

Economics 4650 - John L. Turner

Homework 2 - Answers

1. a. Division I is upstream, because the "flow" of production is from I to II to consumers.

b. The optimal transfer price is given by the outside competitive price of \$1,000. Thus, given that Division II's MRP is $12,000 - y$, this means that they set:

$$1,000 = 12,000 - y$$

and buy $y = 11,000$ total CPUs. They could buy all of these on the open market, but it is cheaper to buy them from Division I up to the point where I's marginal cost is 1,000. This occurs when

$$1,000 = \frac{y_I}{5},$$

or when $y_I = 5,000$. Division II buys the remaining 6,000 CPUs on the outside competitive market.

2. a. Exchange is efficient because total value is larger under exchange ($15M > 10M$). If M is known, then bargaining will lead to exchange, with Friday paying Robinson between $10M$ and $15M$ for the goat. This is a simple application of the Coase Theorem.

b. Robinson will sell if

$$P \geq 10M .$$

Since $M \leq \frac{P}{10}$ whenever Robinson sells, the amount of milk Friday expects to get in any

exchange is uniformly distributed on $[0, \frac{P}{10}]$. The average is just the midpoint of this

interval, $M^* = \frac{P}{20}$.

c. Friday will wish to purchase the goat whenever

$$15M^* \geq P .$$

d. The market will not achieve the efficient outcome, because it is impossible for

$$15M^* \geq P .$$

To see this, plug in $M^* = \frac{P}{20}$, and get

$$15M^* = 15\left(\frac{P}{20}\right) = \frac{3}{4}P \geq P, \text{ impossible!}$$

If instead Friday's surplus is $25M - P$, efficient exchange will be possible, because

$$15M^* = 25\left(\frac{P}{20}\right) = \frac{5}{4}P \geq P.$$

3. Milgrom and Roberts mathematical exercises 3,4,5, p. 164.

***No. 3.** Suppose that Baker reports honestly. Then, when Able's value is actually 100, with probability r confessing that value will result in a payoff of $(100-20)$, which occurs when Baker reports a value of 70 and they share the cost, while with probability $(1-r)$ Able's payoff will be $(100-40)$, which results from Baker's reporting a value of zero and Able's financing the tree alone. Thus, Able's expected payoff when reporting honestly a value of 100 is

$$r(100 - 20) + (1 - r)(100 - 40).$$

If, instead, Able reports a value of zero, his expected payoff will be

$$r(100 - 0) + (1 - r)(0).$$

Thus, incentive compatibility (truth-telling by Able) requires that

$$r(100 - 20) + (1 - r)(100 - 40) \geq r(100 - 0).$$

Solving this yields that $r \leq \frac{3}{4}$.

Now assume that Able tells the truth. For Baker, the expected payoff to reporting truthfully (with a value of 70) is

$$p(70 - 20) + (1 - p)(70 - 40),$$

while the payoff to reporting a value of zero is

$$p(70 - 0) + (1 - p)(0),$$

so incentive compatibility (truth-telling by Baker) requires that

$$p(70 - 20) + (1 - p)(70 - 40) \geq p(70 - 0).$$

Solving this yields that $p \leq \frac{3}{5}$. (Note that there is no reason for either to claim a positive valuation when his or her actual value is zero. This can be easily verified).

Now, let q_A be the probability that the tree is planted when only Able offers to pay, and q_B be the probability that it is planted when only Baker offers to pay. Assume also that $p = .7$ and $r = .8$. Note first that, given the earlier part of the answer, p and r are sufficiently high that truthful reporting will not occur if $q_A = q_B = 1$. These values must be smaller than 1, i.e., some efficiency must be surrendered, in order to achieve truth-telling.

Now, Able's expected payoff from truthful reporting (when he has value 100) is

$$r(100 - 20) + (1 - r)[q_A(100 - 40) + (1 - q_A)(0)].$$

This must be compared to his expected payoff from claiming a value of zero:

$$r[q_B(100 - 0) + (1 - q_B)(0)].$$

Thus, incentive compatibility (truth-telling from Able) requires that

$$r(100 - 20) + (1 - r)[q_A(100 - 40) + (1 - q_A)(0)] \geq r[q_B(100 - 0)].$$

Using $r = .8$ and simplifying, we find

$$16 + 3q_A \geq 20q_B \tag{3.1}$$

Similarly for Baker, the incentive compatibility requirement is

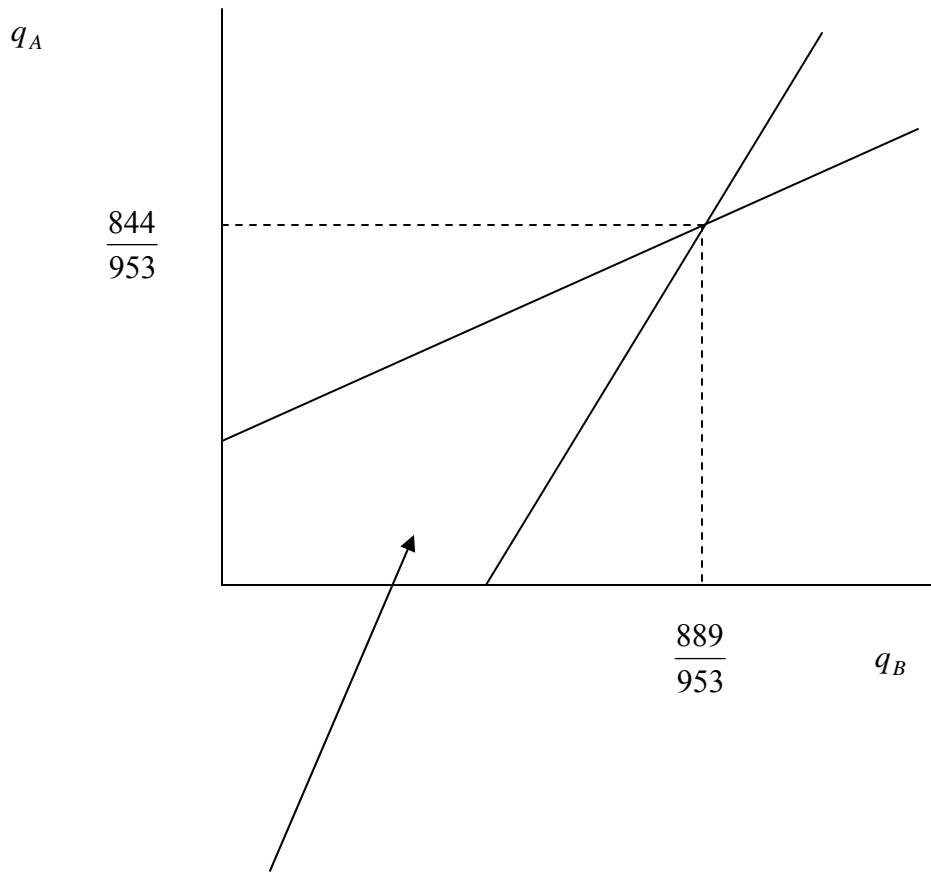
$$p(70 - 20) + (1 - p)[q_B(70 - 40) + (1 - q_B)(0)] \geq p[q_A(70 - 0) + (1 - q_A)(0)] + (1 - p)(0) \tag{3.2}$$

Using $p = .7$ and simplifying, we find:

$$35 + 9q_B \geq 49q_A.$$

Inequalities (3.1) and (3.2) together imply that $q_A \leq \frac{844}{953}$ and $q_B \leq \frac{889}{953}$.

The way to find them is to simply solve (3.1) and (3.2) for the case where they both hold with equality. There are two equations in two unknowns. Just solve for q_A and q_B . The reason this works is that, if you plot (3.1) and (3.2) on a graph, then identify all points consistent with both, you get the following:



Values of q_A and q_B where both Baker and Able tell the truth willingly.

***No. 4.** There are potential gains from trade whenever the car is worth more to the buyer than to the seller.

In this example, if we assume that the valuations are independent (as I wanted you to), there are potential gains from trade in every situation except the one where the car is worth \$750 to the seller and \$500 to the buyer. This situation occurs with probability x^2 , because the probabilities that the car is worth \$750 to the seller and \$500 to the buyer are each x . The probability that gains from trade exist is therefore $1 - x^2$, which is a decreasing function of x .

As in the text, if the buyer reports a low value and the seller a low one, then the price should be set as high as is consistent with the buyer's claimed valuation so that there is a minimal incentive for the buyer to understate the valuation. Similarly, if the seller reports a high value and the buyer a high one, then the price is set as low as possible to discourage the seller from overstating. This leads to a price of \$750 in the first instance and \$500 in the second.

Assuming these prices, the incentive compatibility constraints are the ones that follow the Table

		Buyer	
		\$500 (x)	\$1000 (1-x)
Seller	\$250 (1-x)	500	p
	\$500 (x)	no trade	750

Incentive compatibility for Seller \$250 and Buyer \$1,000 types:

Seller (250, assume buyer tells truth):

Gains from truth \geq Gains from Misrepresent implies

$$(500 - 250)x + (p - 250)(1 - x) \geq x(0) + (750 - 250)(1 - x),$$

which reduces to $p \geq \frac{750 - 1000x}{1 - x}$.

Buyer (1,000, assume seller tell's truth):

Gains from truth \geq Gains from Misrepresent implies

$$(1 - x)(1000 - p) + x(1000 - 750) \geq (1 - x)(1000 - 500) + x(0),$$

which reduces to $p \leq \frac{500 - 250x}{1 - x}$.

It is possible to find a price p to satisfy both constraints when the right-hand side of the second exceeds that of the first:

$$\frac{750 - 1000x}{1 - x} \leq \frac{500 - 250x}{1 - x}, \text{ which implies that } x \geq \frac{1}{3}.$$

If $x = \frac{1}{3}$, then p must be 625. In general, as long as $x \geq \frac{1}{3}$, it is possible to support efficient trade by setting p in the specified range. The buyer and seller avoid misrepresenting their values because, with x relatively large, exaggeration, is too likely to result in no trade occurring. As x falls, the probability that trade is efficient ($1 - x^2$) rises. Eventually, when $x < \frac{1}{3}$, if the other party is going to be honest, then it pays to exaggerate, because it is now relatively unlikely to prevent trade. So, at least one party must find it attractive to exaggerate.

Note that if the valuations are assumed to be perfectly correlated (rather than independent), then both parties are effectively fully informed about the actual valuations. The trade should occur when the buyer's value is \$1000 and the seller's is \$250 - which

now occurs with probability $(1-x)$ - and not in the only other possible circumstance, when the valuations are \$500 for the buyer and \$750 for the seller. There are no incentive problems here.

***No. 5.** In the first case, neither the seller nor the buyer finds it in their individual interests to refuse to trade, because neither loses money when the price is \$10. Honest reporting by the seller is incentive compatible in this case, for the following reason. The seller's payoff from reporting honestly is \$10 if his value is \$0 and it is \$0 if his value is \$10. His payoff from reporting dishonestly is identical, because his report does not affect the terms of the transaction. So, the incentive compatibility conditions reduce to: $10 \geq 10$ and $0 \geq 0$, which are plainly satisfied.

In the second case, since the price depends only on the seller's report, the seller will always find it optimal to report the value that results in the highest price. So, for truthful reporting to always be incentive compatible, the price p must be the same regardless of the seller's report. To see this formally, write the incentive constraints for the seller, allowing different prices, q_0 and q_{10} :

$$\text{value} = 0: \quad q_0 - 0 \geq q_{10} - 0$$

$$\text{value} = 10: \quad q_{10} - 10 \geq q_0 - 10.$$

These imply $q_0 = q_{10}$. In order for trade to be worthwhile for the seller when his value is \$10, it must be that the common price satisfies $p \geq 10$ (this is the "participation" or "individual rationality" constraint). In order for trade to be worthwhile for the buyer, the good must be worth more than p to the buyer on average, that is, that the expected value of the good exceeds the price (this is the buyer's "participation" constraint). If r is the probability that the good is worth \$10 to the seller, then it must be that

$$r15 + (1 - r)5 \geq p \geq 10.$$

Such a price is possible if and only if $r \leq \frac{1}{2}$.

The problem is that if r is too low, then the buyer's participation constraint is inconsistent with the seller's.

4. a. No wealth effects

b. Here is who is the efficient fundraiser depending on hearts being "in it" or "not in it."

Amy's Heart

		In	Not In
Beatrice's Heart	In	Amy	Beatrice
	Not In	Amy	Amy

This is simple, because the total effort cost is always \$100, so efficiency requires picking the person who raises the most money.

c. Throughout this part, assume that Beatrice is telling the truth. Consider first the case where Amy's heart is "in it." If she reports truthfully, then she will be named fundraiser, and her payoff is

$$I + \alpha(20,000) - 100.$$

If she misrepresents, then her payoff is

$$.5(I + \alpha(15,000)) + .5(I + \alpha(20,000) - 100)$$

Thus, she tells the truth if

$$I + \alpha(20,000) - 100 \geq .5(I + \alpha(15,000)) + .5(I + \alpha(20,000) - 100),$$

which simplifies to yield

$$\alpha \geq \frac{50}{2,500}$$

Consider next the case where her heart is "not in it." Then, telling the truth yields a higher payoff if

$$.5(I + \alpha(15,000)) + .5(I + \alpha(10,000) - 100) \geq I + \alpha(10,000) - 100,$$

which simplifies to yield

$$\alpha \geq -\frac{50}{2,500}.$$

Thus, both constraints are satisfied (she always reports truthfully) if

$$\alpha \geq \frac{50}{2,500}$$

d. Now, assume that Amy tells the truth, and let's examine Beatrice's incentive compatibility constraints. If her heart is "in it," then the payoff from telling the truth exceeds the payoff from misrepresenting if

$$.5(I + \alpha(20,000)) + .5(I + \alpha(15,000) - 100) \geq .5(I + \alpha(20,000)) + .5(I + \alpha(15,000)),$$

which simplifies to

$$\alpha \geq \frac{50}{2,500}$$

If her heart is "not in it," then then the payoff from telling the truth exceeds the payoff from misrepresenting if

$$.5(I + \alpha(20,000)) + .5(I + \alpha(10,000)) \geq .5(I + \alpha(5,000) - 100) + .5(I + \alpha(20,000)),$$

which simplifies to yield

$$\alpha \geq -\frac{50}{2,500}.$$

Thus, both constraints are satisfied (she always reports truthfully) if

$$\alpha \geq \frac{50}{2,500}.$$

e. In this part of the problem, you should assume that neither Amy nor Beatrice knows the state of her heart until after the organization is formed (but before the choice of fundraiser is made). Next, assume that Amy and Beatrice tell the truth about their hearts if they form the organization. Then whenever Amy's heart is in it, \$20,000 is raised with Amy as fundraiser. This occurs with probability .5. If Amy's heart is not in it and Beatrice's heart is in it, then \$15,000 is raised with Beatrice as fundraiser. This occurs with probability .25. If neither person's heart is in it, then \$10,000 is raised with Amy as fundraiser. This occurs with probability .25.

The total expected utility from forming the organization is

$$.5(2I + 2\alpha(20,000) - 100) + .25(2I + 2\alpha(15,000) - 100) + .25(2I + 2\alpha(10,000) - 100),$$

which simplifies to

$$2I + 2\alpha(16,250) - 100$$

The total expected utility from not forming the organization is $2I$. Thus, it is optimal to form the organization if

$$2I + 2\alpha(16,250) - 100 \geq 2I,$$

which simplifies to

$$\alpha \geq \frac{100}{32,500}$$

Thus, unless α is very small, so that Amy and Beatrice derive very little benefit from the money they donate to the victims of Katrina, then it is optimal to form the organization.

Since it must be true that

$$\alpha \geq \frac{50}{2,500}$$

in order for truth-telling to be optimal, and

$$\frac{50}{2,500} \geq \frac{100}{32,500},$$

then it is clear that whenever Amy and Beatrice would willingly reveal the states of their hearts truthfully (if the organization were to be formed), they will in fact choose to form the organization.

