survey.

In the duopoly case, we denote demands as  $D_H(P_H, P_I)$  and  $D_I(P_I, P_H)$ . Revenues are  $R_H(P_H, P_I) = P_H D_H(P_H, P_I)$  and  $R_I(P_I, P_H) = P_I D_I(P_I, P_H)$ , respectively. For each firm, profit  $\pi_i^d$  is the sum of revenue and damages received under damage regime  $d$ .

A consumer located at  $x$  buys from one of the two sellers if  $\text{Max}\{V - P_H - tx, V - P_I - t(1 - x)\} \geq 0$ , and buys from the patentee if  $V - P_H - tx \geq V - P_I - t(1 - x)$  also holds. The *pivotal buyer*, who is indifferent between buying from the patentee and the imitator, is

$$\hat{x}(P_H, P_I) = \frac{1}{2} + \frac{P_I - P_H}{2t},$$

provided that  $|P_H - P_I| \leq t$ . All consumers buy from one of the sellers if the *full-coverage constraint* is satisfied:

$$P_H + P_I \leq 2V - t. \tag{2}$$

If (2) holds and  $|P_H - P_I| \leq t$ , then  $D_H(P_H, P_I) = \hat{x}(P_H, P_I)$  and  $D_I(P_I, P_H) = 1 - \hat{x}(P_H, P_I)$ .

The patent covers a brand-new product. If the imitator infringes the patent by making positive sales, the patentee sues and wins damages if the court finds the patent valid and infringed. To benchmark our work against the existing literature, we follow Anton and Yao (2007) in assuming that the patent is found valid and infringed with exogenous probability  $\gamma \in [0, 1]$  and in abstracting from litigation costs.<sup>9</sup>

Through section 3, the timing of the model is as follows. First, the damage regime is exogenously determined. Second, the invention is exogenously made and patented. Third, the imitator decides whether to compete. If the imitator stays out, the patentee operates as a single-price monopolist. If the imitator enters, then the two firms compete in prices and reach a Bertrand-Nash equilibrium. Finally, if the imitator makes sales, then the court determines the patent's validity and infringement, and assigns any damages. In section 4, we include bargaining over a license prior to the imitator's entry decision. In section 5, we endogenize the innovation stage and remove the bargaining stage.

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<sup>9</sup>If litigation costs were positive, firms might prefer to settle out of court to avoid those costs. Payoffs under litigation, which would be profits under competition ( $\{\pi_i^d\}$ ) minus litigation costs, would determine threat points in settlement negotiation. If firms have identical litigation cost  $L$  and the settlement surplus ( $2L$ ) is split evenly, then firm  $i$ 's payoff under settlement [ $\pi_i^d - L + \frac{1}{2}(2L)$ ] would be identical to the payoffs in our paper (making our results robust to settlement). For rich patent litigation models, see Meurer (1989), Choi (1998) or Crampes and Langinier (2002).

## 2.1. Benchmark Cases Under Zero Liability

We first characterize best-response functions and equilibria for two useful benchmark cases of the standard Hotelling duopoly model where imitation results in zero liability (ZL). The analysis illuminates potentially confusing scenarios where the full-coverage constraint binds and multiple equilibria emerge. These outcomes are particularly important in the context of the damage regimes we study. Most notably, cases of multiple equilibria cannot be ruled out (through assumptions on  $V$  and  $t$ ) under reasonable royalty damages.

Profit functions in the duopoly model are, for  $i, j \in \{H, I\}, i \neq j$ ,

$$\pi_i^{ZL} = \text{Max}_{\{P_i\}} R_i(P_i, P_j).$$

For a given  $i$  and  $j$ , the first-order condition yields

$$P_i = \frac{P_j + t}{2}. \quad (3)$$

This function describes firm  $i$ 's best response provided that the full coverage constraint (2) holds. If this constraint does not hold for the responses in (3), then a firm's best response is either to price so that (2) holds with equality or to price like a monopolist, according to (1). When  $V \geq \frac{3t}{2}$ , the best-responses given in (3) jointly yield a unique equilibrium. When  $V < \frac{3t}{2}$ , there are multiple equilibria, determined by the other parts of the best-response functions. We consider benchmark cases illustrating each equilibrium outcome in turn.

First, consider the case  $V \in [2t, 3t]$ , which we refer to as the *Unique Equilibrium Benchmark*. The best-response function for firm  $i$  to firm  $j$ 's price is

$$P_i(P_j) = \begin{cases} \frac{P_j + t}{2} & \text{if } P_j < \frac{4V}{3} - t \\ 2V - t - P_j & \text{if } P_j \in [\frac{4V}{3} - t, V] \\ V - t & \text{if } P_j > V. \end{cases} \quad (4)$$

Best-response functions for both firms are illustrated in Figure 1. When firm  $j$  sets a low price, firm  $i$ 's best response is to price according to (3). As firm  $j$ 's price increases, firm  $i$  optimally responds by increasing  $P_i$ —prices are strategic complements. Once  $P_j$  exceeds

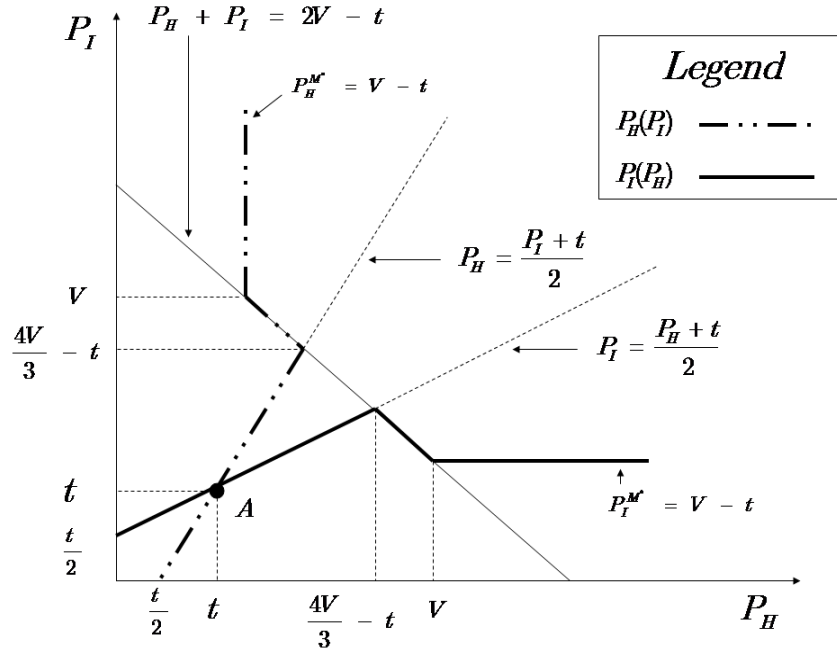


Figure 1: *Best-Response Functions, Unique Equilibrium Benchmark* ( $V \in [2t, 3t]$ )

$\frac{4V}{3} - t$ , however, firm  $i$  is best off reducing its price to the maximum level such that the full coverage constraint (2) holds. Once  $P_j > V$ , firm  $j$ 's price is completely uncompetitive—no consumer would buy from firm  $j$  regardless of what firm  $i$  does. In this case, it is optimal for firm  $i$  to price like a monopolist. Since  $V \geq 2t$ , the optimal price satisfies  $P_i^{M^*} = V - t$  (note that we use this notation for the imitator though it is never technically a monopolist). The first-order conditions determine the unique equilibrium  $P_H^* = P_I^* = t$  (point A).<sup>10</sup>

Next, consider the case  $V \in (t, \frac{3t}{2})$ , which we refer to as the *Multiple Equilibria Benchmark* and illustrate in Figure 2. The best-response functions satisfy:

$$P_i(P_j) = \begin{cases} \frac{P_j + t}{2} & \text{if } P_j < \frac{4V}{3} - t \\ 2V - t - P_j & \text{if } P_j \in [\frac{4V}{3} - t, \frac{3V}{2} - t] \\ \frac{V}{2} & \text{if } P_j > \frac{3V}{2} - t. \end{cases}$$

<sup>10</sup>This equilibrium continues to hold for  $V \in [\frac{3t}{2}, 2t]$  and for  $V > 3t$ , but the best-response functions change in cosmetic ways. For the former case, the monopoly price equals  $\frac{V}{2}$ , and the cutoff value of  $P_j$  such that firm  $i$ 's best response is the monopoly price changes accordingly. For  $V > 3t$ , the best-response functions reflect an additional strategic feature. To secure the entire market, firm  $i$  need only price at least  $t$  below firm  $j$ . Hence, another constraint emerges in the best-response functions:  $|P_i - P_j| \leq t$ . This constraint binds for  $P_i(P_j)$  when  $P_j \in [3t, V]$ .

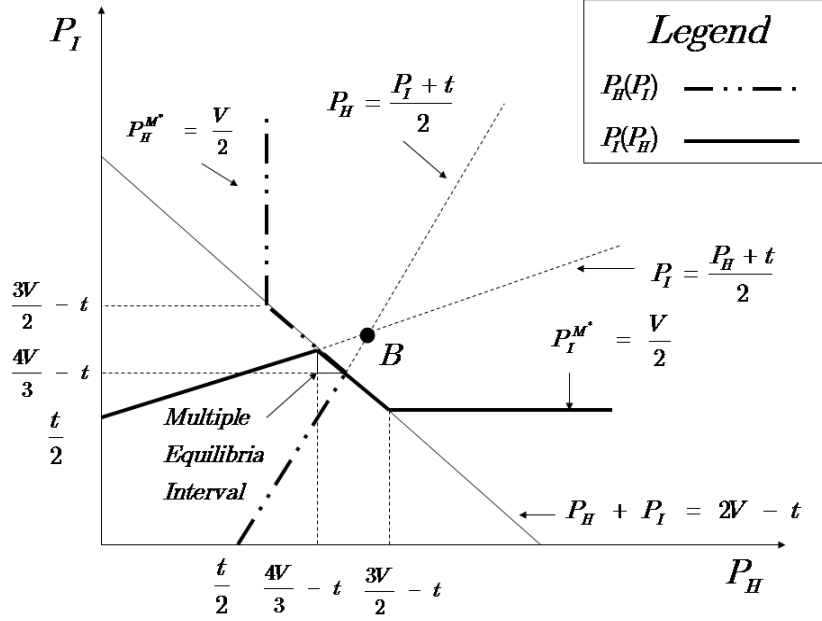


Figure 2: *Best-Response Functions, Multiple Equilibria Benchmark* ( $V \in (t, \frac{3t}{2})$ )

In contrast to the  $V \in [2t, 3t]$  case, firm  $i$  does not serve the entire market as a monopolist and the monopoly price equals  $\frac{V}{2}$ .

Most importantly, the intersection of the functions in (3), point B, no longer constitutes an equilibrium, because the full-coverage constraint is not satisfied for  $P_H = P_I = t$ . Instead, equilibria are found in the region where the best-response functions overlap along the full-coverage constraint, i.e.,  $P_H^* + P_I^* = 2V - t$  and  $P_i^* \in [\frac{4V}{3} - t, \frac{2V}{3}]$  for  $i \in \{H, I\}$ . This interval is symmetric about equal prices  $P_H^* = P_I^* = V - \frac{t}{2}$ . Intuitively, each firm's marginal revenue is increasing in price at each equilibrium price pair provided the full-coverage constraint holds, but is decreasing in price otherwise. Note that, for each pair of equilibrium prices, both firms' prices are at least as high as the monopoly price.

### 3. Market Competition in the Shadow of Expected Damages

Consider now the case where competition ensues in the shadow of expected patent damages. Our primary interest is horizontal settings where, because of fear of possible antitrust consequences, inexperience with licensing or other frictions, firms do not pursue licensing

agreements prior to producing. The analysis also establishes threat-point profits for the case where firms *do* bargain prior to competing.

We focus on the case  $V > 2t$ . This guarantees that, for  $\gamma = 0$ , equilibrium outcomes follow the Unique Equilibrium Benchmark and the monopoly price exceeds duopoly prices. When  $\gamma > 0$ , however, the presence of damage regimes make possible higher prices under duopoly and multiple equilibria similar to those in the Multiple Equilibria Benchmark.

### 3.1. Reasonable Royalty

Under the *reasonable royalty* (RR) damage regime, if the patent is found valid and infringed, then the imitator pays to the patentee fixed fee  $F$  and/or royalties at rate  $r$  per unit sold. The court uses a *hypothetical bargain* exercise to determine these components. We leave the complicated derivation of  $F$  and  $r$  for our discussion of licensing in section 4. For now, we treat these parameters as common knowledge.

Expected profits satisfy:

$$\begin{aligned}\pi_H^{RR} &= \text{Max}_{\{P_H\}} \{R_H(P_H, P_I) + \gamma [F + rD_I(P_I, P_H)]\} \\ \pi_I^{RR} &= \text{Max}_{\{P_I\}} \{R_I(P_I, P_H) - \gamma [F + rD_I(P_I, P_H)]\}.\end{aligned}\tag{5}$$

Assuming the full-coverage constraint holds, first-order conditions yield the following best-response functions:

$$\begin{aligned}P_H &= \frac{P_I + t + \gamma r}{2} \\ P_I &= \frac{P_H + t + \gamma r}{2}.\end{aligned}$$

If  $\gamma r \leq V - \frac{3t}{2}$ , the intersection of these functions gives the unique Nash equilibrium:  $P_I^* = P_H^* = t + \gamma r$ . Figure 3 illustrates this outcome for a “small” expected royalty of  $\gamma r < V - 2t$ .<sup>11</sup>

Symmetry is the most notable feature of this equilibrium. The expected royalty,  $\gamma r$ , affects the pricing decisions of the patentee and imitator in exactly the same way. For the imitator,  $\gamma r$  is an artificial marginal cost of additional market share. For the patentee,  $\gamma r$  is an artificial opportunity cost of additional market share. Because the firms price symmetrically, market shares are always equal.

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<sup>11</sup>If  $\gamma r \in [V - 2t, V - \frac{3t}{2}]$ , the equilibrium remains unique and the same as in Figure 3 but the imitator’s best-response function is cosmetically different. The difference is that the imitator does not wish to cover the entire market as a monopolist, so it charges  $P_I^M = \frac{V + \gamma r}{2}$ . This is highlighted in Figure 4.

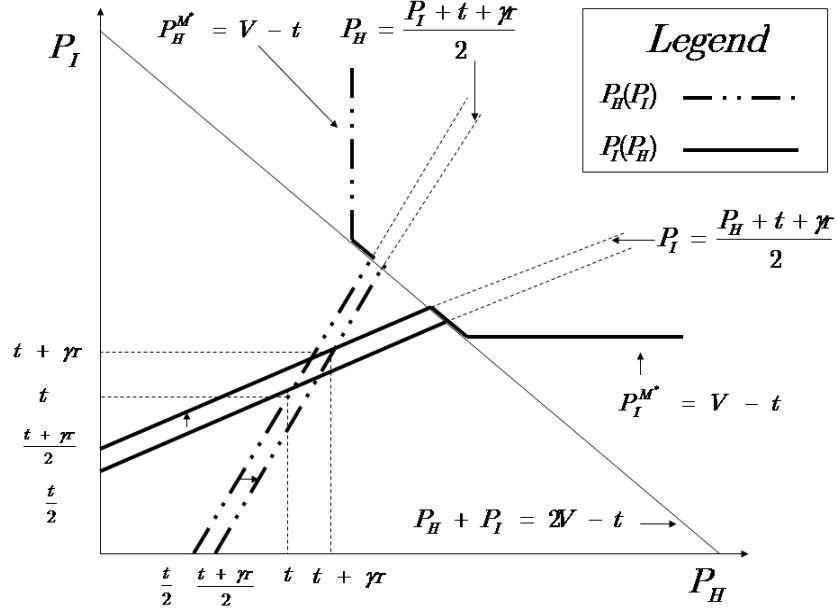


Figure 3: *Best-Response Functions, Reasonable Royalty,  $\gamma r < V - 2t$*

The Unique Equilibrium Benchmark obtains for  $\gamma r = 0$  and is also illustrated in Figure 3. Comparing cases, the expected royalty increases equilibrium prices by shifting out the intercepts of both best-response functions by  $\frac{\gamma r}{2}$ . Since  $\gamma r$  acts like a unit cost for both firms, it enhances the strategic complementarity of prices. The firms pass this artificial cost through completely to consumers, increasing their total expected profit earned by  $\gamma r$ .

Whenever all consumers buy in equilibrium, static welfare is easily calculated as reservation value minus transportation costs. Since transportation costs are minimized under equal market shares, static welfare achieves the maximum  $V - \frac{t}{4}$ . Expected equilibrium profits are asymmetric, because the imitator must pay royalties to the patentee if the patent is found valid and infringed:

$$\begin{aligned}\pi_H^{RR} &= \frac{t}{2} + \gamma(r + F) \\ \pi_I^{RR} &= \frac{t}{2} - \gamma F.\end{aligned}\tag{6}$$

Anticipating these profits, the imitator enters if and only if  $F \leq \frac{t}{2\gamma}$ , and the patentee earns  $\gamma(r + 2F)$  more than the imitator.

For a sufficiently high expected royalty,  $\gamma r > V - \frac{3t}{2}$ , the functions given by the first-order conditions intersect above the full-coverage constraint. This case, which cannot be

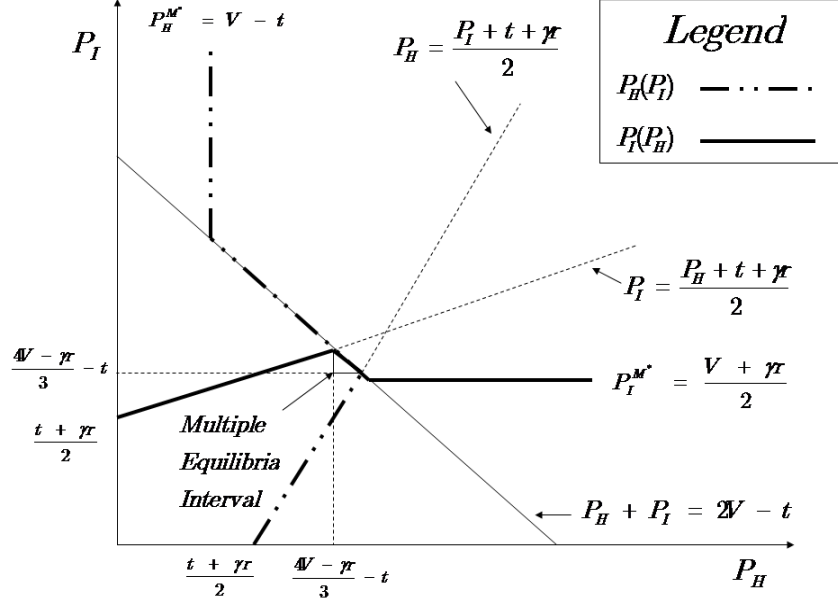


Figure 4: *Best-Response Functions, Reasonable Royalty,  $\gamma r > V - \frac{3}{2}t$*

ruled out by assumptions on  $V$  and/or  $t$ , is illustrated in Figure 4. As in the Multiple Equilibria Benchmark, the best-response functions intersect in an interval along the full-coverage constraint.

**Proposition 1** *Suppose competition ensues in the shadow of the reasonable royalty regime, with components  $\{r, F\}$ . If  $\gamma r \leq V - \frac{3}{2}t$ , the equilibrium is unique, with symmetric prices  $P_H^* = P_I^* = t + \gamma r$  and demands  $D_H = D_I = \frac{1}{2}$ . Static welfare is maximized. If  $\gamma r \in [V - \frac{3t}{2}, V - \frac{6t}{5}]$ , then equilibrium prices satisfy*

- i.  $P_H^* + P_I^* = 2V - t$*
- ii.  $P_i^* \in \left[ \frac{4V - \gamma r}{3} - t, \frac{2V + \gamma r}{3} \right]$  for all  $i \in \{H, I\}$ ,*

*and the market is fully covered. If  $\gamma r \in [V - \frac{6t}{5}, V]$ , then equilibrium prices satisfy*

- i.  $P_H^* + P_I^* = 2V - t$*
- ii.  $P_H^* \in \left[ \frac{4V - \gamma r}{3} - t, \frac{3V - \gamma r}{2} - t \right]$*
- iii.  $P_I^* \in \left[ \frac{V + \gamma r}{2}, \frac{2V + \gamma r}{3} \right]$ ,*

and the market is fully covered. If  $\gamma r > V$ , the patentee operates like a monopolist.

All proofs are in the appendix. The multiple-equilibrium cases here differ from the benchmark case because the “monopoly price” parts of the best-response functions are asymmetric. Because the imitator must pay expected royalties on every unit sold, its optimal monopoly price is  $P_I^{M*} = \frac{V+\gamma r}{2}$ . The patentee’s monopoly price,  $P_H^{M*} = V - t$ , is unaffected by the size of the royalty.<sup>12</sup> For  $\gamma r < V - \frac{6t}{5}$ , as in Figure 4, this asymmetry does not matter because  $P_I^{M*}$  is not in the set of equilibria. For  $\gamma r \in \left[V - \frac{6t}{5}, V\right)$ , however, the level of the horizontal part of the imitator’s best-response function shifts high enough such that  $P_I^{M*}$  is part of an equilibrium (it forms the lower boundary of the set of equilibria), while  $P_H^{M*}$  is not. This yields an interval of equilibria that is not symmetric about the joint-profit-maximizing prices and, for  $\gamma r > V - t$ , the set of equilibria does not include  $P_H = P_I = V - \frac{t}{2}$ . However, we will see in section 4 that the hypothetical bargain precludes the possibility of  $\gamma r > V - t$ .

### 3.2. Lost Profits

The *lost profits* (LP) regime sets damages equal to the difference between monopoly profit and duopoly profit. Expected profit functions under entry and competition are

$$\begin{aligned}\pi_H^{LP} &= \text{Max}_{\{P_H\}} \left\{ R_H(P_H, P_I) + \gamma \left[ \pi_H^M - R_H(P_H, P_I) \right] 1(\pi_H^M > R_H(P_H, P_I)) \right\} \\ \pi_I^{LP} &= \text{Max}_{\{P_I\}} \left\{ R_I(P_I, P_H) - \gamma \left[ \pi_H^M - R_H(P_H, P_I) \right] 1(\pi_H^M > R_H(P_H, P_I)) \right\},\end{aligned}$$

where  $1(\cdot)$  is the indicator function. Assuming the full-coverage constraint holds, the following functions characterize best responses:

$$\begin{aligned}P_H &= \frac{P_I+t}{2} \\ P_I &= \frac{(1+\gamma)P_H+t}{2}.\end{aligned}\tag{7}$$

If these responses do not satisfy (2), then best responses follow the same pattern as in previous sections—a firm prefers either to price on the full-coverage constraint or, facing an entirely uncompetitive price, price like a monopolist.

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<sup>12</sup>As a monopolist, the imitator would prefer to charge  $\frac{V+\gamma r}{2}$  instead of  $V - t$  as long as  $\gamma r > V - 2t$ , which always holds when multiple equilibria are relevant.

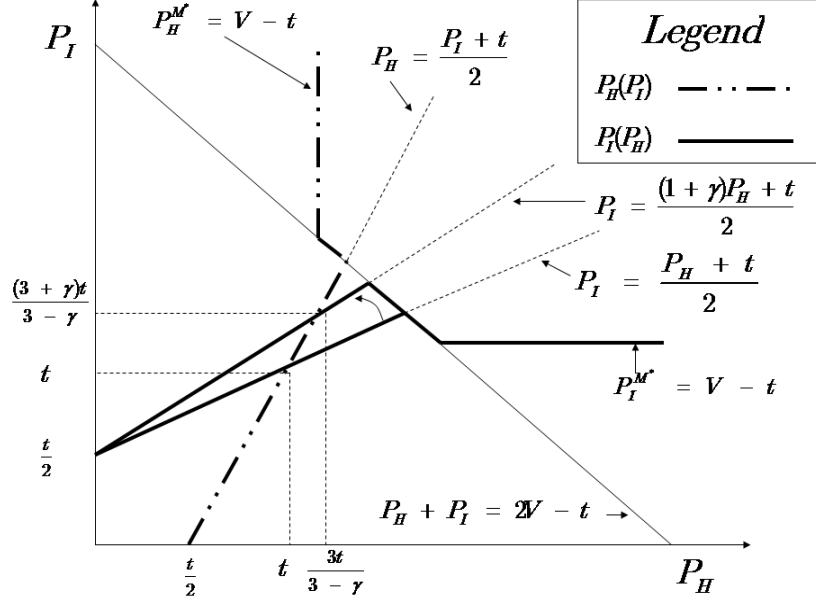


Figure 5: *Best-Response Functions, Lost Profits*

Figure 5 shows best-response functions and equilibria for  $\gamma = 0$  (which coincides with the Unique Equilibrium Benchmark) and for some  $\gamma > 0$ . Except for the constant term  $\pi_H^M$ , the patentee's profit under the lost profits regime is proportional to its profit under zero liability. Hence, the patentee's best-response function,  $P_H(P_I)$ , does not depend on  $\gamma$  and is the same here as under zero liability.

The imitator's best-response function is different. As  $\gamma$  increases, the imitator expects to pay damages with greater likelihood. Thus, it is better off reducing its total sales and increasing the patentee's total sales by choosing a higher price than under zero liability. This effect is magnified as  $P_H$  increases because the patentee's marginal lost revenue (and the corresponding marginal lost profit damages) is higher for higher  $P_H$ . Hence, the slope of the part of  $P_I(P_H)$  given by the first-order conditions in (7) increases. This enhances the strategic complementarity of prices, pushing both prices higher.

For sufficiently low  $\gamma \leq 3 - \frac{9t}{2V}$ , the unique equilibrium is found as the solution of the two first-order conditions. As illustrated in Figure 5, this equilibrium is asymmetric. The

imitator charges a higher price and sells to a smaller market:

$$\begin{aligned} P_H^* &= \frac{3t}{3-\gamma} & D_H(P_H^*, P_I^*) &= \frac{3}{6-2\gamma} \\ P_I^* &= \frac{(3+\gamma)t}{3-\gamma} & D_I(P_I^*, P_H^*) &= \frac{3-2\gamma}{6-2\gamma}. \end{aligned} \quad (8)$$

Since market shares are asymmetric, consumers suffer excessive transportation costs. Total static welfare, which varies monotonically with  $\gamma$  between  $V - \frac{t}{4}$  (when  $\gamma = 0$ ) and  $V - \frac{5t}{16}$  (when  $\gamma = 1$ ), is lower than under reasonable royalty damages for any  $\gamma > 0$ .

When the market is fully covered under duopoly and the equilibrium is unique, which holds for any  $V \geq \frac{9t}{2(3-\gamma)}$ , then lost profits are positive. Hence, infringement is never “passive” in such cases.<sup>13</sup> Intuitively, if the patentee were to price like a monopolist, the imitator’s best response is to price according to (7), which takes market share and revenue away from the patentee (see Figure 5). Passive infringement is possible, however, if  $V$  is low enough so that the patentee would leave part of the market uncovered as a monopolist.<sup>14</sup>

For sufficiently high  $\gamma > 3 - \frac{9t}{2V}$ , when such is possible, there are multiple equilibria.<sup>15</sup> As in the Multiple Equilibrium Benchmark, these equilibria are found as the interval of intersection of the best-response functions along the full-coverage constraint. In contrast to the benchmark case (and to the multiple-equilibrium cases under the reasonable royalty regime), the interval of equilibria under lost profit damages never includes the joint-profit maximizing prices  $P_H = P_I = V - \frac{t}{2}$ . Indeed, the patentee charges a lower price and sells to more consumers in every possible equilibrium.

**Proposition 2** *Suppose competition ensues in the shadow of the lost profits regime. If  $\gamma \leq 3 - \frac{9t}{2V}$ , then prices and demands are asymmetric and follow (8). If  $\gamma > 3 - \frac{9t}{2V}$ , then*

<sup>13</sup>It is important to note that in the process patent setting of Anton and Yao (2007), where there exists a generic alternative production technology, one can think of passive infringement as being relative to prior market competition where the imitator merely uses the generic technology. In our setting, there is no prior market, so our interpretation of infringement (i.e., relative to a hypothetical patentee-monopoly) is distinct.

<sup>14</sup>The technical requirement is that the imitator’s best-response function intersect the full-coverage constraint to the left of the vertical part of the patentee’s best-response function—in such a case the best-response functions would only intersect outside the full-coverage constraint and local monopolies would obtain. This is possible provided the patentee would not cover the entire market as a monopolist ( $P_H^M = \frac{V}{2}$ ) and if  $V \leq \frac{6}{5-\gamma}$ . This latter condition cannot hold if  $V \geq \frac{9t}{2(3-\gamma)}$ .

<sup>15</sup>In contrast to the reasonable royalty regime, there are assumptions on  $V$  and  $t$  that are sufficient to guarantee a unique equilibrium for any  $\gamma$ , namely  $V \geq \frac{9}{4}t$ .

there are multiple equilibria which satisfy

- i.  $P_H^* + P_I^* = 2V - t$
- ii.  $P_H^* \in \left[ \frac{4V-3t}{3+\gamma}, \frac{2V}{3} \right]$
- iii.  $P_I^* \in \left[ \frac{4V}{3} - t, \frac{2V(1+\gamma)-\gamma t}{3+\gamma} \right]$ .

If  $\gamma > 0$ , the patentee charges a lower price and serves a higher demand than the imitator.

Now consider the imitator's entry decision. If the equilibrium follows (8), we have:

$$\begin{aligned}\pi_H^{LP} &= \frac{9t(1-\gamma)}{2(3-\gamma)^2} + \gamma \left( \pi_H^M \right) \\ \pi_I^{LP} &= \frac{9t+6\gamma t-2t\gamma^2}{2(3-\gamma)^2} - \gamma \left( \pi_H^M \right)\end{aligned}\tag{9}$$

The monopolist's profit,  $\pi_H^M$ , grows with  $V$ , but revenues from competition do not. At high levels of  $V$ , the  $\pi_I^{LP}$  above will be negative, and the imitator will choose not to enter.

**Corollary 1** *Under the lost profits regime, for any positive  $\gamma$ , the imitator stays out of the market for sufficiently high  $V > V^{NE}(\gamma) > 2t$ , where the cutoff satisfies  $\pi_I^{LP}(V^{NE}(\gamma)) = 0$ .*

When the imitator does not enter, the patentee earns  $V - t$  and the imitator earns nothing. Consumer transportation costs achieve the maximum level of  $\frac{t}{2}$ , while static welfare totals  $V - \frac{t}{2}$ , the lowest possible level among all damage regimes.

### 3.3. Unjust Enrichment

Under the *unjust enrichment* (UR) regime, the patentee receives the entire revenue earned by the imitator if its patent is found valid and infringed. Expected profit functions are:

$$\begin{aligned}\pi_H^{UR} &= \text{Max}_{\{P_H\}} \{R_H(P_H, P_I) + \gamma R_I(P_I, P_H)\} \\ \pi_I^{UR} &= \text{Max}_{\{P_I\}} \{R_I(P_I, P_H) - \gamma R_I(P_I, P_H)\}.\end{aligned}$$

Apart from the constant term  $\gamma\pi_M$ , the profit functions under unjust enrichment are exactly reversed from the lost profits case. Pricing incentives for the patentee and imitator are similarly reversed. The imitator's best-response function is unchanged from the Unique

Equilibrium Benchmark, while the patentee's best-response curve pivots outward. For  $\gamma \leq 3 - \frac{9t}{2V}$ , equilibrium prices and demands are:

$$\begin{aligned} P_H^* &= \frac{(3+\gamma)t}{3-\gamma} & D_H(P_H^*, P_I^*) &= \frac{3-2\gamma}{6-2\gamma} \\ P_I^* &= \frac{3t}{3-\gamma} & D_I(P_H^*, P_I^*) &= \frac{3}{6-2\gamma}. \end{aligned} \quad (10)$$

We summarize our results with the following.

**Proposition 3** *Suppose competition ensues in the shadow of the unjust enrichment regime. If  $\gamma \leq 3 - \frac{9t}{2V}$ , then the equilibrium is unique, with prices and demands following (10). If  $\gamma > 3 - \frac{9t}{2V}$ , then there are multiple equilibria which satisfy*

$$\begin{aligned} i. \quad & P_H^* + P_I^* = 2V - t \\ ii. \quad & P_I^* \in \left[ \frac{4V}{3} - t, \frac{2V(1+\gamma) - \gamma t}{3+\gamma} \right] \\ iii. \quad & P_H^* \in \left[ \frac{4V-3t}{3+\gamma}, \frac{2V}{3} \right]. \end{aligned}$$

*If  $\gamma > 0$ , the patentee charges a higher price and serves a lower demand than the imitator.*

This set of findings is quite similar to those in Choi (2006) and Anton and Yao (2007), who find an analogous reversal under quantity competition. Now, the patentee has the greater incentive to raise price because its expected profit increases as the revenue of the imitator increases. The imitator's expected profit is proportional to the zero-liability case, so it prices normally. In equilibrium, the imitator garners more than half of the market. For the unique-equilibrium case, profits are independent of  $V$ :

$$\begin{aligned} \pi_H^{UR} &= \frac{9t+6\gamma t-2t\gamma^2}{2(3-\gamma)^2} \\ \pi_I^{UR} &= \frac{9t(1-\gamma)}{2(3-\gamma)^2}. \end{aligned} \quad (11)$$

The imitator always earns a non-negative profit, so infringement is never deterred. Total static welfare is the same as lost profits cases where the imitator enters and competes.

## 4. Bargaining

To complete our analysis of the reasonable royalty regime, we must analyze the hypothetical negotiation. To build a framework for this, consider the following model of bargaining.

Just prior to the imitator's entry decision, the patentee and imitator bargain over a single license. Let the contracting space be a two-part tariff  $\{r, F\}$ , where per-unit royalty  $r$  and fixed fee  $F$  have the same interpretation as in Section 3.1. Under a (perfectly enforceable) contract, we restrict attention to Bertrand-Nash equilibria. Efficient bargaining therefore implements joint-profit-maximizing prices that are equilibria given  $\{r, F\}$ .

Consider prices first. If the pivotal buyer is not held to reservation utility, then the patentee and imitator can both raise prices by the same amount, maintain market shares, and increase joint profit. Hence, joint profit is maximized only if this buyer earns zero utility, i.e., if the full-coverage constraint holds with equality. Factoring in this condition, recalling that we assume  $V > 2t$  and doing a bit of algebra, the firms' problem is

$$\begin{aligned} \text{Max}_{\{P_H, P_I\}} \quad & (V - \frac{1}{2}t) - \frac{1}{2t}(P_I - P_H)^2 \\ \text{s.t.} \quad & P_H + P_I = 2V - t. \end{aligned} \tag{12}$$

Joint profit is maximized if and only if the efficient bargain yields  $P_H = P_I = V - \frac{1}{2}t$ .

The firms must also choose a royalty rate such that neither wishes to deviate from the efficient prices. As seen in the analysis of the reasonable royalty regime, a well-chosen  $r$  can achieve this because it enhances the strategic complementarity of prices. The following result shows that a continuum of per-unit royalties achieve an efficient contract.

**Lemma 1** *The bargain is efficient only if it yields  $P_H = P_I = V - \frac{1}{2}t$ . These prices are a Bertrand-Nash equilibrium if and only if the per-unit royalty  $r^* \in [V - \frac{3}{2}t, V - t] \equiv \mathcal{R}^*$ . Total profit is  $\Pi_T = V - \frac{t}{2}$  and total welfare equals the maximum  $V - \frac{t}{4}$ .*

The intuition for this result follows the analysis of reasonable royalty damages for  $\gamma = 1$  (recall Proposition 1). For sufficiently low  $r < V - \frac{3}{2}t$ , even though the firms expect perfect enforcement they each prefer to cut price below  $V - \frac{1}{2}t$  to (try to) increase market share. For sufficiently high  $r > V - t$ , the imitator prefers to reduce royalty payments by charging a higher price than  $V - \frac{1}{2}t$  and earning a smaller market share than the patentee.

We denote the profits for the patentee and imitator from their market activity (including royalty revenue) under an efficient bargain as  $\Pi_H^{B_d}$  and  $\Pi_I^{B_d}$ . Factoring in the equilibrium

per-unit royalty payment of  $\frac{r^*}{2}$ , we can write profits for a given  $d$ :

$$\begin{aligned}\Pi_H^{B_d} &= \frac{1}{2}(V - \frac{t}{2}) + \frac{r^*}{2} + F_d^* \\ \Pi_I^{B_d} &= \frac{1}{2}(V - \frac{t}{2}) - \frac{r^*}{2} - F_d^*.\end{aligned}\tag{13}$$

The equilibrium fixed payment  $F_d^*$  remains to be determined.

The expected profits from market competition in absence of a license,  $\pi_i^d$  for  $i \in \{H, I\}$ , form the *threat points* for bargaining. The patentee and imitator choose  $F_d^*$ , conditional on  $r^*$ , to evenly split the *bargaining surplus*,  $S_B^d = V - \frac{t}{2} - (\pi_H^d + \pi_I^d)$ , achieved when a contract is implemented in the shadow of regime  $d$ —i.e., they have equal bargaining power. In efficient bargaining, each party gets its threat-point profit plus half of the extra surplus:

$$\begin{aligned}\Pi_H^{B_d} &= \pi_H^d + \frac{S_B^d}{2} \\ \Pi_I^{B_d} &= \pi_I^d + \frac{S_B^d}{2}.\end{aligned}\tag{14}$$

The equilibrium  $F_d^*$  sets the profits in (14) equal to those from (13), given  $r^* \in \mathcal{R}^*$ :

$$F_d^* = \frac{1}{2}(\pi_H^d - \pi_I^d - r^*).\tag{15}$$

#### 4.1. The Hypothetical Negotiation

US court precedent says that reasonable royalty damages are to be calculated,

“...based upon a hypothetical negotiation between the patent owner and the infringer, at the time the infringement began, with both parties to the negotiation assuming that the patent is valid and would be infringed but for the license.”

(*Northlake v. Glaverbel*, 72 F.Supp. 2d 893 [ND Ill. 1999])

In applying the hypothetical negotiation (HN) inquiry, courts have generally treated the assumption of validity and infringement, which directly implies a mutual belief of  $\gamma = 1$ , as also mandating an injunction for hypothetical threat point competition.<sup>16</sup> In our model, this

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<sup>16</sup>The recent case of *eBay v. MercExchange*, 547 U.S. 388 (2006) ended the general rule that an injunction should issue following a finding that a patent is valid, instead giving the court discretion in determining whether to issue an injunction. It is unclear if and how this ruling will affect the hypothetical negotiation (Blackburn and Meyer 2007). We discuss this further in the conclusion.

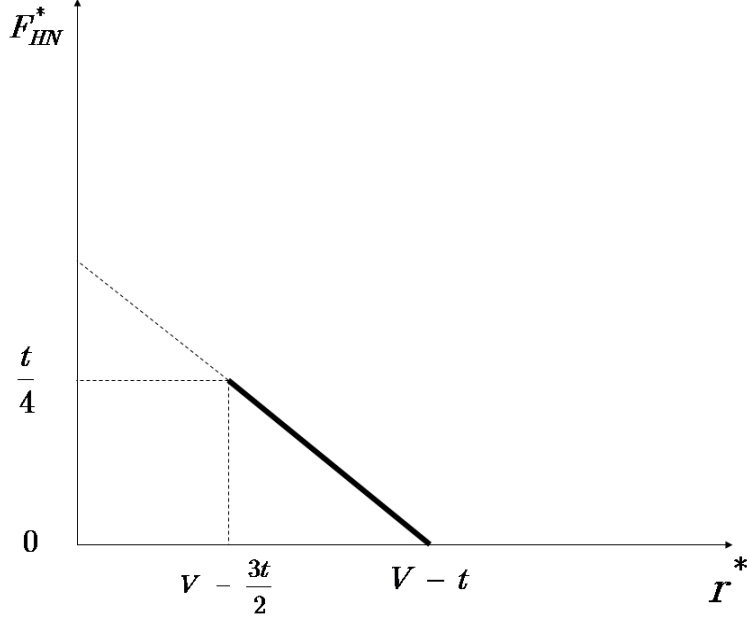


Figure 6: *Possible Reasonable Royalty Rates under the Hypothetical Negotiation*

implies that the threat points for the hypothetical negotiation are that the imitator would not enter and the patentee would operate as a monopolist:

$$\begin{aligned}\pi_H^{HN} &= V - t \\ \pi_I^{HN} &= 0.\end{aligned}\tag{16}$$

Plugging these values into (15), we find the following result.

**Proposition 4.** *Following the hypothetical negotiation, the reasonable royalty components  $\{r^*, F_{HN}^*\}$  satisfy  $r^* \in \mathcal{R}^*$  and*

$$F_{HN}^* + \frac{r^*}{2} = \frac{t}{4} + \frac{1}{2} \left( V - \frac{3t}{2} \right)\tag{17}$$

The schedule of possible  $\{r^*, F_{HN}^*\}$  combinations, which pins down the imitator’s *total* payment to the patentee exactly, is shown in Figure 6. While there are multiple royalty rates that satisfy the hypothetical negotiation, note that the equilibria that yield these rates do not depend on the firms’ beliefs about what the court would choose as reasonable royalty damages—i.e., there is no “circularity” problem. Rather, the patentee and imitator believe

that threat points reflect the profits in (16), which importantly *do not depend on  $F$  and  $r$* .<sup>17</sup> Indeed, the court can avoid the circularity problem altogether by simply using lost profit damages to construct threat points in the hypothetical negotiation.

Now consider the implications of  $\{r^*, F_{HN}^*\}$  for the analysis of the reasonable royalty regime in absence of a bargain (recall Section 3.1). For  $\gamma r^* \leq V - \frac{3t}{2}$ , the equilibrium is unique and expected profits follow (6). Since the imitator's profit from (6) is between  $\frac{t(2-\gamma)}{4}$  and  $\frac{t}{2}$ , entry is always optimal. Expected profit increases in  $r^*$  for both firms. A higher  $r^*$  yields higher prices, so there is more total profit. A higher  $r^*$  also necessitates a *lower*  $F_{HN}^*$ , so the imitator's expected profit increases.

For  $\gamma r^* > V - \frac{3t}{2}$ , the equilibrium is not unique. Since  $r^* \leq V - t$ , however, it follows that  $\gamma r^* \leq V - t$ , so that symmetric pricing is always an equilibrium (Proposition 1). Under symmetric pricing, profits follow the right-hand sides of the equals signs in (13) and are invariant to royalty rates. Under asymmetric pricing, the size of  $r^*$  affects the range of possible equilibrium prices. The imitator's profit is always non-negative, so this regime does not preclude entry.<sup>18</sup> Such prices are not functions of  $r^*$ , however, so we cannot perform comparative statics on other effects of  $r^*$  and  $F_{HN}^*$  without additional restrictions.

## 5. Incentives to Innovate

The goal of a patent system is to stimulate innovation without excessively harming static welfare. Having considered welfare, we now compare incentives to innovate across damage regimes for the case  $\gamma > 0$ . To avoid a taxonomy and to fix ideas, we focus on cases where unique equilibria obtain. Hence, our discussion centers on the case  $V \geq \frac{9t}{2(3-\gamma)}$  and, for the reasonable royalty regime, on the case where the court chooses components  $\{r^* = V - \frac{3t}{2}, F_{HN}^* = \frac{t}{4}\}$ . These assumptions allow us to highlight each regime's potential strengths.

Consider a standard “memoryless” patent race model styled on Reinganum (1983). Initially, neither firm has innovated and both firms earn zero profits per unit time. Each

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<sup>17</sup>In an *actual* bargain, were reasonable royalty to be specified as the damage regime ( $d = RR$ ), the equilibrium  $r^*$  would not depend on what particular royalty rates the firms believe the court would choose. However,  $F_{RR}^*$  would depend on beliefs about  $F_{HN}^*$  (recall (15)), and it is possible that  $F_{RR}^* \neq F_{HN}^*$ .

<sup>18</sup>The tedious and uninteresting proof of this is available from the authors upon request.

chooses a flow of investment  $x_k, k \in \{1, 2\}$ , which yields continuous probability of innovation  $h(x_k)$  and costs  $x_k$ . Further, let  $h$  be twice continuously differentiable, strictly increasing and strictly concave in  $x$  for any non-negative  $x$ , and let it satisfy Inada conditions.

The firm that innovates first becomes the patentee and earns present-discounted profit  $W_H^d$ , while the other firm becomes the imitator and earns  $W_I^d$  for damage regime  $d$ . Assuming, for convenience, that the infringement suit lasts for the life of the innovation, we can write  $W_i^d = \frac{\pi_i^d}{\rho}$ , where  $\rho$  is the discount rate.

The expected payoff to firm 1 when it chooses investment  $x_1$  and firm 2 chooses  $x_2$  is

$$\begin{aligned} U_1(x_1, x_2) &= \int_0^\infty [h(x_1)W_H^d + h(x_2)W_I^d - x_1] e^{-\rho t} e^{-[h(x_1)+h(x_2)]t} dt \\ &= \frac{h(x_1)W_H^d + h(x_2)W_I^d - x_1}{\rho + h(x_1) + h(x_2)}. \end{aligned}$$

Taking first-order conditions and imposing symmetry, we find

$$h'(x^*)[h(x^*)(W_H^d - W_I^d) + \rho W_H^d + x^*] - [\rho + 2h(x^*)] = 0, \quad (18)$$

where  $x_1 = x_2 \equiv x^*$  is the equilibrium investment (second-order conditions hold, as shown in the proof of Proposition 5).

Using the implicit function theorem on (18), we can show the following.

**Proposition 5.** *The equilibrium investment  $x^*$  is strictly increasing in the patentee's profit  $W_H^d$  and in the difference in profits,  $W_H^d - W_I^d$  (for constant  $W_H^d$ ).*

Hence, if a damage regime generates both a higher  $W_H^d$  and a higher  $W_H^d - W_I^d$ , then it generates superior incentives to innovate. Moreover, if  $W_H^d - W_I^d$  is equal across two regimes but  $W_H^d$  is unequal, then the regime with the higher  $W_H^d$  generates superior incentives to innovate.

Assuming entry is deterred, the patentee's expected (flow) profit under the lost profits regime is  $V - t$  and the imitator's profit is 0, so that the difference in profits is  $V - t$ . Under the reasonable royalty regime, the patentee's profit is  $\frac{t}{2} + \gamma \left( V - \frac{5t}{4} \right)$  and the imitator's profit is  $\frac{t(2-\gamma)}{4}$ , so that the difference in profits is  $\gamma(V - t)$ . Thus, if  $\gamma < 1$ , the difference in profits is bigger under lost profits. As  $V$  increases, the patentee's profit under lost profits increases

at rate 1 but its profit under reasonable royalty increases at rate  $\gamma$ . For sufficiently high  $V$ , the patentee's profit is bigger under lost profits and, therefore, lost profits generates superior incentives to innovate. Unjust enrichment, where the patentee's profit is at most  $\frac{13t}{8}$  and does not depend on  $V$ , never produces the strongest incentives.

Next, fix  $V$  and let  $\gamma$  approach 1. The difference in profits under reasonable royalty approaches  $V - t$ , the same as under lost profits. The patentee's profit approaches  $V - \frac{3t}{4}$ , however, which is higher than the patentee's profit under lost profits. Hence, incentives to innovate are higher under reasonable royalty.<sup>19</sup>

## 6. Conclusion

Focusing on product patents in a differentiated, duopoly setting, we find that maximizing static welfare does not necessarily preclude high incentives to innovate. When the likelihood of patent enforcement is high, the reasonable royalty regime maximizes both static welfare and incentives to innovate. If patent enforcement is uncertain and product value is high, however, then the lost profits regime, which generates the standard sacrifice of static welfare, maximizes incentives to innovate.

In practice, it is not always possible to calculate lost profits (Coolley 1993) and the court-imposed reasonable royalty rate may differ greatly from what firms would have agreed to in bargaining (Reitzig et al. 2006). However, constructions of damage awards do rely on economic benchmarks (Werden et al. 1999; Werden et al. 2000) and our results may be useful in that context. For example, when products are differentiated and a monopolist would serve all demand, duopoly prices under infringement may be higher than the monopoly price. Hence, courts should use caution in basing damages on hypothetical "price erosion."

Perhaps more importantly, our analysis of the hypothetical negotiation also shows how the construction of threat points is crucial to whether the circularity problem identified

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<sup>19</sup>If one were to treat bargaining (as in section 4) as the equilibrium, then lost profits would generate precisely the same profits as reasonable royalty when  $\gamma = 1$ —the patentee would get  $\frac{1}{2}$  of the bargaining surplus for a total profit of  $V - \frac{3t}{4}$ . In that case, incentives to innovate would be maximized under either regime. For  $\gamma < 1$ , the same profits would obtain under lost profits provided that entry would be deterred absent a license. Under reasonable royalty, however, the patentee's profit would be lower and the imitator's would be higher. Thus, lost profits would maximize incentives to innovate for  $\gamma < 1$  and high  $V$ .

by Schankerman and Scotchmer (2001) emerges. Given recent events, this has potentially important implications. In *eBay v. MercExchange* (2006), the Supreme Court overruled what had been the standard legal precedent in patent cases that a ruling of “valid and infringed” necessitates an injunction. As Blackburn and Meyer (2007) point out, the Court failed to consider ramifications for the hypothetical negotiation. If courts eliminate the injunction requirement from the threat points in the hypothetical negotiation, they argue, the circularity problem could reemerge. Our analysis shows, however, that the circularity is avoided by specifying that lost profit damages be used to determine threat points.

We leave several interesting extensions for future work. By modeling the imitator as weighing a more favorable horizontal location versus a greater likelihood of infringement, one could study the impact of damage regime on product differentiation. Alternatively, by introducing additional periods of competition one could study injunctions beyond their impact on hypothetical negotiations. This would be particularly interesting in the presence of consumer switching costs, as the expectation of an injunction after period one would (asymmetrically) affect how aggressively patentees and imitators compete for first-period market share. We look forward to further progress.

## Appendix

**Proof of Proposition 1.** We begin by identifying the patentee’s best-response function:

$$P_H(P_I) = \begin{cases} \frac{P_I+t+\gamma r}{2} & \text{if } P_I < \frac{4V-\gamma r}{3} - t \\ 2V - t - P_I & \text{if } P_I \in [\frac{4V-\gamma r}{3} - t, V] \\ V - t & \text{if } P_I > V. \end{cases}$$

When the full-coverage constraint (2) holds, then the profit function in (5) is concave (the second derivative is  $-1/t$ ), so the first-order condition  $P_H = \frac{P_I+t+\gamma r}{2}$  yields the best response. Pricing according to this expression yields prices that satisfy (2) for any  $P_I < \frac{4V-\gamma r}{3} - t$ . For  $P_I \in [\frac{4V-\gamma r}{3} - t, V]$ , pricing according to the first-order condition yields both a price that exceeds the patentee’s optimal monopoly price  $V - t$  and incomplete market coverage. Since profit is concave in price in the monopolist’s problem when the market is not fully covered, profit increases as price is cut to the point that (2) holds. It is not optimal to cut price further, because marginal profit is *increasing* in  $P_H$  whenever (2) holds and  $P_H < \frac{P_I+t+\gamma r}{2}$ .

If  $P_I > V$ , then no consumer will buy from the imitator regardless of  $P_H$ , so the patentee essentially operates as a monopolist.

Next, consider the imitator. First, note that royalties matter to the imitator in its (hypothetical) monopoly problem, in contrast to the monopolist. Indeed,  $\pi_I^M = (P_I - \gamma r) \left( \frac{V - P_I}{t} \right)$  provided that the market is incompletely covered and  $\pi_I^M = P_I - \gamma r$  if the market is completely covered. If  $\gamma r \leq V - 2t$ , the imitator's optimal monopoly price is  $V - t$ , the same as the patentee, so its best-response function is the same as the patentee's. For  $\gamma r > V - 2t$ , the imitator would not wish to serve the entire market as a monopolist. Rather,  $P_I^{M*} = \frac{V + \gamma r}{2}$ . The imitator's best-response function is then:

$$P_I(P_H) = \begin{cases} \frac{P_H + t + \gamma r}{2} & \text{if } P_H < \frac{4V - \gamma r}{3} - t \\ 2V - t - P_H & \text{if } P_H \in \left[ \frac{4V - \gamma r}{3} - t, \frac{3V - \gamma r}{2} - t \right] \\ \frac{V + \gamma r}{2} & \text{if } P_H > \frac{3V - \gamma r}{2} - t. \end{cases}$$

Bertrand-Nash equilibria obtain for price pairs such that both best-response functions are satisfied. For  $\gamma r \leq V - \frac{3t}{2}$ , pricing according to the first-order conditions (first lines in the best-response functions) yield  $P_H^* = P_I^* = t + \gamma r \leq \frac{4V - \gamma r}{3} - t$ . In this case,  $\hat{x}(P_H^*, P_I^*) = \frac{1}{2}$  and demands satisfy  $D_H = D_I = \frac{1}{2}$ . Transportation costs are  $2 \int_0^{\frac{1}{2}} t x dx = \frac{t}{4}$ .

For  $\gamma r \in \left( V - \frac{3t}{2}, V \right]$ , equilibria are found as the intersection of the best-response functions along the full-coverage constraints (see the second lines of the best-response functions). For  $\gamma r > V$ , the imitator's profit is negative if  $P_I < V$  but its demand is zero if  $P_I > V$ . Hence, it is uncompetitive and the patentee operates essentially as a monopolist. **QED**

**Proof of Proposition 2.** The patentee's best-response function follows (4) from the Unique Equilibrium Benchmark. Conversely, the imitator's best response function is:

$$P_I(P_H) = \begin{cases} \frac{(1+\gamma)P_H + t}{2} & \text{if } P_H < \frac{4V - 3t}{3 + \gamma} \\ 2V - t - P_H & \text{if } P_H \in \left[ \frac{4V - 3t}{3 + \gamma}, V \right] \\ V - t & \text{if } P_H > V. \end{cases}$$

If the full-coverage constraint (2) is non-binding, then each party will price according to (8). These prices satisfy (2) if  $\gamma \leq 3 - \frac{9t}{2V}$ . Clearly,  $P_H^* < P_I^*$  and  $D_H > D_I$  for such equilibria.

If the prices in (8) violate the full-coverage constraint (2), then each firm's best response is to price on the constraint (though not below the monopoly price,  $V - t$ ). The best-response functions overlap along (2) when  $P_H \in \left[ \frac{4V - 3t}{3 + \gamma}, \frac{2V}{3} \right]$  and  $P_I \in \left[ \frac{4V}{3} - t, \frac{2V(1+\gamma) - \gamma t}{3 + \gamma} \right]$ . The highest possible patentee price,  $\frac{2V}{3}$ , is lower than the lowest imitator price,  $\frac{4V}{3} - t$ , if  $V > \frac{3t}{2}$ . Since  $V > 2t$ , we find  $P_H^* < P_I^*$  and therefore  $D_H > D_I$  in any equilibrium. **QED**

**Proof of Corollary 1.** Infringement is deterred if and only if  $\frac{9t+6\gamma t-2\gamma^2 t}{2(3-\gamma)^2} < \gamma(V-t)$ . This is equivalent to  $V > \frac{9t+24\gamma t-14\gamma^2 t+2\gamma^3 t}{2\gamma(3-\gamma)^2} \equiv V^{NE}(\gamma)$ . A bit of algebra shows that this is equivalent to  $9 > 12\gamma - 10\gamma^2 + 2\gamma^3$ , which holds for any  $\gamma \in [0, 1]$ . The condition  $V^{NE}(\gamma) > 2t$  is similarly shown. Clearly,  $V^{NE}(\gamma)$  is finite for any  $\gamma > 0$ . **QED**

**Proof of Proposition 3.** This follows immediately from the fact that, except for the constant term  $\gamma\pi_M$ , the profit functions for the patentee and imitator are reversed from the lost profits case. Hence, the equilibrium prices and market shares are reversed. **QED**

**Proof of Lemma 1.** It is clear from (12) that total profit is maximized only if  $P_H = P_I = V - \frac{1}{2}t$ . Moreover, Proposition 1 shows that these prices are an equilibrium if and only if  $\gamma r \in [V - \frac{3t}{2}, V - t]$ . Since any bargain is perfectly enforceable, equilibrium pricing under a bargain follows Proposition 1 for  $\gamma = 1$ . Hence  $r \in [V - \frac{3t}{2}, V - t]$  is necessary. **QED**

**Proof of Proposition 4.** The proof follows directly from the discussion in the text. Simply substitute the threat points from the hypothetical negotiation into (14) and solve. The limits on  $r$  follow directly from Lemma 1. **QED**

**Proof of Proposition 5.** Given  $h''(x_k) < 0$ , the second-order condition for  $x_1$  satisfies

$$\frac{d^2U(x_1, x_2)}{dx_1^2} \Big|_{x_1=x_2=x^*} = \frac{h''(x_1) [h(x_2)(W_H^d - W_I^d) + \rho W_H^d + x_1]}{[\rho + h(x_1) + h(x_2)]^2} < 0.$$

Hence, the objective function is strictly concave local to  $x^*$ . Rewriting (18) as

$$F(x^*, W_H^d, W_I^d) = h'(x^*)[h(x^*)(W_H^d - W_I^d) + \rho W_H^d + x^*] - [\rho + 2h(x^*)] = 0,$$

it is clear that  $\frac{dx^*}{dW_H^d}$  and  $\frac{dx^*}{d(W_H^d - W_I^d)} \Big|_{W_H^d=K}$  have the same sign as  $\frac{dF}{dW_H^d}$  and  $\frac{dF}{d(W_H^d - W_I^d)} \Big|_{W_H^d=K}$ , respectively. Then we have  $\frac{dF}{d(W_H^d - W_I^d)} = h'(x^*) [h(x^*) + \rho] > 0$  and  $\frac{dF}{dW_H^d} \Big|_{W_H^d - W_I^d = K} = h'(x^*)\rho > 0$ , completing the proof. **QED**

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