Economics 4650 - John L. Turner

Homework 4 Answers.

This problem addresses some, but not all, of the issues covering incentive contracting. Notably, it does not address the risk/incentive tradeoff. In the interest of helping you understand this, I address what would happen if the agent were risk averse in this answer key.

a. The manager's expected benefit is 200Tβ, while his cost is T^2. Equating marginal benefit to marginal cost, we find:

$$T = 100\beta$$

gives the optimal number of transactions.

b. This question assumes that the principal's (Enran) expected payoff is the company profit, 200T, minus the wage paid to the division manager, $W_0 + \beta 200T$. By substituting $T = 100\beta$ in for T, we get:

$$\text{Expected Payoff} = 20{,}000(\beta - \beta^2) - W_0$$

c. It is easy to show that β=.5 maximizes the above expression. It does not maximize total value, which is:

$$200T - T^2,$$

or, substituting in β,

$$20{,}000\beta - 10{,}000\beta^2.$$

This is maximized by setting β=1. The intuition for why β=.5 is inefficient is that the agent is paying the full cost of effort, but is only receiving a β share of the profit. In order to induce the efficient effort β=1 is optimal.

In this problem, when the efficient level of incentives are provided, the principal's payoff is $-W_0$, so that in essence the agent is paying the principal for the right to be a division manager. This is a quirk of the problem that stems from the assumption of risk neutrality. So consider the case where the agent is risk averse, with risk tolerance r. Then the risk premium,

$$\frac{1}{2} r\beta^2 Var(\psi) = \frac{1}{2} r[C'(T)]^2 Var(\psi)$$

enters the total certain equivalent. To find the optimal T and β, we must write the risk premium with the substitution 200β = C'(T) = 2T. Then the total certain equivalent becomes:

$$200T - T^2 - \frac{1}{2} r (\frac{T}{100})^2 Var(\psi) .$$

Maximizing this with respect to T, we have

$$200 - 2T - \frac{rTVar(\psi)}{10{,}000} = 0$$

Then, substituting $T = 100\beta$ back into the expression, we have

$$200 - 200\beta - \frac{rVar(\psi)}{100} \beta = 0 .$$

This yields

$$\beta = \frac{200}{200 + \frac{rVar(\psi)}{100}} .$$

The right-hand side of this is clearly smaller than 1 whenever *rVar(ψ)* is positive, so if the agent is risk averse and there is some risk to being a division manager, then it is optimal to lower the incentives accordingly.

Compare this solution for the optimal β to the formula for the Incentive Intensity Principal from the book and the last day of class:

$$\beta = \frac{P'(e)}{1 + rC''(e)Var(x + \gamma y)}$$

If we think of effort as being represented by transactions, then we can relate these two expressions with a bit of work. It is clear that the analog of P'(e) here is 200, as the company's profit increases by 200 with each transaction. In the problem in the book, the agent was only rewarded \$β for each unit of effort. Here, by contrast, she gets \$200β. So the "1" in the denominator of the above expression is therefore replaced by a "200."

The final piece is the C"(e) term, and the \$200β compensation comes up again. Since, in equating marginal benefits to marginal cost, the agent sets C'(T) = 200β, we have that

$$\beta = \frac{C'(T)}{200}$$

The derivative of this term,

$$\frac{C''(T)}{200} = \frac{1}{100},$$

is what shows up in the expression

$$\beta = \frac{200}{200 + \frac{rVar(\psi)}{100}}.$$

d. Under risk neutrality, this is very easy. With β=.5, the agent optimally executes 50 transactions. She gets a fixed component of 1,000, a "transaction" component of .5(10,000) = 5,000, and suffers an effort cost of 50(50) = 2,500. So her expected payoff is 3,500, clearly better than the 2,500 outside option.

If she were risk averse, then her choice would factor in the risk premium,

$$\frac{1}{2} r\beta^2 Var(\psi) = \frac{1}{2} r(.25)(5{,}500^2) = 3{,}781{,}250r.$$

If this is smaller than 1,000, then the agent will take the job. This occurs whenever

$$r < .000264.$$