

Problem Set #9 Solutions  
Econ 2106H, J.L. Turner

1)

		<b>Tim</b>	
		Confess	Deny
<b>Paul</b>	Confess	3,3	0,4
	Deny	4,0	2,2

The shaded area is the Nash Equilibrium.

2)

		<b>B</b>		
		3	4	6
<b>A</b>	3	18,18	15,20	9,18
	4	20,15	16,16	8,12
	6	18,9	12,8	0,0

The shaded area is the Cournot-Nash equilibrium.

3) a) **monopoly situation**

i) Since the firm is a monopolist, it sets  $MR=MC$

$$53-8Q=5$$

$$48=8Q$$

$$Q=6$$

Substituting this value into the demand equation yields:

$$P=53-4(6)$$

$$=53-24$$

$$=29$$

ii) Profit =  $TR-TC$ . Total revenue is  $P*Q=29*6$ . This is equal to 174.

Total Cost is  $5Q=5(6)=30$ . So, profit is  $174-30=144$ .

iii) The consumer surplus is the area below the demand curve and above the price paid by consumers.

This area is  $.5(53-29)(6)=72$ .

b) **duopoly situation**

i) and ii) A duopolist sets its own  $MR$  equal to its own  $MC$ . Doing this for both firms, and solving for each firm's  $Q$  yields each firm's reaction curve.

$$\text{Firm 1: } MR = 53-8Q_1-4Q_2 ; MC=5$$

$$53-8Q_1-4Q_2 = 5$$

$$* \quad Q_1 = 6 - (1/2)Q_2$$

$$\text{Firm 2: } MR = 53-8Q_2-4Q_1 ; MC=5$$

$$53-8Q_2-4Q_1 = 5$$

$$** \quad Q_2 = 6 - (1/2)Q_1$$

Equations \*, and \*\* are the reaction curves. Substituting  $Q_2$  into the equation for  $Q_1$  above, yields

$$Q_1 = 6 - (1/2)(6 - (1/2)Q_1)$$

$$= 6 - 3 + (1/4)Q_1$$

$$(3/4)Q_1 = 3$$

$$Q_1 = 4$$

This will be the Cournot output level for firm 1. Putting this quantity into firm 2's reaction function yields:

$$Q_2 = 6 - (1/2)(4) = 4$$

This will be the Cournot output level for firm 2. (Notice this is a symmetric duopoly, they both produce the same output).

So, the total output in the industry will be  $4 + 4 = 8$ .

$$\text{This price is } P = 53 - 4Q$$

$$= 53 - 4(8)$$

$$= 21$$

iii) Profit = TR - TC. For each firm,  $TC = 5Q$ , which becomes  $TC = 5(4) = 20$

Total Revenue is  $PQ$ , which is  $21(4) = 84$ .

So, Each firm's individual profit is  $84 - 20 = 64$ .

(Note that the profit of the two firms together is less than that of a monopoly).

iv) The consumer surplus is the area below the demand curve and above the price paid by consumers.

This area is  $.5(53 - 21)(8)$

### c) competition

i) The supply curve is simply the marginal cost curve, which is equal to 5.

ii) The market equilibrium is found by equating Marginal cost with demand, as this is a competitive market. Doing this yields:

$$53 - 4Q = 5$$

$$48 = 4Q$$

$$Q = 12$$

which is the equilibrium quantity. The equilibrium price can be found by substituting the equilibrium price into the demand equation. Doing this yields:

$$P = 53 - 4(12)$$

$$= 53 - 48$$

$$= 5$$

which is the equilibrium price.

iii) the consumer surplus is:

$$CS = (1/2)(53 - 5)(12) = 288$$

d)

Structure	Consumer Surplus
Monopoly	72
Duopoly	128
Competition	288

4)

### a) Monopoly Case

i) A monopolist sets  $MR = MC$ , doing this yields:

$$Q = 24 - 2Q$$

$$3Q = 24$$

$$Q^* = 8$$

The price at this level of output can be found by placing this quantity in the demand function.

$$P=24-(8)$$

$$P^*=16$$

ii) The Profit =TR-TC, evaluated at the equilibrium quantities.

$$TR=P^*Q^*=16*8=128$$

$$TC=(1/2)(8)^2=32$$

$$\text{Profit}=128-32=96$$

iii) Consumer surplus is the area below the demand curve and above the price charged by the monopolist.

$$CS=(1/2)(24-16)(8)=32$$

## b) Duopoly Case

i) A duopolist sets its own MR equal to its own MC. Doing this for both firms, and solving for each firms' Q, yields the reaction curves.

$$\text{Firm 1: } MR=24-2Q_1-Q_2 ; MC=Q_1$$

$$24-2Q_1-Q_2 = Q_1$$

$$* \quad Q_1 = 8-(1/3)Q_2$$

$$\text{Firm 2: } MR=24-2Q_2-Q_1 ; MC=Q_2$$

$$MR=24-2Q_1-Q_2 =Q_2$$

$$** \quad Q_2 = 8-(1/3)Q_1$$

\* and \*\* are the reaction functions for firm 1 and firm 2, respectively.

ii) To solve for the Cournot quantities, substitute \*\* into \* to get the expression:

$$Q_1 = 8-(1/3)[8-(1/3)Q_1]$$

$$Q_1 = 8-(8/3)+(1/9)Q_1$$

$$(8/9)Q_1 = (16/3)$$

$$Q_1 = (9/8)(16/3)$$

$$= 6$$

substituting this quantity into equation \*\* yields...

$$Q_2 = 8-(1/3)(6)$$

$$Q_2 = 6$$

Since these are the only two producers in the market,  $Q_1+Q_2=12$  is the market quantity. Substituting this number into the demand curve yields the equilibrium price  $P = 24-12 = 12$

iii) Profit=TR-TC (for each firm)

$$TR = P^*Q_i = (12)(6)=72$$

$$TC = (1/2)(6)^2 = 18$$

$$\text{Profit} = 72-18 = 54$$

this is the profit for each firm in the duopoly

iv) As before, Consumer Surplus is the area below the demand curve and above the price charged.

$$CS=(1/2)(24-12)(12)$$

$$= 72$$

## c) Competitive case



e)

Regime	Consumer Surplus	Total Profit	Total Welfare
Monopoly	32	96	128
Duopoly	72	108	180
Competition	200	40	240