Economics 4650 - John L. Turner

Homework 1 Answers

1.a. Milgrom and Roberts Mathematical Problem 1., p. 53 Given the data in the problem, the total benefit enjoyed by all the parties in the problem is

$$21y - 2\frac{1}{2}y^2$$

Since the cost is *y*, the net benefit, or "total value," is

$$21y - 2\frac{1}{2}y^2 - y = 20y - 2\frac{1}{2}y^2$$

This is maximized at a point where the derivative is zero

$$20 - 5y = 0,$$

or $y = 4$. The total value or "net benefit" at this point is 40. You could arrive at this answer, with a bit more work, by trial-and-error (i.e try y = 1,2,3,4,..... and see which makes the total value the largest).

1.b. Milgrom and Roberts Mathematical Problem 2, p. 53. Assuming each family contributes 1/4 of the total cost *y*, the only acceptable levels of *y* are those at which the value (benefit minus cost) to the family is at least zero. At higher levels, they would prefer not to take part in the project. Mathematically,

$$\textit{Families}\,1\,\textit{and}\;2: 4\frac{3}{4}y - \frac{1}{2}y^2 \geq 0 \text{ or } y \leq 9\frac{1}{2}$$

$$\textit{Family}\;3: 6\frac{3}{4}y - \frac{1}{2}y^2 \geq 0 \text{ or } y \leq 13\frac{1}{2}$$

$$\textit{Family}\;4: 3\frac{3}{4}y - y^2 \geq 0 \text{ or } y \leq 3\frac{3}{4}$$

Family 4 is thus unwilling to contribute its 1/4 share to support the efficient level of $y = 4$. The most that all families would agree to with equal cost is $y = 3\frac{3}{4}$. At that level, the families net benefits are:

$$Families\,1\,and\;2: 4\frac{3}{4}\left(3\frac{3}{4}\right) - \frac{1}{2}\left(3\frac{3}{4}\right)^2 = 10.78$$

$$Family\;3: 6\frac{3}{4}\left(3\frac{3}{4}\right) - \frac{1}{2}\left(3\frac{3}{4}\right)^2 = 18.28$$

$$Family\;4: 3\frac{3}{4}\left(3\frac{3}{4}\right) - \left(3\frac{3}{4}\right)^2 = 0$$

$$Total,\,4\;Families = 39.84$$

It is possible to add (40-39.84)/4 = .04 to the welfare of each family by increasing the choice of y to 4 and arranging payments as follows: Families 1 and 2 each pay 1.18, family 3 pays 1.68, and family 4 receives .04. The resulting family payoffs are 10.82 for families and 2, 18.32 for family 3, and .04 for family 4. Other schemes will clearly do.

2. a. F = 2 for each person. Friday thus spends 2 hours catching fish, and 10 hours making 5 units of C. Robinson spends 6 hours catching fish, and has 6 hours left to make 2 units of C.

b. The efficient production plan involves Friday producing all fish, and Robinson specializing in clothes-making.

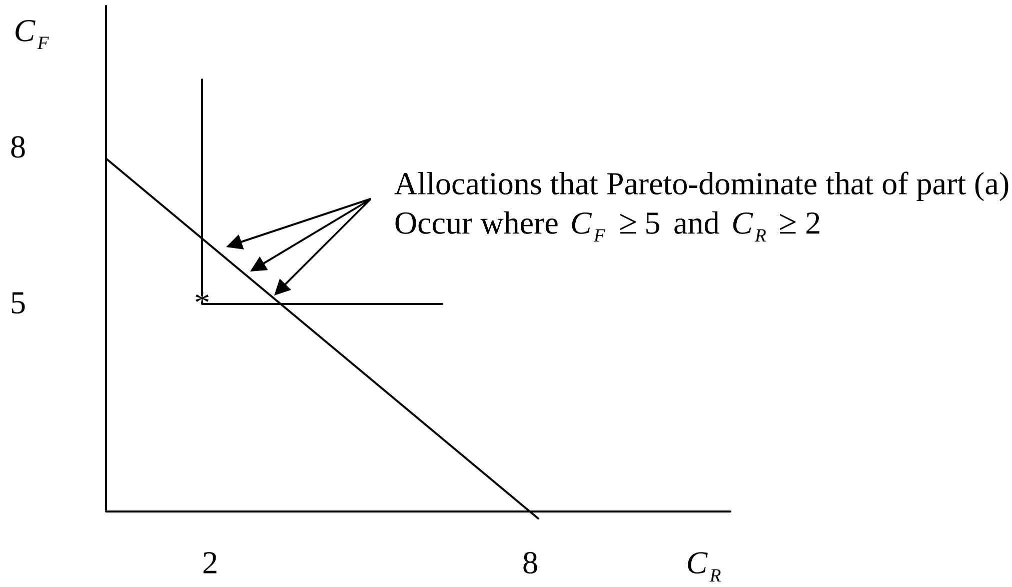
$$Friday: F = 4, C = 4$$

$$Robinson: F = 0, C = 4$$

This production plan yields 8 units of C, so the possible allocations of clothing to Friday, C_F, and Robinson C_R, must simply satisfy the following equation:

$$C_F + C_R = 8$$

This is plotted on the graph below, along with the allocation of part (a)



3. a. The bees are a positive externality, because Farmer Smith benefits from bees whenever they are present, even if Farmer Jones does not factor that in to his decision-making.

b. If Farmer Jones makes artificial honey, he earns a profit of

$$10(70) - 10(24 + 20) = 700 - 440 = 260.$$

If instead he uses bees, he earns a profit of

$$10(70) - 10(10 + 36) = 700 - 460 = 240.$$

Thus, it is in Jones' best interest to use artificial honey. In that case, Smith must hire 16 hours of labor, so his profit is

$$20(10) - 16(12) = 200 - 192 = 8.$$

c. This is clearly not the efficient production plan. Noting that total profit making artificial honey (part (b)) is 260 + 8 = 268, we then calculate total profit when bees are used. Obviously, Jones earns 240. With 5,000 bees present, Smith saves 8 hours of labor, earning a profit of

$$20(10) - 8(12) = 200 - 96 = 104,$$

So total profit is 240 + 104 = 344 > 268. Since total profit is larger with bees, this is the efficient production plan. It will be implemented by a bargain between Smith and Jones.

Since Jones loses 20 in profit directly when he uses bees, Smith must pay him at least 20 in this bargain. Since Smith earns an extra 104 - 8 = 96 in profit when Jones uses bees, he is willing to pay up to 96. Thus, the bargain involves Smith paying Jones between 20 and 96 to use bees.

d. This is simple value maximization stuff, but requires a solid understanding of that concept and of the Coase Theorem. If the price of honey, P_H, falls far enough, it is no longer efficient to produce honey. This occurs when the total profit from producing honey with bees and producing apples is less than the total profit of just producing apples (without bees), that is,

$$10P_H - 460 + 104 < 8.$$

This occurs for $P_H < 36.40$.

If the price of apples, P_A, falls far enough, it is no-longer efficient to produce apples, even with bees present, so apples should be abandoned ***and*** Jones should switch to making artificial honey. This occurs when the total profit of producing honey with bees and producing apples is less than the profit of just producing artificial honey, that is

$$240 + 10P_A - 96 < 260.$$

This occurs for $P_A < 11.60$.

Afterthought to Problem 3: This is a classic Coase Theorem problem, based on the "Fable of the Bees." James Meade (1952, *Economic Journal*) argued that, because of the positive externality existing between beekeepers and apple orchards, the government should subsidize beekeeping. Years later, Cheung (1973, *Journal of Law and Economics*), working at the University of Washington, studied apple orchards and beekeepers in Washington State. He identified well-developed markets, in which beekeepers and apple growers regularly bargained over honey production, “internalizing” the externalities Meade identified, so that no market failure occurred. Despite this, the US government ran a "US Honey Program," subsidizing beekeeping, for more than 50 years. The wastefulness of this program is discussed by Muth, Rucker, Thurman and Chuang (2004, *Journal of Law and Economics*).

4. a. This is a standard "part a" of which I've spoken repeatedly. Here, the key is to just find the value-maximizing plan. Profit without the promotion is 40,000, while profit with the promotion is 60,000, so doing the promotion is efficient. The part of the question about the contract is a red herring. While the "presence of a contract" may affect whether the promotion gets done, it does not affect the fact that doing the promotion ***is*** efficient.

b. Here, the 40 Watt doesn't know how much it is going to be "held up" by Britney and Kevin. It needs to compare expected profits (we are implicitly assuming Britney and

Kevin's costs are the same regardless of whether the 40 Watt does promotion). If it does not do the promotion, then 40 Watt profits are:

$$\text{Britney and Kevin have a "bad" week:} \quad .1(20{,}000) + 20{,}000 = 22{,}000$$
$$\text{Britney and Kevin have a "good" week:} \quad .5(20{,}000) + 20{,}000 = 30{,}000$$

Thus, with a 50/50 chance of Britney/Kevin having a "good"/"bad" week, the 40 Watt expects .5(22,000) + .5(30,000) = 26,000 in profit.

$$\text{Britney and Kevin have a "bad" week:} \quad .1(50{,}000) + 20{,}000 - 10{,}000 = 15{,}000$$
$$\text{Britney and Kevin have a "good" week:} \quad .5(50{,}000) + 20{,}000 - 10{,}000 = 35{,}000$$

Thus, with a 50/50 chance of Britney/Kevin having a "good"/"bad" week, the 40 Watt expects .5*(15,000) + .5(35,000) = $25,000 in profit. Thus, the 40 Watt will not do advance promotion.

When the probability of a "bad" week is 20%, the 40 Watt does the promotion (expected profit with promotion = 31,000 > 28,400 = expected profit w/o promotion). When the probability is 90%, it does not (expected profit with promotion = 17,000 < 22,800 = expected profit w/o promotion). If the 40 Watt knew in advance whether the week would be "good" or "bad," it would do promotion if it was going to be "good" (b/c 35,000 > 30,000) and not do promotion if it was going to be "bad" (b/c 22,000 > 15,000).

Afterthought on Problem 4: This is a hold-up problem. The 40 Watt is right to be nervous about doing advance promotion with no contract. Whether Britney/Kevin have a "bad" week does not affect the efficient production plan, but may affect whether investment is efficient in equilibrium.