A Spatial Model With Discrete Policy Choices That May Not Match

Gregory A. Trandel*
University of Georgia†

December, 2006

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*I thank Todd Nesbit for helpful comments on an earlier version of this paper. I bear full responsibility for any errors contained in it.

*Mailing address: Greg Trandel, Department of Economics, Terry College of Business, University of Georgia, Athens, GA 30602-6254. e-mail: trandel@terry.uga.edu. web-page: www.terry.uga.edu/~gtrandel/

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I. Introduction

In 2006, a store located in South Carolina advertised its products on a billboard in Athens, Georgia. Athens is roughly fifty miles from the Georgia-South Carolina border. The advertising store sold fireworks; its goal was to attract customers to buy types of fireworks that are legally sold in South Carolina, but are not officially available in Georgia.

In addition to Georgia and South Carolina, there are many other instances where U.S. states sharing a border impose very different regulations on the sale of fireworks — examples include Iowa-Missouri, Kentucky-Tennessee, Arizona-New Mexico, Ohio-Pennsylvania, New Jersey-Pennsylvania, and Connecticut-New York.[1] In many cases, the differing state policies — including those between states that don't appear to have widely dissimilar political environments — have been in existence for many years.

It is possible, of course, that the best explanation for such state-to-state policy variation is that the voters or office holders in the relevant states simply have differing preferences. This short paper instead uses a spatial model to investigate conditions under which jurisdictions with identical preferences and identical ex ante opportunities may make differing policy choices.[2]

Various authors have already used theoretical spatial models to consider why the policy choices (concerning, say, tax rates) of neighboring jurisdictions might differ. Abstracting away from possible differences in preferences or political considerations, these authors — examples include Kanbur and Keen (1993) and Nielsen (2001) — have explained varying policies by looking at the incentives that result, for instance, from jurisdictions having differing sizes or population densities.[3] If the jurisdictions in these models have matching parameter values, the analysis predicts that the policy choices of the jurisdictions will match.

There is also an empirical literature that shows, controlling for other influences, that differences in policy choices tend to lessen over time. Case, Rosen, and Hines (1993), for

[1]See www.fireworksafety.com/laws.htm, at the National Council of Fireworks Safety website.

[2]Spatial models capture situations in which consumers must physically travel to the location of a seller in order to purchase a product.

[3]Empirical analyses of these issues are conducted by Hansen and Kessler (2001).

example, demonstrate that U.S. state expenditure policies positively affect the policies of "neighboring" states. In the case of discrete policy choice — the focus of this paper — several authors have shown that states are more likely to adopt a lottery if neighboring states have already done so; Alm, McKee, and Skidmore (1993) and Caudill, Ford, Mixon, and Peng (1995) are examples.[4]

This paper differs from the theoretical spatial models described above in two major ways. First, the model doesn't consider governments choosing a continuous variable like a tax rate. Rather, it analyzes governments that make a discrete choice about whether or not to legalize a certain activity (like the sale of fireworks) that has a negative externality associated with it. More specifically, this paper considers a case in which the externality is large enough that the net welfare effect of use by a jurisdiction's own residents is negative. In spite of this harmful impact, though, a welfare-maximizing government might legalize if it has the potential to collect tax revenue (and, in a model with market power, if its firms can earn profits) from border-crossing residents of another jurisdiction, while not experiencing the (full) externality created by their consumption. That externality is instead felt by the home jurisdiction of the border crossers.

Second, the paper considers jurisdictions with identical ex ante characteristics, but allows one jurisdiction to — for whatever reason — make its policy choice first.[5] In the model detailed below, this first-mover status affects ultimate outcomes only if firms possessing market power arrange themselves (with some permanence) in a manner that depends on early policy choices. Firm locations can be expected to have some degree of permanence if a seller (which has fixed costs) locates adjacent to a border that separates a legalizing from a non-legalizing jurisdiction. Even if the non-legalizing government later switches its policy, no seller of similar size may want to locate just inside that jurisdiction.

In a model with a negative externality, a first mover, and some permanence of location choice, otherwise-identical neighboring jurisdictions can make differing policy choices that persist — some jurisdictions legalize, while others don't.

In addition to the firweworks example presented above, this model may be relevant for activities such as casino gambling, as long as *(i)* some of the pathologies associated with gambling occur where the participant lives rather than where he or she gambles, and *(ii)* a jurisdiction that becomes a gambling center before its neighbors is likely to remain a particularly prominent tourist attraction.

II. Model With Competitive Firms

Consumers are arranged with uniform density equal to unity along (following Braid, 1987) a line of infinite length. That line is divided into jurisdictions, all of which are of the same size. Each jurisdiction decides whether or not to allow sale of a certain good. If legalized, sales of that good within a jurisdiction are subject to the same sales tax t that is imposed on all other transactions. Since the tax isn't unique to the studied good, its level is assumed exogenous — and identical in all jurisdictions — throughout this paper.

[4]See Coughlin, Garrett, and Hernández-Murillo (2006) for a survey of the literature on lottery adoption.

[5]Wang (1999) considers a model in which a Stackelberg-leader jurisdiction is the first of two jurisdictions — with differing population densities — to choose its tax policy.

Every consumer is interested in buying only a single unit of the good, and will do so if the effective price (monetary price plus travel expense) to him or her is less than some exogenous maximum value v, which is the same for all consumers.

The use of a good creates a uniform negative externality e; this externality affects the jurisdiction in which the consumer resides, not the one in which he or she buys the good. In the fireworks illustration noted above, the externality can capture either the annoyance to, or the possible endangerment of, neighbors, or the extent to which a jurisdiction's taxpayers are responsible for some of the medical expenses that result from use of the product.[6]

In this section of the paper, it is assumed that consumers can buy the good (or participate in the activity) at any location along the line at which such a transaction is legal. [By assumption, there is no fixed cost associated with supplying the product.] It is also assumed that any quantity of customers can be accommodated at any point along the line. In combination, these two assumptions imply that the good will always be sold at its constant and uniform marginal cost c. It is assumed that consumers value for the good exceeds its tax-inclusive price: $v > c + t$.

If any jurisdiction doesn't legalize selling the good, people interested in obtaining it must travel to the nearest location (i.e., just across the border) in a neighboring jurisdiction that has legalized. Travelling to (and back from) the purchasing location requires any buyer to pay a travel cost equal to k per unit distance.

When a jurisdiction decides whether or not to legalize a good, the potential net benefit from consumption by one of its residents is $v - c - e$ (where surplus and tax revenue are assumed to be valued equally, so the presence of the tax has no effect on welfare). Since this paper considers situations in which a welfare-maximizing government might or might not legalize, it is assumed that $v - c - e < 0$. If borders are closed (so that legalization means selling only to a jurisdiction's own residents (and selling to all of them)), therefore, a welfare-maximizing government will never choose to legalize.

When borders can be crossed, however, a jurisdiction may gain tax revenue — and not experience any externality cost — from sales made to residents of a neighboring non-legalizing jurisdiction. Residents of such a location will travel into a legalizing jurisdiction as long as the private benefit from consumption exceeds the tax-inclusive, travel-inclusive cost of the good.

Consider a jurisdiction A that considers legalizing when its neighbors haven't so done. Although all jurisdictions are in ex ante identical situations, political and bureaucratic characteristics that differ among the jurisdictions may allow one to act before others. Suppose that this jurisdiction's borders are 0 and some positive n (so that its population is n). At the 0 border, some consumers located to the left of the 0 point will travel into the jurisdiction to purchase the good; the last consumer willing to do this is the one located at $\bar{b} < 0$, which is defined by $v - c - t - |\bar{b}|k = 0$. Therefore, $|\bar{b}| = (v - c - t)/k$. The legalizing jurisdiction will gain tax revenue from border-crossing consumers, but will not feel the externality (nor the consumer value) associated with the usage of the good. A corresponding formula applies at the n border. When neither neighboring jurisdiction has legalized, therefore, choosing to legalize brings in $2t|\bar{b}|$ in tax revenue from non-residents.

[6]The possibility of any medical costs experienced by the consumer himself are assumed to be imbedded into the v term.

Depending on the relative values of the various parameters, a jurisdiction that would not legalize in the absence of border crossers may do so in their presence. Specifically, this will occur if

$$n(v - c - e) + 2t|\bar{b}| > 0. \tag{1}$$

Consider now the case of a jurisdiction that hasn't legalized, but has seen its two neighbors so do. Such a jurisdiction has some of its residents travelling across each border to buy the good. The jurisdiction's welfare is the sum of three elements. First, ignoring their cost of travelling, the $2|\bar{b}|$ people who buy across a border pay a monetary price of $c + t$; each therefore experiences $v - c - t$ personal net benefit. The cost of travel for a border-crosser is $b^{'}k$, where $b^{'}$ ranges from 0 to $(v - c - t)$. The total travel cost of the both sets of border-crossers is thus $2|\bar{b}|(1/2)(v - c - t) = (v - c - t)^2/k$. Finally, border-crossers impose a total externality cost of $2|\bar{b}|e$ on their home jurisdiction, but contribute no tax revenue at home. A non-legalizing jurisdiction thus experiences welfare equal to

$$\begin{aligned} W_n &= 2|\bar{b}|(v - c - t) - \frac{(v - c - t)^2}{k} - 2|\bar{b}|e \\ &= 2|\bar{b}|(v - c - t - e) - \frac{(v - c - t)^2}{k} \end{aligned}$$

If the jurisdiction legalized, welfare would be $n(v - c - e)$. Given that a jurisdiction's neighbors have legalized, its government will raise jurisdiction welfare by also legalizing if

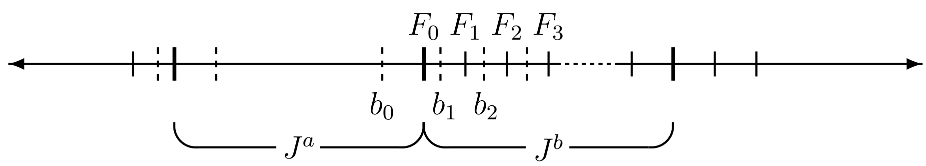
$$\begin{aligned} n(v - c - e) &> 2|\bar{b}|(v - c - t - e) - \frac{(v - c - t)^2}{k} \\ (n - 2|\bar{b}|)(v - c - e) + 2t|\bar{b}| + \frac{(v - c - t)^2}{k} &> 0 \end{aligned} \tag{2}$$

Remembering that $(v - c - e) < 0$, inequality (2) will hold whenever inequality (1) does. Equations (1) and (2) thus establish the (minor) result of this section. When the good is sold at all locations, and when first-moving jurisdictions gain by legalizing, then later-moving jurisdictions will also gain by legalizing. It is also possible, of course, that inequality (1) is not satisfied, in which case no jurisdiction will choose to legalize. In either case, equilibrium policy choices are consistent — either all jurisdictions legalize or none do so. In the all-legalize outcome, each jurisdiction experiences a lower welfare than it would have had none legalized. When $n(v - c - e) + 2t|\bar{b}| > 0$ holds, in other words, decisions about whether or not to legalize have the characteristics of a prisoners' dilemma.

III. Model With Discrete Firms

In this section, it is assumed that the existence of fixed costs mean that the product will be available at specific locations. Once a firm exists at such a location, it remains there. Firms are located inside legalizing jurisdictions at unit intervals along the line — Firm 0 (F_0) at point 0, Firm 1 at point 1, etc. When one jurisdiction legalizes before its neighbor does so, a firm locates (effectively) on the relevant border, just inside the legalizing jurisdiction. The solid hash marks in Figure 1 illustrate this situation, where the heavy marks are firm locations that correspond to a border. Each segment of the line that makes up a separate jurisdiction is marked with a J^i. The broken hash marks are the market borders between firms (b_i). The figure is drawn under the assumption that jurisdiction J^a hasn't legalized, but that both of its neighbors have done so.

Figure 1



As in the previous section, it is assumed that legalizing sales of the good when borders cannot be crossed is welfare reducing. In this section, however, the net benefit of purchasing the good must include the effects of travel cost. Legalization with closed borders implies that consumers travel between a distance between 0 and 1/2 to get to the nearest firm; the average travel distance is therefore 1/4 unit, and the average travel cost is $k/4$. The assumption that legalization without border crossing is welfare reducing thus amounts to assuming that $v - c - e - (k/4) < 0$.

Within a legalizing jurisdiction, consumer value v is assumed to always exceed the tax-inclusive, travel-cost inclusive cost so that all within-jurisdiction consumers purchase the good. The market border b_1, for example, between the firms located at points 0 and 1 is the point at which a consumer is indifferent between buying at the two locations. In other words, the full price — monetary plus travel expense — of buying from those two firms must be equal, which implies that

$$\begin{aligned} p_0 + t + kb_1 &= p_1 + t + k(1 - b_1) \\ b_1 &= \frac{1}{2} + \frac{p_1 - p_0}{2k} \end{aligned}$$

Other market borders are defined similarly. Firm 1 — which has active firms located on both sides of it — sells to all customers located between b_1 and b_2, so its profit is

$$\begin{aligned} \Pi_1 &= (b_2 - b_1)(p_1 - c) \\ &= \left(\frac{3}{2} + \frac{p_2 - p_1}{2k} - \frac{1}{2} - \frac{p_1 - p_0}{2k}\right)(p_1 - c) \end{aligned}$$

Differentiating the above expression with respect to p_1 and setting the resulting expression equal to zero produces the first-order condition for profit maximization. When rearranged, this expression yields Firm 1's profit-maximizing reaction function for price:

$$p_1 = \frac{c}{2} + \frac{k}{2} + \frac{p_0}{4} + \frac{p_2}{4} \tag{3}$$

The reaction functions of Firms 2, 3, etc. are defined similarly.

Firm 0 faces a different situation. This firm faces no competing firm on its left, so its market border on that side is the point at which a consumer gets no surplus from travelling to and buying from Firm 0. This occurs at location b_0, where $v - t - p_0 - |b_0|k = 0$, which implies that this border is defined as

$$b_0 = -\frac{v - t - p_0}{k} < 0.$$

Firm 0 sells to all consumers located between b_0 and b_1, so its profit is

$$\begin{aligned}\Pi_0 &= (b_1 - b_0)(p_0 - c) \\ &= \left(\frac{1}{2} + \frac{p_1 - p_0}{2k} + \frac{v - t - p_0}{k}\right)(p_0 - c)\end{aligned}$$

Differentiating, setting equal to zero, and rearranging produces Firm 0's reaction function:

$$p_0 = \frac{v - t}{3} + \frac{c}{2} + \frac{k}{6} + \frac{p_1}{6} \qquad (4)$$

Similar to the result in Braid (1987), the reaction functions given above have a closed-form solution, which can be expressed as a weighted average of two potential profit-maximizing prices. The first potential price is the price that would hold in equilibrium if there was an active firm at every integer spot on the line. If all firms have reaction functions of the form given by (3), the market equilibrium price is $\hat{p} = c + k$.[7] The second is the price that would be charged by a single profit-maximizing firm that is unconstrained by any neighboring firms (in other words, a monopoly firm), which is $(v - t + c)/2$.[8] [Intuitively, the relevance of these two prices arises from the fact that Firm 0 effectively faces the monopoly situation on one side of its location, and faces the the fully-filled line on the other side.] The Appendix shows that the equilibrium prices of Firms i, for $i \geq 0$ are:

$$p_i^* = \alpha_i \left(\frac{v - t + c}{2}\right) + (1 - \alpha_i)(c + k), \qquad \text{where } \alpha_i = \frac{4}{4 + \sqrt{3}}(2 - \sqrt{3})^i. \qquad (5)$$

Again consider the possibility that jurisdictions don't make simultaneous decisions about legalization. If one jurisdiction legalizes before a neighbor does, the firm found at the border between the two jurisdictions is assumed to be located in the legalizing jurisdiction. Crucially, it is also assumed that the firm would remain in the same jurisdiction even if the neighboring jurisdiction later legalized.[9]

Consider the choices facing a welfare-maximizing government. If no other governments have legalized, a jurisdiction that also doesn't legalize will experience welfare equal to 0.

[7] An in-jurisdiction consumer who travels the maximum distance to reach a firm incurs a travel cost of $k/2$. The derivation of the equilibrium price $\hat{p}$ is correct only if consumer value for the good v exceeds the travel-cost, tax-inclusive price $c + k + (k/2) + t$ for all buyers; this relationship is assumed to hold.

[8] Note that in this model, at some parameter values, the "monopoly" price can be lower than the "oligopoly" price. This outcome can occur because a monopoly faces more elastic demand than does a firm with neighboring sellers.

[9] If universal legalization leads to eventual adjustments in firm location, the results in this paper can be interpreted as analysis of short-run government incentives.

In contrast, if it does legalize, all its home consumers will purchase the good, as will some border-crossers from the two neighboring jurisdictions. On sales to its own residents, a jurisdiction's welfare is expressed as: consumer value minus firm price minus tax payment minus travel cost plus firm profit plus tax revenue minus externality cost, summed over all jurisdiction residents. In total, the expression equals $n(v - c - e - (k/4))$. On sales to the two sets of border crossers, a jurisdiction's firms earn profit, and the government collects tax revenue: the total impact is $2|b_0|(p_0 - c + t)$. If consumption by a jurisdiction's own residents has only a slightly negative impact, the gain from border crossers might be large enough to make legalization welfare-improving. This situation arises if the welfare of a first-moving jurisdiction W_F is positive:

$$W_F = n(v - c - e - \frac{k}{4}) + 2|b_0|(p_0 - c + t) > 0 \tag{6}$$

Now consider a jurisdiction acting after its two neighboring jurisdictions have already legalized.[10] Some of its residents already cross the border, ($|b_0|$ in either direction), and in the aggregate they experience — temporarily ignoring travel cost — a benefit of $2|b_0|(v - p_0 - t)$. Travel cost ranges from (effectively) 0 (for consumers located at the border) to $v - p_0 - t$ (for consumers just at the margin of buying). The average travel cost for border-crossers is thus $(v - p_0 - t)/2)$, and aggregate travel cost is $2|b_0|(v - p_0 - t)/2$. The cross-border buyers also impose an aggregate externality cost of $2|b_0|e$ on their home jurisdiction. A jurisdiction that doesn't legalize, even though its neighbors have done so, thus experiences welfare of $W_N = |b_0|(v - p_0 - t - 2e) < 0$.

If a jurisdiction in this situation chooses to legalize, its n residents will experience $n(v - \hat{p} - t)$ personal benefit (ignoring travel cost), will create externality cost of ne, and will experience travel cost of $nk/4$. Since this jurisdiction moved latter than did others, the firms at its borders are located in neighboring jurisdictions, so that only $(n - 1)$ firms are located inside a jurisdiction of size n. Those firms collect a total profit of $(n - 1)(\hat{p} - c)$ and a total tax of $(n - 1)t$. Thus, a jurisdiction that is last to legalize experiences welfare of $W_L = n(v - c - e - k/4) - (\hat{p} - c + t) < 0$. Given that other jurisdictions have legalized, the remaining one will experience higher welfare by not legalizing if

$$W_N = |b_0|(v - p_0 - t - 2e) > n(v - c - e - \frac{k}{4}) - (\hat{p} - c + t) = W_L. \tag{7}$$

At any parameter values that satisfy both both expressions (6) and (7), there can be long-term differences in policy choices between jurisdictions. Jurisdictions that can legalize early may find that the opportunity to affect firm location, and to draw in profits and tax revenue (but not externality costs) from some neighboring residents is attractive. In contrast, a jurisdiction that acts late, and that — even if it legalizes — will still see some of its residents purchase elsewhere, may decide not to experience the negative welfare effect of enabling all its residents to consume.

[10] The case in which both neighboring jurisdictions have legalized is studied because this is the situation in which a decision by a follower not to legalize is most unlikely (since the jurisdiction is already experiencing the welfare-reducing impact of border-crossing consumption by two distinct groups of its residents.

For many combinations of parameter values, equations (6) and (7) (along with the other parameter restrictions introduced throughout the paper) are not simultaneously satisfied. For such parameter values, the prisoners' dilemma nature of the model illustrated in Section II also holds in this version of the model.

There do exist, however, parameter combinations for which inequalities (6) and (7) (and the other relevant parameter restrictions) hold. For example, a some-but-not-all-legalize outcome arises for the following parameter values: $v = 2.8$, $c = 1$, $k = 1$, $t = .2$, $e = 1.6$, and $n = 24$. These values imply that $p_0 = 1.539$, $|b_0| = 1.061$, $W_F = .368 > 0$ and $W_N = -2.27 > -2.40 = W_L$.

An outcome in which jurisdictions choose different policies is more likely when *(i)* the welfare effect of legalization per unit population when borders are closed is, while negative, small in absolute value, and *(ii)* jurisdictions are of intermediate size. If jurisdictions are "small", a late-moving jurisdiction is already experiencing the negative externality from enough of its population that it gains by legalizing. In contrast, if jurisdictions are "large", the tax revenue from border crossers is relatively too small to entice any government into legalizing.

IV. Conclusion

This paper has used a spatial model to consider whether otherwise identical jurisdictions may act in differing ways based entirely on the order in which policy decisions are made. If early policy decisions have no impact on the locations of sales, early legalization is always followed by universal legalization. On the other hand, if the first jurisdiction to legalize an activity gains a long-lasting advantage in firm location — if firms locate where their product is legal and (at least for awhile) stay there even if neighboring jurisdictions legalize — government policies may differ. With this assumption about firm location, an early-moving government may legalize because of the ability to attract nonresident consumers, while a later-moving government may not legalize because even if it did so some of its residents would continue purchasing outside its borders.

This paper began with a discussion of retail products that might have negative externalities associated with them. Considering some possible examples, it seems reasonable that a jurisdiction that first legalizes casino gambling could gain a long-lasting advantage by becoming a tourism center. In regards to fireworks, when two jurisdictions have dramatically different laws, large fireworks stores tend to locate just inside the border of the legalizing jurisdiction. It is possible that those stores might well maintain some of their advantage in size or selection, and thus might continue to draw some cross-border buyers even if legalization became universal. To at least some degree, then, the model in this paper might explain why differences in policy concerning these activities could persist.

In contrast, the technology to sell state lottery tickets can be easily placed in many small retail stores. For lottery tickets therefore, universal legalization would quickly eliminate any first-mover advantage, and the paper's model unambiguously predicts policy convergence.

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Appendix

Equations (x) and (y) in the body of the paper show that $p_0 = (v-t)/3+c/2+k/6+p_1/6$ and $p_i = c/2+k/2+p_{i-1}/4+p_{i+1}/4$ (for $i \geq 1$) are the reaction functions for firm prices. This appendix establishes that these reaction functions are satisfied at the p_i ($i \geq 0$) equilibrium prices that are given by the following weighted average:

$$p_i^* = \alpha_i \left(\frac{v-t+c}{2}\right) + (1-\alpha_i)(c+k), \quad \text{where } \alpha_i = \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^i. \tag{A.1}$$

Showing that this is indeed the formula for the equilibrium prices entails showing that plugging both p_0^* and p_i^* (for $i \geq 1$) into the relevant reaction functions produces consistent results.

Considering p_0^* first, equation (A.1) with $i = 0$ yields

$$p_0^* = \frac{2}{4+\sqrt{3}}(v-t+c) + \frac{\sqrt{3}}{4+\sqrt{3}}(c+k).$$

Similarly,

$$p_1^* = \frac{2(2-\sqrt{3})}{4+\sqrt{3}}(v-t+c) + \left(\frac{-4+5\sqrt{3}}{4+\sqrt{3}}\right)(c+k).$$

Substituting the latter expression into the p_0 reaction function produces

$$p_0 = \frac{v-t}{3} + \frac{c}{2} + \frac{k}{6} + \left(\frac{2-\sqrt{3}}{3(4+\sqrt{3})}\right)(v-t+c) + \left(\frac{-4+5\sqrt{3}}{6(4+\sqrt{3})}\right)(c+k)$$

Setting p_0^* equal to p_0 yields

$$p_0^* = \frac{2(v-t)}{4+\sqrt{3}} + \frac{2c}{4+\sqrt{3}} + \frac{\sqrt{3}(c+k)}{4+\sqrt{3}} = \left(\frac{1}{3} + \frac{2-\sqrt{3}}{3(4+\sqrt{3})}\right)(v-t) +$$
$$\left(\frac{1}{2} + \frac{2-\sqrt{3}}{3(4+\sqrt{3})} + \frac{-4+5\sqrt{3}}{6(4+\sqrt{3})}\right)c + \left(\frac{1}{6} + \frac{-4+5\sqrt{3}}{6(4+\sqrt{3})}\right)k = p_0$$

or

$$p_0^* = \frac{2(v-t)}{4+\sqrt{3}} + \frac{(2+\sqrt{3})c}{4+\sqrt{3}} + \frac{\sqrt{3}k}{4+\sqrt{3}} = \left(\frac{6}{3(4+\sqrt{3})}\right)(v-t) +$$
$$\left(\frac{3(4+\sqrt{3})+2(2-\sqrt{3})-4+5\sqrt{3}}{6(4+\sqrt{3})}\right)c + \left(\frac{4+\sqrt{3}-4+5\sqrt{3}}{6(4+\sqrt{3})}\right)k = p_0$$

or

$$p_0^* = \frac{2(v-t)}{4+\sqrt{3}} + \frac{(2+\sqrt{3})c}{4+\sqrt{3}} + \frac{\sqrt{3}k}{4+\sqrt{3}} = \frac{2(v-t)}{(4+\sqrt{3})} + \frac{(12+6\sqrt{3})c}{6(4+\sqrt{3})} + \frac{(6\sqrt{3})k}{6(4+\sqrt{3})} = p_0.$$

The equality between the left-hand side and right-hand side of the above expression establishes that the equilibrium prices p_0^* and p_1^* are consistent with reaction function p_0.

Turning now to equilibrium price p_i^* $(i \geq 1)$, equation (A.1) equals

$$p_i^* = \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^i \left(\frac{v-t+c}{2}\right) + \left[1 - \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^i\right](c+k) \tag{A.2}$$

Plugging p_{i-1}^* and p_{i+1}^* into the p_i reaction function produces

$$\begin{aligned} p_i = \frac{c}{2} + \frac{k}{2} + \frac{1}{4}\left[\frac{4}{4+\sqrt{3}}(2-\sqrt{3})^{i-1}\left(\frac{v-t+c}{2}\right)\right] + \frac{1}{4}\left[1 - \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^{i-1}\right](c+k) \\ + \frac{1}{4}\left[\frac{4}{4+\sqrt{3}}(2-\sqrt{3})^{i+1}\left(\frac{v-t+c}{2}\right)\right] + \frac{1}{4}\left[1 - \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^{i+1}\right](c+k) \end{aligned}$$

Rewriting this formula yields

$$\begin{aligned} p_i = \frac{c}{2} + \frac{k}{2} + \left(\frac{1}{4+\sqrt{3}}\right)\left(\frac{v-t+c}{2}\right)\left[(2-\sqrt{3})^{i-1} + (2-\sqrt{3})^{i+1}\right] \\ + \frac{1}{4}\left[1 - \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^{i-1} + 1 - \frac{4}{4+\sqrt{3}}(2-\sqrt{3})^{i+1}\right](c+k) \end{aligned}$$

or

$$\begin{aligned} p_i = \left(\frac{1}{4+\sqrt{3}}\right)\left(\frac{v-t+c}{2}\right)(2-\sqrt{3})^{i-1}\left[1 + (2-\sqrt{3})^2\right] \\ + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{4} - \left(\frac{(2-\sqrt{3})^{i-1}}{4+\sqrt{3}}\right)\left(1 + (2-\sqrt{3})^2\right)\right](c+k) \end{aligned}$$

or

$$p_i = \left(\frac{1}{4+\sqrt{3}}\right)\left(\frac{v-t+c}{2}\right)(2-\sqrt{3})^{i-1}(8-4\sqrt{3}) + \left[1 - \left(\frac{(2-\sqrt{3})^{i-1}}{4+\sqrt{3}}\right)(8-4\sqrt{3})\right](c+k)$$

or

$$p_i = \left(\frac{4(2-\sqrt{3})^i}{4+\sqrt{3}}\right)\left(\frac{v-t+c}{2}\right) + \left[1 - \frac{4(2-\sqrt{3})^i}{4+\sqrt{3}}\right](c+k).$$

This last expression matches equation (A.2) which establishes that, for all $i \geq 1$, the formula for equilibrium price p_i^* satisfies reaction function p_i.