

Efficient Dissolution of Partnerships and the Structure of Control

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Abstract

Past work has shown that asymmetric information and asymmetric ownership affect the possibility of efficient dissolution of partnerships. We show in this paper that *control* is also a central determinant of the possibility of efficient implementation. We demonstrate this point by analyzing a benchmark case of asymmetric control where a single partner exercises complete control under the status quo partnership. We show that two-person partnerships cannot be dissolved efficiently with any incentive compatible, individually rational mechanism, regardless of the ownership structure, but that this impossibility result can be reversed if the number of partners is sufficiently large. We also show that equal-shares partnerships are not generally the easiest to dissolve when the distribution of control is asymmetric.

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1. Introduction

When a partnership is to be dissolved, the partners ideally wish to capture the full gains from trade by allocating the partnership to the most capable partner. How easy is it to do this? Theoretical inquiries into this question have shown that both asymmetric information and asymmetric ownership structure make it difficult and sometimes impossible to design individually rational mechanisms to implement efficient dissolution. In this paper, we show that asymmetric control also affects the possibility of efficient dissolution.

Akerlof (1970) provides the fundamental intuition for the effects of asymmetric information in an extreme-ownership setting, and Myerson and Satterthwaite (1983) prove general impossibility results for bilateral exchange of an asset under private information. Cramton, Gibbons and Klemperer (1987, henceforth “CGK”) show that, if partners have independent and identically distributed types that represent their valuations for the asset, then the Myerson-Satterthwaite impossibility result extends to partnerships where the partners’ shares are unbalanced though not necessarily extreme. CGK also point out, however, that equal-shares partnerships — as well as unequal-shares where ownership is not “too unbalanced” — can always be dissolved efficiently.

In recent work, Moldovanu (2002), Fieseler, Kittsteiner and Moldovanu (2003) and Jehiel and Paudner (2006) consider the case where partners’ valuations depend on others’ types as well as on their own. These papers show that interdependence of valuations can affect the set of dissolvable partnerships significantly. Moldovanu and Fieseler et al. find that when information is ex ante symmetric, a partnership is more difficult to dissolve if a given partner’s valuation is increasing in the types of the other partners, while the opposite is true if the partner’s valuation is decreasing in the types of the other partners. In the former case, the equal-shares partnership may not be dissolvable efficiently, while in the latter case even efficient bilateral exchange may be possible. Jehiel and Paudner focus on cases where only one partner is informed about the value of the co-owned asset. In such a setting, they identify a wide class of situations where efficient dissolution is unachievable.

In the models of the papers cited above, individual rationality requires that any partner aware of his private type expects to earn, via the dissolution mechanism, at least as much as his share times his expected valuation for the asset. This modelling choice neglects one potentially crucial feature of partnerships — that an asset’s gross value, from an individual partner’s perspective, may depend on the very existence of the partnership. The reason is that when an agent buys out his partners, the asset’s gross value can change for him.

Circumstances where this occurs are natural in many trading situations. Consider the case where the asset is a firm whose value stems from the profit it generates. It is natural to assume, as is common in the literature, that the independent type of each partner represents the profit he would earn with full ownership of the firm’s assets. Now, when the firm is owned by a partnership, the ownership shares determine how the firm’s profit is to be split among them, but the distribution of *control* is assigned separately.^{1,2} The assignment of control gives rise to a particular profit level, which is the value of the firm under the partnership and is common to all partners. In this setting, individually rational participation in a dissolution mechanism requires that any partner aware of his private type expects to earn at least as much as his share times his expected valuation for the firm *were it to remain a partnership*.³

We develop and analyze a tractable model of dissolution of partnerships that are initially characterized by asymmetric control. We term this type of structure a “silent partnership,” where one “active” partner manages the firm while all other “silent” partners simply share in the profits.⁴ Each partner’s independent private type represents the firm’s profit (and his own payoff) under his sole ownership, while the active partner’s type determines the firm’s profit under the partnership. This last feature of our setting implies that the partners’ payoffs are interdependent. Thus, our structure can be viewed as a hybrid of the independent private values case studied by CGK and the cases of interdependent valuations (Moldovanu 2002; Fieseler et al. 2003; Jehiel and Paudner 2006).

Building on methodological contributions by Williams (1999) and Krishna and Perry (2000), among others, we use Vickrey-Clarke-Groves mechanisms to study the possibility of efficient dissolution of silent partnerships. Our results are quite distinct from those in the received literature on partnership dissolution. We show, first, that it is impossible to dissolve efficiently silent partnerships that consist of only two partners.⁵ While surprising, this finding

¹Our definition of control is similar to Aghion and Tirole’s (1997) definition of *real authority*. We employ different terminology because we focus on distinct issues. For example, we do not attempt to explain how real authority is acquired or when it is likely to be detached from formal authority.

²In early work in the literature on property rights, ownership and control were closely associated. Grossman and Hart (1986, p. 694), for instance, “define ownership as the power to exercise control.” Hart (1995, p. 64), however, notes that “residual income and residual control do not have to be bundled together on a one-to-one basis” and describes several situations characterized by the lack of a one-to-one relationship.

³A related but distinct approach is taken by Jehiel and Paudner (2006), who allow for “increasing returns to scale” in the ownership shares. In that case, each partner’s valuation for the asset increases disproportionately with the partner’s share of the asset, so valuations depend on the ownership structure.

⁴This structure resembles a very common characteristic of regular businesses, which are often owned by several partners but effectively run by only a few of them. The term “silent partnerships” is frequently used in business environments, particularly in some countries (e.g. Japan), to denote partnerships where the distribution of control is significantly asymmetric.

⁵This result is of particular importance given that two-partner partnerships are quite prevalent. Hauswald

highlights how the assignment of control, shown in the property rights literature to be a key element of optimal integration decisions,⁶ affects the possibility of efficient dissolution of partnerships. The asymmetry of control, emphasized at the extreme in a silent partnership, emerges as another potential stumbling block in implementing efficient dissolution.

We show, however, that an increase in the number of partners can mitigate the problems stemming from asymmetric control. We provide intuition for this result by showing how a simple mechanism, involving side payments and a standard first-price auction, could be used to achieve efficient dissolution as the number of partners becomes sufficiently large. We also show that ownership symmetry, which facilitates efficient dissolution in the settings of CGK and Fieseler et al. (2003), play no such role when asymmetry of control is present. In fact, we show that with more than two partners even partnerships with extreme ownership structures can be dissolvable, as long as the owner of the business is a silent partner; if the owner is instead the active partner, the Myerson-Satterthwaite impossibility result obtains. We summarize our findings with an example for the case of uniform types.

2. The Model

2.1. Preliminaries

Consider a partnership that jointly owns a firm. Before it is dissolved, there are $n > 1$ risk-neutral partners indexed by $i \in \{1, \dots, n\}$. Partner i owns share $r_i \in [0, 1]$ of the firm and shares sum to one ($\sum_{i=1}^n r_i = 1$). Each partner i has private information $v_i \in [\underline{v}, \bar{v}]$, which represents his “managerial capacity.” Each v_i is drawn independently from distribution F_i , which is common knowledge and has positive continuous density f_i . Each v_i represents the firm’s flow of profits under partner i ’s full control. Let $v \equiv (v_1, \dots, v_n)$.

To study the effects of asymmetric control on the possibility of efficient dissolution, we assume a single partner exercises full control under the status quo partnership. This type of partnership has the virtues of being both intuitive and analytically tractable.

and Hege (2003), for instance, show that around 80% of a sample of US joint ventures announced between 1985 and 2000 were two-partner joint ventures. This is also the case emphasized by much of the literature (e.g. Myerson and Satterthwaite 1983, De Frutos and Kittsteiner 2005, Jehiel and Paudner 2006).

⁶Grossman and Hart (1986) and Hart and Moore (1990) treat control as a “residual right” to decide how to use an asset. They show that the optimal firm structure often depends crucially upon the assignment of such rights.

Definition 1. A **Silent Partnership** $SP \langle n, r, F_1, F_2 \rangle$ is defined as follows. Let partner 1 have full control over the jointly owned business; call him the **active** partner and all other $n - 1$ partners the **silent** partners. Let r denote the active partner's share of the partnership; thus, the silent partners' shares sum to $1 - r$. Let the active partner's type v_1 be drawn from distribution F_1 and the silent partners' types $\{v_2, \dots, v_n\}$ be each drawn from distribution F_2 . The profit of the firm under the partnership is $\pi(v) = v_1$.

Using the revelation principle, we focus on a direct revelation game where partners report simultaneously their types and a mechanism allocates shares $s(v) = \{s_1, \dots, s_n\}$ such that $\sum s_i(v) = 1$, and determines transfer payments $t(v) = \{t_1, \dots, t_n\}$ to the partners. We refer to $\langle s, t \rangle$ as a *trading mechanism*. If truthful revelation of types is a Bayesian-Nash equilibrium under mechanism $\langle s, t \rangle$, then we say it is *incentive compatible*. A mechanism is (ex post) *efficient* if the firm is allocated to the partner with the highest valuation. A mechanism is (ex ante) *budget balanced* if the mechanism designer does not expect to subsidize the partners, i.e. $E\{\sum t_i(v)\} \leq 0$.

Under mechanism $\langle s, t \rangle$, partner i obtains utility $v_i s_i + t_i$ and expects, conditional on his type, to receive shares and transfers $S_i(v_i) \equiv E_{-i}\{s_i(v)\}$ and $T_i(v_i) \equiv E_{-i}\{t_i(v)\}$, respectively, where $E_{-i}\{\cdot\}$ denotes the expectation operator with respect to v_{-i} and $v_{-i} \equiv \{v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n\}$. Partner i 's interim expected utility from the mechanism is therefore $M_i(v_i) = v_i S_i(v_i) + T_i(v_i)$. Under the partnership, it is $P_i(v_i) = r_i E_{-i}\{v_1\}$.

The key aspect of our setting that distinguishes it from the received literature is the individual rationality constraint that arises from our class of partnerships. The conventional notion of interim individual rationality is that any partner aware of his type must expect to earn, via the dissolution mechanism, at least as much as his share times his type, $M_i(v_i) \geq r_i v_i$. In the present context, each partner i requires instead at least his share r_i of his expectation over the firm's expected profit to participate in a dissolution mechanism. That is, in our setting interim individual rationality requires $M_i(v_i) \geq P_i(v_i)$ for each partner.

As a result, the interim *expected net utility* from dissolving instead of maintaining the partnership, $U_i(v_i) \equiv M_i(v_i) - P_i(v_i)$, is different for the active and silent partners:

$$\begin{aligned} U_1(v_1) &= v_1 S_1(v_1) + T_1(v_1) - r_1 v_1, \\ U_i(v_i) &= v_i S_i(v_i) + T_i(v_i) - r_i E_{-i}\{v_1\} \quad \text{for all } i \neq 1. \end{aligned}$$

The *worst-off type* of partner i in $\langle s, t \rangle$ is defined by

$$U_i(v_i^*) \leq U_i(v_i) \text{ for all } v_i.$$

Formally, we call a mechanism (interim) *individually rational* if expected net utility is non-negative for all partners, i.e. if $U_i(v_i^*) \geq 0$.

2.2. Efficient Dissolution

Since our problem falls within the general class analyzed by Williams (1999), it suffices to restrict attention to Vickrey-Clarke-Groves (VCG) mechanisms. Let $\tilde{v} = \max_i \{v_1, \dots, v_n\}$. The efficient allocation rule assigns shares so that

$$s_i(v) = \begin{cases} 1 & \text{if } v_i = \tilde{v} \\ 0 & \text{if } v_i < \tilde{v}. \end{cases} \quad (1)$$

This implies that the expected share function satisfies $S_i(v_i) = G_i(v_i) \equiv \prod_{j \neq i} F_j(v_i)$.

In our setting, the VCG class of mechanisms specifies the following transfers:

$$t_i(v) = \begin{cases} -k_i & \text{if } s_i(v_i) = 1 \\ \tilde{v} - k_i & \text{if } s_i(v_i) = 0, \end{cases} \quad (2)$$

where k_i is a real number. VCG mechanisms are incentive compatible, and any two yield, up to a constant, the same expected transfers. In our setting, we can express this as follows:

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} \left[S_i(u) - r_i \frac{dE_{-i}\{v_1\}}{dv_i} \right] du \text{ for all } i. \quad (3)$$

Clearly, $U_i(v_i)$ is strictly convex in v_i ,⁷ so the first-order condition for (3),

$$S_i(v_i^*) = r_i \frac{dE_{-i}\{v_1\}}{dv_i}, \quad (4)$$

fully characterizes the worst-off types. In an *SP*, we have that $v_1^* = G_1^{-1}(r)$ and $v_i^* = \underline{v}$ for all $i \neq 1$. Thus, the worst-off type of active partner expects to be neither a buyer nor a seller under the mechanism, while the worst-off type of silent partner expects to sell with probability one.

We say that a partnership can be *dissolved efficiently* if there exists an ex post efficient mechanism $\langle s, t \rangle$ that is incentive compatible, interim individually rational and that satisfies

⁷It is clear from (1) that $S_i(v_i)$ is strictly increasing and has full range $[0, 1]$.

ex ante budget balance. The following proposition gives the condition governing whether an SP can be dissolved efficiently.⁸ See the appendix for the proof.

Proposition 1. *An $SP \langle n, r, F_1, F_2 \rangle$ can be dissolved efficiently if and only if*

$$\left\{ \int_{G_1^{-1}(r)}^{\bar{v}} [1 - F_1(u)] u dG_1(u) - \int_{\underline{v}}^{G_1^{-1}(r)} F_1(u) u dG_1(u) \right\} + (n-1) \int_{\underline{v}}^{\bar{v}} [1 - F_2(u)] u dG_2 \geq (1-r) \int_{\underline{v}}^{\bar{v}} u dF_1(u), \quad (5)$$

where $G_i(v_i) \equiv \prod_{j \neq i} F_j(v_i)$.

The left-hand side of the inequality gives the sum of the expected transfers to the worst-off types of partners. The term on the right-hand side is the sum of the expected profits that would accrue to the silent partners were the partnership to remain intact. Note that only the size, and not the distribution, of the $1-r$ share of the partnership among the silent partners matters for dissolvability. Since the worst-off type of silent partner sells his shares with certainty, this term equals exactly the minimum total compensation that the worst-off types of silent partners need to receive to participate in the mechanism.

In proving our results, we often use the following simplified version of condition (5). See the appendix for the proof.

Lemma 1. *An $SP \langle n, r, F_1, F_2 \rangle$ can be dissolved efficiently if and only if*

$$\int_{G_1^{-1}(r)}^{\bar{v}} [r - G_1(u)] du + \int_{\underline{v}}^{\bar{v}} [1 - r - (n-1)F_2(u)^{n-2}(1 - F_2(u))] F_1(u) du \geq 0. \quad (6)$$

In characterizing our results, we make frequent use of the following definition.

Definition 2. *Given two silent partnerships SP and SP^* , we say that SP^* is **easier to dissolve** than SP if and only if the left-hand side of (6) is larger for SP^* than for SP .*

Thus, if SP is dissolvable and SP^* is easier to dissolve, then SP^* must also be dissolvable. By the same token, if SP^* is easier to dissolve than SP but SP^* is not dissolvable without a positive outside subsidy, then SP requires at least the same outside subsidy to be dissolved.

⁸This is the same condition that would emerge if ex post budget balance were required. Using a technique similar to that in the “If” part of the proof of Lemma 4 from CGK (p. 628), it would be straightforward to specify the constant terms in (2) so that ex post budget balance is satisfied.

3. Characterization

3.1. Main Results

Under partnership structure $SP \langle n, r, F_1, F_2 \rangle$, the asymmetry of control affects crucially the possibility of constructing efficient dissolution mechanisms. Intuitively, this stems from the partners' asymmetric incentives. Generally, if the active partner's type is high (low), he will wish to buy out (sell to) the silent partners. A silent partner with a high type will also wish to buy out his partners. However, if his type is low, he may not wish to sell; he may prefer to continue to free-ride on the active partner by keeping the partnership intact.

When there are only two partners, this asymmetry renders efficient dissolution impossible regardless of the distribution of ownership shares. On the other hand, unless the active partner owns the entire firm ($r = 1$), efficiency becomes possible if the number of partners is sufficiently large. We present and discuss these results in turn.

Proposition 2. *An $SP \langle n = 2, r, F_1, F_2 \rangle$ cannot be dissolved efficiently.*

Proof. We will show that inequality (6) does not hold in this case. Note that in a SP with $n = 2$, $G_1(u) = F_2(u)$. We can then rewrite condition (6) as

$$\int_{F_2^{-1}(r)}^{\bar{v}} [r - F_2(u)] du + \int_{\underline{v}}^{\bar{v}} [F_2(u) - r] F_1(u) du \geq 0.$$

This inequality can be rearranged as

$$\int_{F_2^{-1}(r)}^{\bar{v}} [r - F_2(u)][1 - F_1(u)] du + \int_{\underline{v}}^{F_2^{-1}(r)} [F_2(u) - r] F_1(u) du \geq 0. \quad (7)$$

It is easy to see that both terms in the left-hand side of this inequality are non-positive and at least one must be strictly negative. Specifically, since $F_2(u) > r$ when $u > F_2^{-1}(r)$,

$$\int_{F_2^{-1}(r)}^{\bar{v}} [r - F_2(u)][1 - F_1(u)] du \leq 0,$$

where the inequality is strict if $r < 1$. Similarly, since $F_2(u) < r$ when $u < F_2^{-1}(r)$,

$$\int_{\underline{v}}^{F_2^{-1}(r)} F_1(u)[F_2(u) - r] du \leq 0,$$

where the inequality is strict if $r > 0$. Therefore, for any $r \in [0, 1]$, neither term in the left-hand side of inequality (7) is positive and at least one is strictly negative. It follows that (6) does not hold for $n=2$. **QED**

Intuitively, dissolution is impossible when $n = 2$ because the worst-off type of silent partner expects to sell to the active partner with certainty. He will wish to participate only if he expects to be paid a price (per share) at least as large as the active partner's (expected) type. But since it is impossible to get truthful revelation from the active partner without giving him some informational rent, a positive outside subsidy is necessary to make participation individually rational for all types.

An *SP* can, however, be dissolved efficiently when the number of partners is sufficiently large, provided that the active partner does not have full ownership.

Proposition 3. *An $SP \langle n, r, F_1, F_2 \rangle$ can be dissolved efficiently for a sufficiently large n if $r < 1$, but it cannot be dissolved efficiently for any n if $r = 1$.*

Proof. We first show that condition (6) can always be satisfied when n becomes arbitrarily large if $r < 1$. In that case, $G_1^{-1}(r) \equiv (F_2^{n-1})^{-1}(r)$ becomes arbitrarily close to \bar{v} , $G_1(u) \equiv F_2^{n-1}(u)$ becomes arbitrarily close to zero, and the first integral in (6) vanishes. Furthermore, since $\lim_{n \rightarrow \infty} (n-1)F_2(u)^{n-2}[1-F_2(u)]F_1(u) = 0$, the second integral in (6) specializes to $\int_{\underline{v}}^{\bar{v}} (1-r)F_1(u)du$. Thus, as $n \rightarrow \infty$ (and $r < 1$), condition (6) simplifies to

$$(1-r) \int_{\underline{v}}^{\bar{v}} F_1(u)du \geq 0,$$

which is satisfied for any distribution $F_1(u)$. When $r = 1$, however, condition (6) reduces to

$$-(n-1) \int_{\underline{v}}^{\bar{v}} F_1(u)F_2(u)^{n-2}[1-F_2(u)]du \geq 0, \quad (8)$$

and this inequality is not satisfied for any finite $n > 1$. Thus, if $r = 1$, an *SP* cannot be dissolved efficiently even if n is arbitrarily large. **QED**

The main difficulty in dissolving silent partnerships, relative to other cases analyzed in the literature, is that each silent partner requires a price per share that is at least as large as the active partner's (expected) type, $E_{-i}\{v_1\}$, to sell his shares. If the number of partners is small (as in Proposition 2), such compensation may not be feasible. However, as n increases, $E\{\tilde{v}\}$ approaches \bar{v} and it becomes eventually possible to compensate all partners.

Note also that, when $r = 1$, our setting becomes equivalent to a buyer(s)/seller setup with $n - 1$ buyers. Thus, dissolution is impossible in that case for precisely the same reasons as in Myerson and Satterthwaite (1983).⁹ Proposition 3 shows, however, that extreme ownership structures can be dissolved efficiently in the other extreme, when the partnership is owned entirely by a silent partner ($r = 0$), provided that n is large enough.

3.2. A Simple Mechanism

The best way to understand the intuition for propositions 2 and 3 is probably to consider an explicit mechanism. We consider a simple first-price auction with additional fixed payments and revenue sharing. For analytical simplicity, we let $F_2 = F_1$ in this subsection.

In this auction, each of the n partners places a bid (B_i) and the highest bidder, whose bid we denote by \tilde{B} , wins the firm. Thus, letting $1(\bullet)$ represent the indicator function, the partners' payoffs are:

$$\begin{aligned} \text{Active:} \quad & 1(B_1 = \tilde{B})(v_1 - B_1) + r\bar{v} + \frac{1}{n} \left[\tilde{B} - r\bar{v} - (1 - r) \int_{\underline{v}}^{\bar{v}} u dF_1(u) \right] \\ \text{Silent :} \quad & 1(B_i = \tilde{B})(v_i - B_i) + r_i \int_{\underline{v}}^{\bar{v}} u dF_1(u) + \frac{1}{n} \left[\tilde{B} - r\bar{v} - (1 - r) \int_{\underline{v}}^{\bar{v}} u dF_1(u) \right]. \end{aligned}$$

For each partner, the first term represents his net surplus from buying the firm if he wins. The second term is a fixed payment that is at least as large as his expected income were the partnership to stay intact. The third term is the product of $1/n$ and the revenue from the auction, net of the total fixed payments. Thus, the net revenue is returned to the partners in equal-share amounts. This ensures ex post budget balance, which guarantees ex ante budget balance. Note that the terms in brackets may be negative if \tilde{B} is small.

Individual rationality requires that each type of each partner expects the above payoff to exceed his payoff were the partnership to stay intact:

$$\begin{aligned} \text{Active:} \quad & E_{-1} \left\{ 1(B_1 = \tilde{B})(v_1 - B_1) + \frac{1}{n} \left[\tilde{B} - r\bar{v} - (1 - r) \int_{\underline{v}}^{\bar{v}} u dF_1(u) \right] \right\} \geq r(v_1 - \bar{v}) \\ \text{Silent :} \quad & E_{-i} \left\{ 1(B_i = \tilde{B})(v_i - B_i) + \frac{1}{n} \left[\tilde{B} - r\bar{v} - (1 - r) \int_{\underline{v}}^{\bar{v}} u dF_1(u) \right] \right\} \geq 0. \end{aligned}$$

Except for the fixed payments, which do not affect the bids (since they do not affect incentive compatibility), this auction is identical to the $k + 1$ -price auction studied by CGK (for $k = 0$)

⁹This is related also to the results of Gresik and Satterthwaite (1989), who extend the Myerson-Satterthwaite result to situations with larger numbers of buyers and sellers and identify the rate of convergence of these markets to efficiency as the markets grow large.

and is a special case of a class of auctions studied by Engers and McManus (2004) and Goeree et al. (2005). Bidders are ex ante identical and equilibrium bids are

$$B_i(v_i) = v_i - \int_{\underline{v}}^{v_i} \left(\frac{F_1(u)}{F_1(v_i)} \right)^n du$$

for both the active and the silent partners. Since this bidding function is increasing in v_i , the outcome of this auction is efficient. Note that when the partners bid according to this function, each partner's bid is no larger than his type and is strictly smaller if $v_i > \underline{v}$.

Partner i 's conditional expectation of the winning bid, $E_{-i}\{\tilde{B}|B_i(v_i)\}$, is increasing in $B_i(v_i)$ and hence v_i , so the type of bidder who expects the lowest \tilde{B} is the \underline{v} type, who knows with certainty that he will not win the auction. For this type of partner, the expected revenue is the expected highest bid among the remaining $n - 1$ partners, who bid as if there were n partners with a chance to win the firm.

For $n = 2$, the \underline{v} type of silent partner bids \underline{v} and expects the active partner to win for sure with bid B_1 , so his individual rationality constraint is

$$E_{-2}\{B_1\} - r\bar{v} - (1 - r) \int_{\underline{v}}^{\bar{v}} u dF_1(u) \geq 0. \quad (9)$$

Because $B_1(v_1) < v_1$ for all $v_i \in (\underline{v}, \bar{v}]$, it is easily seen that $E_{-2}\{B_1\} < \int_{\underline{v}}^{\bar{v}} u dF_1(u) \leq r\bar{v} + (1 - r) \int_{\underline{v}}^{\bar{v}} u dF_1(u)$, so that condition (9) is not satisfied. Thus, the silent partner is unwilling to participate in this auction.

For general n , however, since equilibrium bids are increasing in n , the \underline{v} type of silent partner expects revenue $E_{-i}\{\tilde{B}|B_i(\underline{v})\}$ to be at least as large as the ex ante expected revenue from an auction with $n - 1$ partners participating. Since the ex ante expected revenue from this auction is at least that of a standard independent private value auction (where no revenue is returned to the bidders), and the revenue from a standard IPV auction approaches \bar{v} as $n \rightarrow \infty$, we have that $\lim_{n \rightarrow \infty} E_{-i}\{\tilde{B}|B_i(\underline{v})\} = \bar{v}$.¹⁰ Thus, as long as $r < 1$, individual rationality is satisfied for sufficiently large n .

¹⁰Consider the class of auctions where, in addition to possibly winning the item, each bidder gets $\alpha\tilde{B}$ in utility from the auction. With respect to bidding functions and revenue, the case we look at here is in this class, with $\alpha = 1/n$. It is well known that the expected revenue from a standard IPV auction ($\alpha = 0$) is the expected second-highest order statistic. Goeree et al. (2005) additionally show that (1) the expected revenue is increasing in α , and (2) the expected revenue for $\alpha = 1$ is the first-highest order statistic. Thus, the revenue for $\alpha = 1/n$ is between the second- and first-highest order statistics. Since both order statistics converge to \bar{v} as $n \rightarrow \infty$, by the Squeeze Theorem, $\lim_{n \rightarrow \infty} E_{-i}\{\tilde{B}|B_i(\underline{v})\} = \bar{v}$.

Note that, when n increases, the additional “partners” need not own positive shares in the firm. In fact, they may simply represent additional bidders for the firm. Thus, the owners of a two-person partnership, for example, may overcome the impossibility result in Proposition 2 by simply allowing outsiders to buy the firm as well (provided that $r < 1$).

In the mechanism described in this subsection, dissolution is least demanding when $r = 0$ and becomes monotonically easier with n . However, such comparative statics results do not hold generally — note that we assumed $F_2 = F_1$ and that the mechanism is not optimal in the first place. In the next subsection, we show in particular that dissolvability is generally not easiest when $r = 0$.

3.3. Ownership Structure and Dissolution

Recall that, in the setting of CGK, the equal-shares partnership is the easiest to dissolve. Fieseler et al. (2003) show that the same is true in some cases where valuations are interdependent: the set of shares for which efficient dissolution is possible is symmetric around the equal-ownership distribution, although the set can also be empty if valuations are positively interdependent. The next proposition proves that the equal-shares case is not generally the easiest to dissolve in a silent partnership, although it is still true that the extreme-ownership settings ($r = 0$ and $r = 1$) are never the easiest to dissolve.¹¹

Proposition 4. *Let $\mu_1 \equiv \int_{\underline{v}}^{\bar{v}} u dF_1(u)$. Partnership $SP \langle n, r = G_1(\mu_1), F_1, F_2 \rangle$ is easier to dissolve than any other partnership $SP \langle n, r, F_1, F_2 \rangle$.*

Proof. It is sufficient to show that the left-hand side of condition (6) is maximized when $r = G_1(\mu_1)$. Differentiating that expression with respect to r , we find

$$\begin{aligned} \frac{dLHS(6)}{dr} &= \int_{G_1^{-1}(r)}^{\bar{v}} du + \frac{dG_1^{-1}(r)}{dr} [r - G_1(G_1^{-1}(r))] - \int_{\underline{v}}^{\bar{v}} F_1(u) du \\ &= \bar{v} - G_1^{-1}(r) - (\bar{v} - \mu_1) \\ &= \mu_1 - G_1^{-1}(r). \end{aligned}$$

Since this expression is decreasing in r ($dG_1^{-1}(r)/dr > 0$), the left-hand side of condition (6) is maximized when $\mu_1 = G_1^{-1}(r)$ —or equivalently, when $r = G_1(\mu_1)$. **QED**

¹¹Jehiel and Paudner (2006) show that, in a 2-person partnership where one partner is informed about the other partner’s valuation, extreme ownership settings can actually be the “easiest” to dissolve, in the sense of requiring the minimal subsidy to implement an efficient allocation.

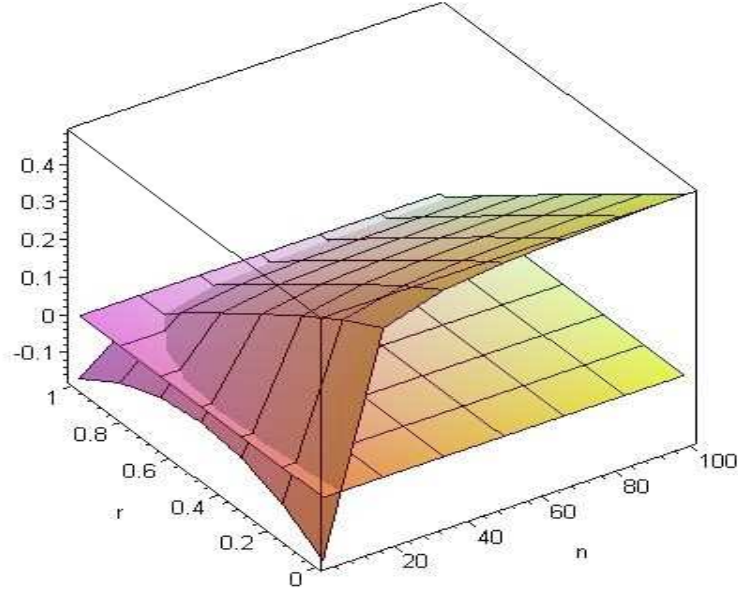


Figure 1: *Efficient Dissolution with Uniform Distributions.*

Ownership shares affect dissolution in two different ways in our setting. On the one hand, a greater r decreases the expected transfer to the worst-off type of active partner at a rate $G_1^{-1}(r)$, just as it would do under CGK's setting. The worst-off types of silent partners are, however, qualitatively distinct. Their expected transfers do not increase with r (i.e. as their $1 - r$ share decreases), as they would under CGK's setting. Rather, their participation costs fall with r , at a constant rate μ_1 . Proposition 4 shows that the conditions for efficient dissolution are most facilitated when these two rates are equalized.¹² Since $0 < G_1(\mu_1) < 1$ for any finite n , it is clear that the most extreme ownership structures are never the easiest to dissolve in this setting. However, the equal-shares partnership is generally not the easiest to dissolve either, as $G_1(\mu_1)$ depends on F_1 , F_2 and n .

To illustrate our main results, consider the case where all partners' types are *iid* uniform on $[0, 1]$. Condition (5) then reduces to

$$(n - 1) \left(\frac{1}{1 + n} - \frac{r^{\frac{n}{n-1}}}{n} \right) - \frac{1 - r}{2} \geq 0. \quad (10)$$

The curved surface in Figure 1 plots the left-hand side of the above expression, for $r \in [0, 1]$

¹²In CGK, the two correspondent rates (for the 2-player case) would be $G^{-1}(r)$ and $G^{-1}(1 - r)$, since they consider identical distributions. Thus, dissolution is "easiest" when $G^{-1}(r) = G^{-1}(1 - r)$, i.e. when $r = \frac{1}{2}$.

and $n = \{2, 3, \dots, 100\}$. The flat plane is at zero for all r and n . Hence, when the surface is above the plane, the inequality above is satisfied.

When $n = 2$, inequality (10) is not satisfied for any distribution of shares. When $n = 3$, all partnerships with $r \leq \frac{9}{16}$ are dissolvable, while all partnerships with $r > \frac{9}{16}$ are not. Note that the range of dissolvable partnerships is not symmetric about the equal-shares partnership. Hence, there are ownership structures where partnerships are dissolvable here but not dissolvable in the setting of CGK and vice versa. Most notably, the inequality above is always satisfied when $n > 2$ and $r = 0$, so this “extreme ownership” partnership is dissolvable unless $n = 2$.¹³ On the other hand, even though the upper bound of the range of dissolvable partnerships increases with n , it is not satisfied for any n when $r = 1$.

4. Conclusion

We have demonstrated that the asymmetry of control of a firm’s operations is a potential obstacle to the efficient dissolution of partnerships. Our results suggest that partnerships in which one partner effectively dominates the management of the firm will often encounter problems when they attempt to dissolve. This problem is most acute if there is only one other partner. However, firms can mitigate this problem if they are willing, during the dissolution process, to entertain bidding by outsiders.

Numerous papers in the property rights literature have addressed the determinants of control in organizations. Papers in the mechanism design literature have studied the forces shaping the efficient dissolution of partnerships. However, none of the contributions in each of these lines of research has analyzed the effects of the structure of control on the design of efficient dissolution mechanisms. This paper starts to bridge this gap. We explore in detail a form of partnership characterized by an extreme but common form of control structure.

Our framework can be extended to other partnership structures where the value of the asset is common under the partnership. Further applications will help us access in more detail how the allocation of control within organizations affects the prospects of efficient dissolution. In particular, it would be interesting to know how far the results obtained for silent partnerships extend to other asymmetric but less extreme control structures.

¹³By contrast, the partnership with ownership shares $\{r_1 = \frac{5}{8}, r_2 = \frac{3}{8}, r_3 = 0\}$, for example, is dissolvable under CGK but not here, since $\frac{5}{8} > \frac{9}{16}$.

Appendix

Proof of Proposition 1. Williams (1999, Theorem 3, p. 166) shows that an interim individually rational, ex ante budget balanced mechanism exists if and only if $(n - 1)$ times the expected total value under efficient implementation is no greater than the sum of expected net utilities of the worst-off types in a *basic Groves mechanism*, where $k_i = 0$ for all i for the transfers defined in (2). This yields the condition

$$(n - 1)E\{\tilde{v}\} \leq [v_1^*G_1(v_1^*) + E_{-1}\{\tilde{v}1(v_1^* < \tilde{v})\} - rv_1^*] \\ + [(n - 1)E_{-i \neq 1}\{\tilde{v}1(\underline{v} < \tilde{v})\} - (1 - r)E_{-i \neq 1}\{v_1\}],$$

where $1(\bullet)$ is the indicator function, the first bracket of terms on the right-hand side is $U_1(v_1^*)$ and the second is $\sum_{i \neq 1} U_i(\underline{v})$. From the definition of v_1^* we have that $G_1(v_1^*) = r$. Thus, the expression above can be rewritten as

$$(n - 1) \int_{\underline{v}}^{\bar{v}} ud[F_1(u)F_2(u)^{n-1}] \leq \int_{v_1^*}^{\bar{v}} udF_2(u)^{n-1} \\ + (n - 1) \int_{\underline{v}}^{\bar{v}} ud[F_1(u)F_2(u)^{n-2}] - (1 - r)E\{v_1\}.$$

After multiplying out the $d[F_1(u)F_2(u)^{n-1}]$ and $d[F_1(u)F_2(u)^{n-2}]$ terms, doing a bit of algebra and substituting back in for v_1^* , we have

$$\left\{ \int_{G_1^{-1}(r)}^{\bar{v}} [1 - F_1(u)]udG_1(u) - \int_{\underline{v}}^{G_1^{-1}(r)} F_1(u)udG_1(u) \right\} + (n - 1) \int_{\underline{v}}^{\bar{v}} [1 - F_2(u)]udG_2 \\ \geq (1 - r) \int_{\underline{v}}^{\bar{v}} udF_1(u), \quad (11)$$

which is condition (5). **QED**

Proof of Lemma 1. Rewrite condition (5) as

$$\int_{G_1^{-1}(r)}^{\bar{v}} udG_1(u) + \int_{\underline{v}}^{\bar{v}} u \{ (n - 1) [1 - F_2(u)] dG_2(u) - F_1(u)dG_1(u) - (1 - r)dF_1(u) \} \geq 0.$$

Integrating by parts each of the integrals above, we can rearrange the inequality as

$$\bar{v} - G_1^{-1}(r)r - \int_{G_1^{-1}(r)}^{\bar{v}} G_1(u)du + \int_{\underline{v}}^{\bar{v}} \{ 1 - r - (n - 1)F_2(u)^{n-2} [1 - F_2(u)] \} F_1(u)du - (1 - r)\bar{v} \geq 0,$$

which simplifies easily to yield condition (6). **QED**

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