Making the Indifference-Curve Approach to Excess Burden More Understandable

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Abstract: This paper presents a simple method by which students can calculate numeric values for the excess burden created by an excise tax. So doing should enable students to better grasp the concept of excess burden. Furthermore, by considering different tax rates and different utility functions, a student can explicitly see how the size of a tax's excess burden depends both on the size of the tax and on the extent to which the tax distorts consumer behavior.

Key words: excess burden, deadweight loss, excise tax, indifference curve

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INTRODUCTION

The excess burden (or the deadweight loss) created by a tax is a key concept in public-sector economics. It seems very likely that most instructors of classes in this field teach the concept. Rosen and Gayer (to cite one source) define excess burden as "a loss of welfare [due to a tax] above and beyond the tax revenues collected" (2008, p. 331). The importance of excess burden to the public-economics field is underscored by the fact that all eight of the undergraduate public textbooks checked (see below) cover the topic.

From both theoretical and empirical perspectives, students must understand excess burden — and the factors that influence its size — in order to fully understand both the inefficiencies created by taxation and the resulting implications for the design of (relatively) efficient tax systems.

At an undergraduate level, two primary methods are used to explain the excess burden concept — one relies on a indifference-curve-budget-constraint diagram; the other uses a demand-curve diagram.

This paper argues that it can be beneficial to have students apply the indifference-curve-budget-constraint approach, work with specific utility functions, and compute numeric values for the excess burden created by an excise tax. Students who calculate excess burden are likely to better understand the concept than are students who merely read or hear a description.

The approach described here takes advantage of the special characteristics both of Cobb-Douglas and Stone-Geary utility functions and of the demand functions derived from them. The recommended technique also enables a student to see how the magnitude of excess burden varies with changes both in the size of the tax and in the degree to which consumers are willing to shift their consumption from a taxed to an untaxed good.

This paper first discusses how various public-sector economics textbooks introduce students to the concept of excess burden. It then details the suggested numeric approach, after which it explains how the technique can be varied in order to demonstrate to students how consumer preferences affect the size of excess burden.

METHODS OF PRESENTING EXCESS BURDEN

While undergraduate textbooks as a group explain excess burden using both the indifference-curve-budget-constraint approach and the demand-curve approach, not all texts use both methods. Some undergraduate books (Anderson (2003), Ulbrich (2003), and Gruber (2007)) use only a triangle-under-the-demand-curve technique. One major drawback of presenting the concept in this way is that correctly calculating excess burden requires the use of a compensated demand curve. A natural way to explain to students this requirement is to refer to a diagram showing that excess burden is related to a movement along an indifference curve (i.e., is related to a substitution effect) — the very effect captured by moving along a compensated demand curve. Without the use of an IC-BC diagram, however, it is a challenge to explain the importance of using compensated demand.

Holcombe (2006), Hyman (1999), and Bruce (2001) explain excess burden using both IC-BC and DC diagrams; the first two authors explain the IC-BC approach in a different way than does the third writer. Holcombe states that an "individual is better off with [a] lump-sum tax than with [a] tax on [a single] good" and that the "difference in the individual's well-being is the difference in utility between the indifference curve [under the single-good tax] and the higher indifference curve [under the lump-sum tax]" (p. 212). Hyman's approach is similar: "the excess burden of the price-distorting tax is the reduction in well-being of the taxpayer from [indifference curve] U_2 to U_1 when the price-distorting tax is used instead of the lump-sum tax" (p. 405). In contrast, Bruce (2001) — who places his IC-BC material in an appendix — notes that a lump-sum tax that has the same effect on utility as does a distorting tax will "resul[t] in the government getting more tax revenue", and states that the "extra revenue is the excess burden" (p. 387) of the distorting tax. When using the indifference-curve-budget-constraint approach, however, none of Holcombe (2006), Hyman (1999), and Bruce (2001) explain that excess burden can be calculated on a cardinal scale by measuring the length of a specific line segment in an IC-BC figure.

Of the eight texts under consideration, only Stiglitz (2000) and Rosen and Gayer (2008) explain why excess burden equals a particular distance in an indifference-curve-budget-constraint diagram. Rosen and Gayer's use of the IC-BC approach ends with that statement. The authors do not provide a numeric example of the IC-BC calculation of excess burden, nor do any of their chapter-end questions address the exact measurement of excess burden in such a diagram. Like Rosen and Gayer, neither Bruce (2001), Holcombe (2006), Hyman (1999), nor Stiglitz (2000) provide any IC-BC-based excess-burden questions.

The excess burden concept is a complicated one. Even if students read a clear explanation of the idea, it seems probable that their grasp of it would be strengthened by working a problem or two using — or at by least seeing a numeric illustration of — the topic.

Another way in which numeric examples can deepen understanding is by allowing students to see how varying the extent of the behavioral response to a tax alters the calculated value of excess burden. The magnitude of the excess burden created by a tax depends on "the extent to which consumers and producers change their behavior to avoid the tax" (Gruber, 2007, p. 581). One natural way to think about such willingness to alter behavior is by considering the shape of a person's indifference curves for a taxed and an untaxed good. Indifference curves with shapes that imply a greater willingness to substitute between such goods mean that a given tax creates a larger excess burden.[1]

Of the books listed above, three — Holcombe (2006), Bruce (2001) and Stiglitz (2000) — include in their indifference-curve-budget-constraint material some statement addressing how preferences (as illustrated by the shapes of indifference curves) affect the size of excess burden. Holcombe (2006) states "[t]he closer indifference curves are to right angles, the less people will substitute because of price changes, which will reduce the excess burden. Individuals will substitute more when indifference curves have little curvature, which will make the excess burden larger." Similarly, Bruce (2001) states that "[i]f the consumer's indifference curves were right angles, there would be no excess burden" (p. 387). These authors do not provide any illustrations or problems to reinforce their statements. Stiglitz (2000) provides an diagram that illustrates his statement that when "indifference curves are L-shaped ... there is no substitution effect, and, it is apparent, there is no deadweight loss" (p. 524). Stiglitz does not make any more general statement. Once again, none of these authors follow up their statements about indifference-curve shape with any end-of-chapter questions.

In addition to these textbook approaches, both Trandel (2003) and Liu and Rettenmaier (2005) have offered suggestions about how to teach the excess-burden concept. Neither paper utilizes an approach like the one used in this paper. Trandel's (2003) discussion connects the demand-curve approach with the algebraic formula for excess burden. Liu and Rettenmaier (2005) relate an inventive way to link the indifference-curve-budget-constraint approach with the demand-curve approach. At many schools, however, their technique might be better understood by public-economics instructors than by the typical student.

NUMERIC CALCULATION OF EXCESS BURDEN

Three different utility functions — which imply differing degrees of substitutability between goods — are used in the following presentation.

Students are asked to consider a single consumer who can spend income I buying quantities of goods x and y. The pre-tax prices of the goods are $p_x^{'}$ and p_y. A per-unit excise tax of t_x can be imposed on good x; good y is not taxed. With a tax in place, post-tax prices are $p_x = p_x^{'} + t_x$ and p_y.

The baseline case employs a Cobb-Douglass utility function in which goods x and y are weighted equally:

$$U = \sqrt{x \cdot y}. \tag{1}$$

This function implies that the consumer's demand functions are

$$x = \frac{I/2}{p_x} \quad \text{and} \quad y = \frac{I/2}{p_y}\,. \tag{2}$$

These utility functions can be directly derived using the Lagrangian-multiplier technique in calculus. A somewhat simpler approach — but one that still requires calculus — is described in the appendix.

[1] Demand-curve slope and/or demand elasticity can, of course, also be used to illustrate behavioral changes. Employing such tools, however, again runs into the need to explain the use of compensated demand.

Public-economics students who have not taken calculus can still be largely convinced that equations (2) describe a consumer's utility-maximizing demand functions. One approach for so doing is to ask students to compare the utility values that result from consumption at various points along a consumer's budget constraint. For example, if $I = 60$, $p_y = 1$, and $p_x = 1 + t_x = 1 + .5 = 1.5$, students could be asked to compute utility at the following affordable combinations of x and y: ($x = 18$, $y = 33$), (19,31.5), (20,30), (21,28.5), and (22,27). The observation that the (20,30) point yields the highest (affordable) level of satisfaction reinforces the claim that equations (2) do indeed describe how a consumer should allocate his or her budget.

Demand functions (2) indicate that a consumer who has utility function (1) allocates half of his or her budget to each good, and then buys as many units of each good as he or she can afford.

For particular price and tax values, students can compute the excess burden experienced by a representative consumer by using an equivalent-variation approach. To begin the calculation, a student is first asked to assume consumer utility function (1) and a price-distorting tax t_x on good x. They are then asked to use equations (2) to find the consumer's choice of x and y, and to compute the consumer's resulting utility.

A student must then find a way to measure — using a dollar (rather than a utility) scale — the extent of the burden imposed on a consumer by the tax on good x. He or she can do this by finding the size of a hypothetical lump-sum tax that would lower the consumer's utility to the same level as that felt under the distorting tax. The tax revenue that would be collected by such a lump-sum tax is a measure of the burden imposed on the consumer by the tax. The difference between that hypothetical tax revenue and the tax revenue collected by the excise tax is the excess burden of the tax — it is a dollar measure of the loss of consumer well-being that is not offset by government tax collections.

Specifically, consider the Cobb-Douglas utility function given above, along with the following parameter values: $I = 60$, $p_y = 1$, and $p_x = 1+t_x = 1.5$. With the excise tax in place, equations (2) tell us that consumption levels are $x = 20$ and $y = 30$, so that the consumer's utility equals $\sqrt{20 \cdot 30} = 24.495$. Tax revenue is $t_x x = .5 \cdot 20 = 10$.

This situation is illustrated in Figure 1, where BC_N is the consumer's budget constraint when there is no tax in place, BC_E is the budget constraint when an excise tax is imposed on good x, and the quantities of x and y chosen by the consumer are shown. Indifference curves IC_N and IC_T (the shapes of which are consistent with utility function (1)) are, respectively, the highest curves that the consumer can reach at the relevant consumption bundles with no tax in place and with the excise tax imposed. In other words, IC_N and IC_T are tangent to BC_N and BC_E respectively.

Now consider removing the excise tax (so that $p_x = p_y = 1$, implying that the consumer buys equal quantities of x and y), and instead imposing a hypothetical lump-sum tax, set at a level so that the consumer can just achieve the same utility as he or she attains in the excise-tax situation. In particular, since utility equal to 24.495 would be attained if $x = y = 24.495$, the hypothetical lump-sum tax equivalent must must leave the consumer with the resources needed to just afford that consumption bundle; i.e., the lump-sum tax must equal $60-(2 \cdot 24.495) = 11.010$. Imposing such a lump-sum tax would reduce the vertical and horizontal intercepts of the consumer's budget constraint to $60 - 11.010 = 48.990$. The consumer in Figure 1 would therefore face budget constraint BC_L (which is tangent to IC_T).[2]

As noted above, the excess burden of the price-distorting tax equals the difference between the hypothetical lump-sun tax just computed (a dollar measure of the burden imposed by the excise tax) and the (excise) tax revenue collected. With the above parameters, the excess burden is therefore $11.010 - 10 = 1.010$.[3]

The fact that a distorting tax creates an excess burden means that the tax's negative effect on consumer well-being exceeds the loss that directly results from the amount of money taken away from away from the consumer. Students asked to compute a numeric answer for excess burden will likely better understand this concept than would a student who only reads or hears a description of excess burden.

A natural follow-up question (for homework, perhaps) is to ask students to repeat the excess burden calculation using a tax that has been increased by, for example, 10% (i.e., from $t_x = .5$ to $t_x = .55$), The student can then determine whether the resulting excess burden has also increased by 10%, by less than this percentage, or (as shown to be the case in Table 1 below) by more. The disproportionate extent to which excess burden grows when a tax rate is increased has (as all public-economics instructors know) important theoretical implications for government tax and spending policies.

CONSUMER RESPONSIVENESS AND EXCESS BURDEN SIZE

The excess burden created by an excise tax arises from a consumer (optimally) responding to the tax by reducing his or her purchases of the taxed good. Other things equal, the greater this distortion in consumption choices, the greater the excess burden. It is possible to make this point — and three textbooks noted above do so — by considering the shape of indifference curves. However, textbook discussions of the size of excess burden (even in the three books noted) are typically built around the elasticity of a (compensated) demand curve.

By working numeric problems with a variety of utility functions, however, students can directly see how a consumer's underlying preferences — and his or her resulting willingness to substitute between taxed and untaxed goods (as illustrated by the shape of the consumer's indifference curves) — affect the size of a tax's excess burden. In particular, students will

[2]For the use of interested instructors, an example of a homework assignment (based on the approach in this paper), which might be appropriate after students have been introduced to the general concept of excess burden, is available on the author's webpage at xxx.xxx.edu/xxxxxxx/papers.htm. The material available also includes versions of Figures 1–3 with fewer values labelled (and in some cases fewer budget-constraint lines). The LaTeX code used to produce the three figures is also available upon request from the author.

[3]In Figure 1, this excess burden is the vertical distance between the point at (20,30) and the point on BC_L directly below it. This relationship holds because the vertical distance between BC_N and BC_E at $x = 20$ measures tax revenue, while the vertical distance between BC_N and BC_L measures the burden imposed by the tax. With the specific parameters in use, the hypothetical lump-sum tax equals 11.01, so that the y-coordinate of the relevant point on BC_L is $48.99 - 20 = 28.99$. The relevant point's coordinates are therefore (20,28.99), and the vertical distance between this point and (20,30) matches the 1.01 value for excess burden found above. This vertical distance is the measure of excess burden described by Rosen and Gayer (2008) and Stiglitz (2000).

see that a greater willingness to substitute — as shown by the person's indifference curves having relatively little curvature — implies a larger excess burden.

The see this, students who have already calculated excess burden using utility function (1) can be asked to consider a consumer who possesses the following Stone-Geary utility function:

$$U = \sqrt{(x-20)\cdot(y-20)}. \tag{3}$$

Note that for this utility function to be meaningful, a consumer must have the financial resources to consume at least twenty units of each of goods x and y.

Two indifference curves that result from utility function (3) are illustrated in Figure 2 (where all curves are labelled as they were in Figure 1). As the diagram suggests, and as numeric calculations will confirm, a consumer with this utility function is much less willing than is the consumer considered earlier to alter his or her consumption choices in response to an excise tax. This pattern is intuitive: if a consumer (whenever possible) must always buy at least twenty units of each of x and y, his or her consumption choices will show relatively little flexibility.

A consumer with utility function (3) has demand functions equal to

$$x = 20 + \frac{(I - 20p_x - 20p_y)/2}{p_x} \quad \text{and} \quad y = 20 + \frac{(I - 20p_x - 20p_y)/2}{p_y}. \tag{4}$$

The appendix to this paper shows how functions (4) can be derived from the partial derivatives of function (3). As noted in the appendix, the demand functions derived from utility function (1) can be computed in a similar, but simpler, way.

While many economics students will probably not be able to compute these demand functions directly, calculating utility at a few affordable points along the budget constraint can again convince them of the accuracy of the demand functions. The intuition for these demand curves is clear: the consumer first spends what is needed to consume twenty units of each of goods x and y; once that goal is met, the consumer divides his or her remaining spending power equally between the two goods.

The difference in preferences between consumers with utility functions (1) and (3) can be expected to alter the size of a given tax's excess burden. Again considering $I = 60$, $p_y = 1$, and $p_x = 1 + t_x = 1.5$, equations (4) imply that $x = 23.33$ and $y = 25$. A tax imposed only on good x causes the consumer to cut back on purchases of both x and y. Compared to the above case, consumer behavior is distorted by a relatively small amount; i.e., there is much less of a relative shift in consumption away from the taxed good. Plugging the relevant consumption levels into utility function (3) reveals that $U = \sqrt{(23.33-20)\cdot(25-20)} = 4.082$.[4] Tax revenue is $t_x x = .5 \cdot 23.33 = 11.667$.

The consumption levels with undistorted prices (and utility function (3)) that would produce $U = 4.082$ are, of course, $x = y = 24.082$. These quantities would be chosen by a consumer facing a hypothetical lump-sum tax equal to $60 - (2 \cdot 24.082) = 11.835$. Excess burden — the difference between the lump-sum tax just computed and the tax revenue raised

[4] Instructors should emphasize to their students that the differences among the numeric values of utility created by utility functions (1), (3), and (5) (below) are of no significance.

by the distorting tax — thus equals $11.835 - 11.667 = .168$. When a consumer's preferences are described by utility function (3), an excise tax distorts consumer behavior by a relatively small amount. In such a situation, therefore, a tax creates a excess burden much smaller than that found when the consumer has utility function (1).[5]

A contrast to this situation can be found by considering the following utility function (which is defined only for positive values of x and y):

$$U = \sqrt{(x + 20) \cdot (y + 20)}. \tag{5}$$

The indifference curves that result from this utility function are illustrated in Figure 3. These curves — which display relatively less curvature than those considered previously — are those of a consumer who is quite willing to respond to a change in relative prices by dramatically shifting his or her consumption choices.

Utility function (5) results in the following demand functions,

$$x = -20 + \frac{(I + 20p_x + 20p_y)/2}{p_x} \quad \text{and} \quad y = -20 + \frac{(I + 20p_x + 20p_y)/2}{p_y}, \tag{6}$$

which can be found using a straightforward modification of the approach in the appendix.

Using the above values for I, p_x and p_y, equations (6) imply that $x = 16.667$ and $y = 35$. A tax imposed only on good x causes the consumer to buy much less x and more y — there is a large shift in consumption from the taxed to the untaxed good. The relevant consumption levels create a utility level of $U = \sqrt{(16.67 + 20) \cdot (35 + 20)} = 44.907$. Tax revenue is $t_x x = .5 \cdot 16.667 = 8.333$.

With undistorted prices, the consumption levels that would produce $U = 44.907$ are $x = y = 24.907$. These quantities would be chosen by a consumer facing a lump-sum tax equal to $60 - (2 \cdot 24.907) = 10.186$. Excess burden in this case thus equals $10.186 - 8.333 = 1.853$.

Compared to the results from either of the two earlier cases, an excise tax imposed on a consumer with utility function (5) substantially distorts consumer behavior, and therefore creates a relatively large excess burden.

Table 1 displays the excess burden calculations described in this paper; the table shows the way in which excess burden varies as the tax rate and consumer preferences vary.

CONCLUSION

This paper presents a simple method that allows students to calculate numeric values for the excess burden created by an excise tax. So doing should enable students to better grasp the fundamental excess-burden concept. Furthermore, by considering different tax rates and

[5]Instructors might want to note that the "full burden" on the consumer of the excise tax (as measured by the size of the utility-equivalent lump-sum tax) is larger for a consumer with utility function (3) than it is for a person with function (1). This is because a tax's direct impact on a consumer is greater the larger the extent to which his or her preferences lead the person to continue buying the taxed good. With such behavior, however, more of the tax's impact on the individual is offset by the government's tax revenue, thus resulting in a relatively small excess burden.

different utility functions, students can derive results that illustrate two important points. First, the magnitude of a tax's excess burden rises proportionally faster than does the size of the tax. Second, a tax's excess burden is larger the greater the extent to which the tax distorts consumer behavior.

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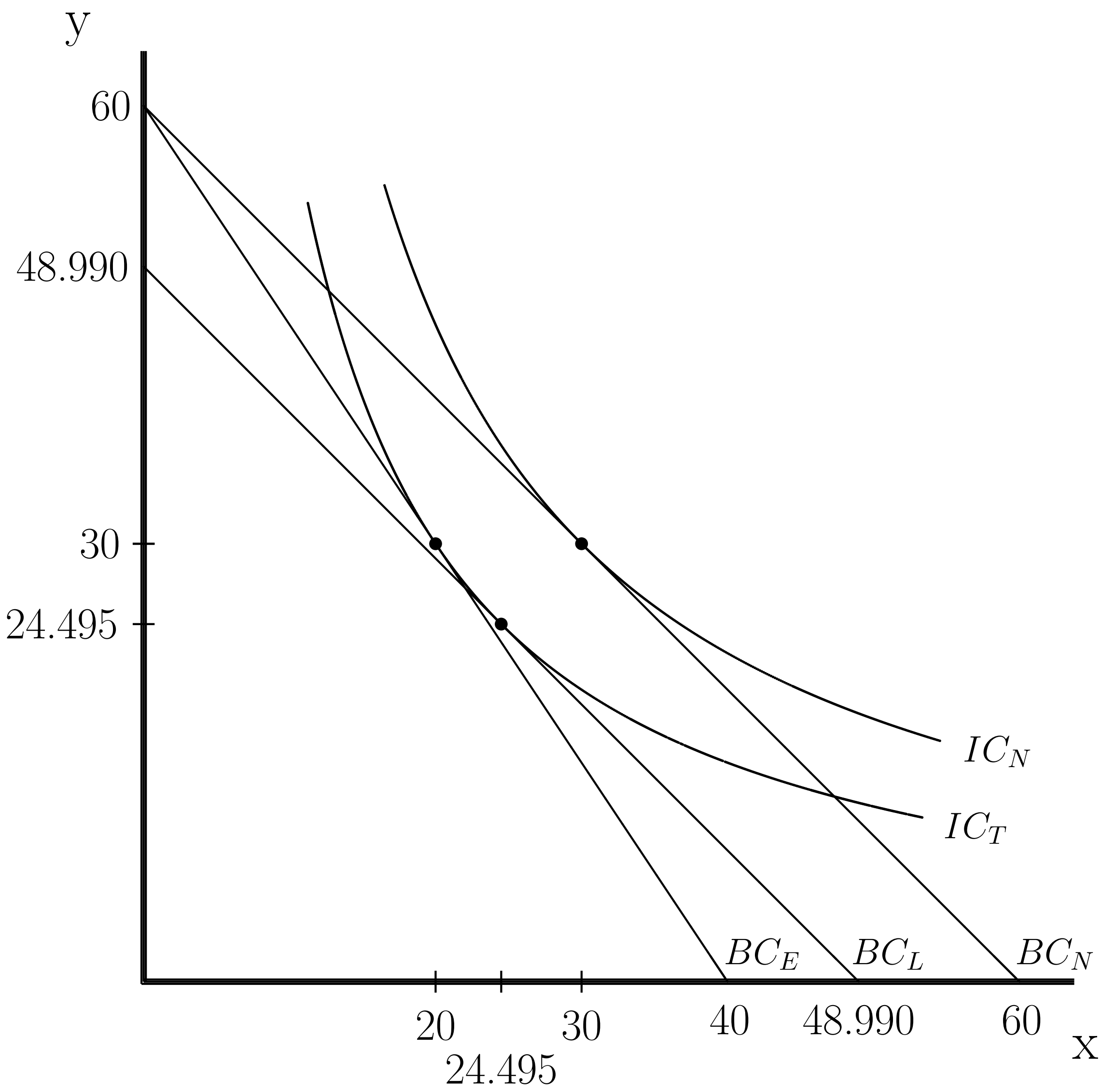

Figure 1: moderate substitution between x and y

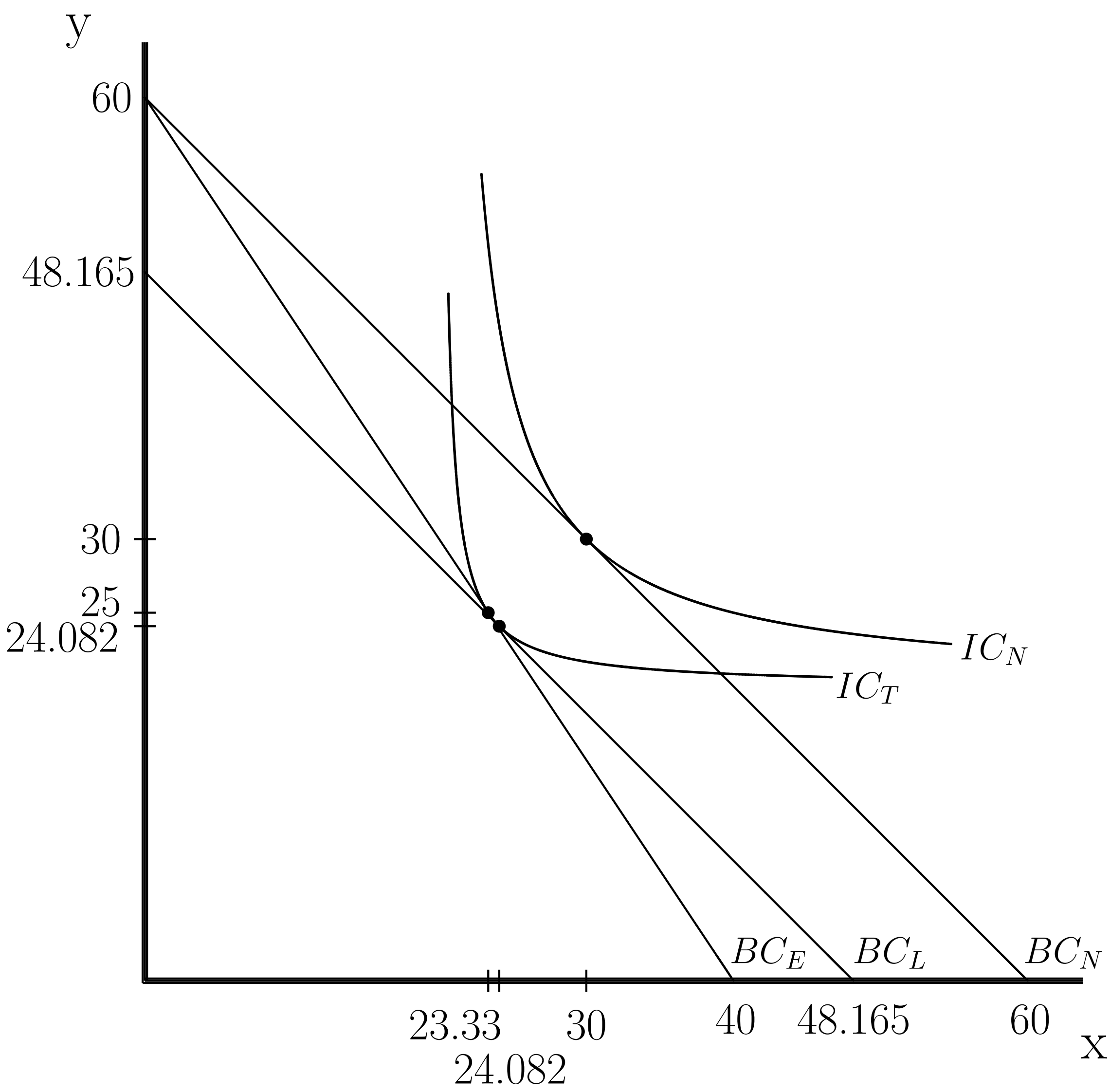

Figure 2: little substitution between x and y

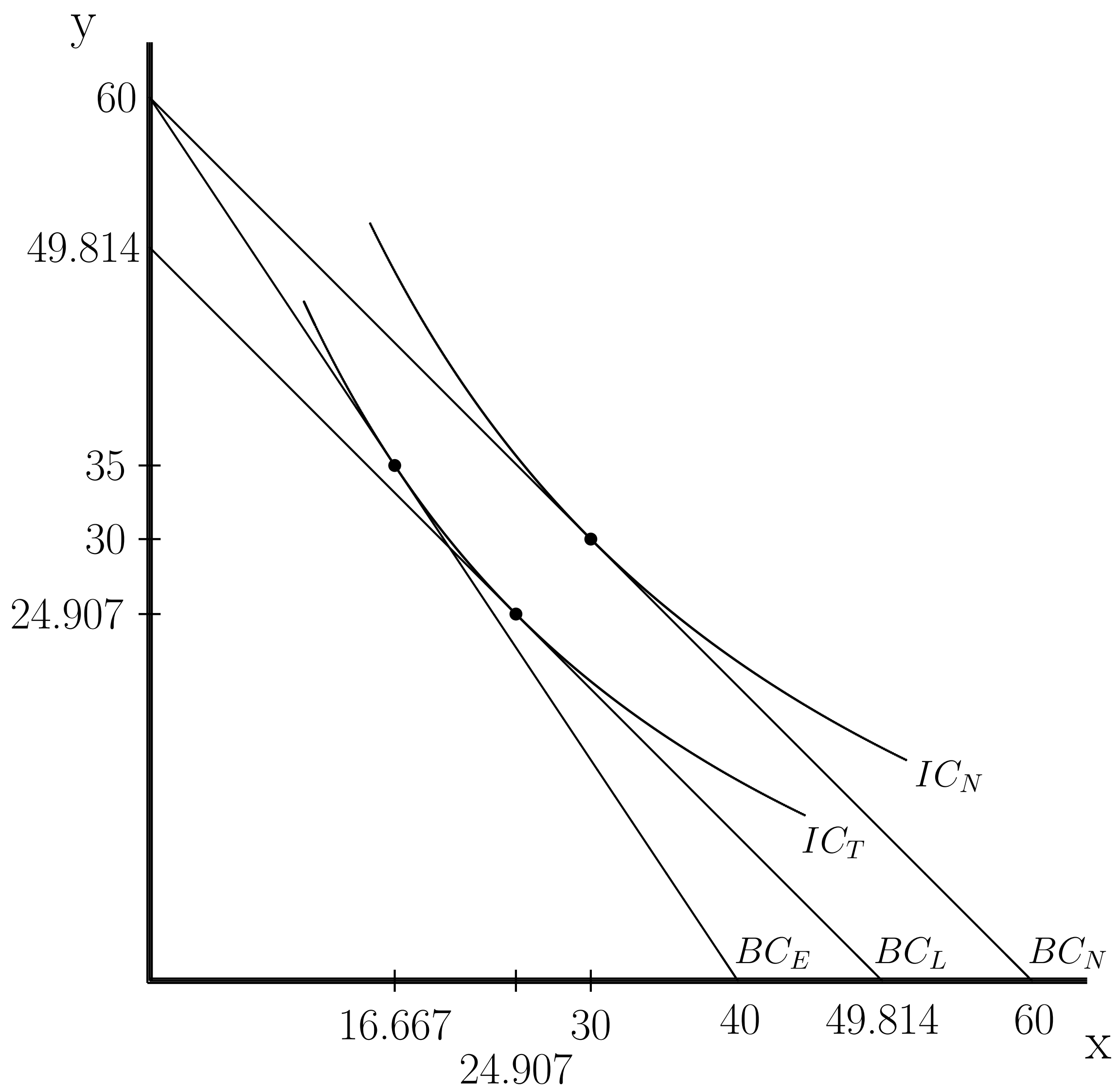

Figure 3: much substitution between x and y

Table 1: calculation of excess burden

utility function	t_x	x	y	U	exc. tax revenue	lump-sum tax	excess burden
$\sqrt{x \cdot y}$	.5	20	30	24.495	10	11.010	1.010
$\sqrt{x \cdot y}$	.55	19.355	30	24.097	10.645	11.807	1.162
$\sqrt{(x-20) \cdot (y-20)}$	.5	23.333	25	4.082	11.667	11.835	.168
$\sqrt{(x+20) \cdot (y+20)}$	.5	16.667	35	44.907	8.333	10.186	1.853

APPENDIX

This appendix shows how demand functions (4) are derived (using calculus) from utility function (3). The procedure for deriving equations (2) and (6) are straightforward variations of this approach.

Taking partial derivatives of the utility function given by (3) produces

$$\begin{aligned}
\frac{\partial U}{\partial x} &= \frac{1}{2}((x-20)(y-20))^{-(1/2)}(y-20) \\
&= \frac{1}{2}\frac{\sqrt{y-20}}{\sqrt{x-20}} \\
\frac{\partial U}{\partial y} &= \frac{1}{2}((x-20)(y-20))^{-(1/2)}(x-20) \\
&= \frac{1}{2}\frac{\sqrt{x-20}}{\sqrt{y-20}}
\end{aligned}$$

One dollar of spending on good x yields $(1/p_x)$ unit of the good; that dollar of spending in turn produces a change in utility equal to

$$\Delta U = \Delta x \cdot \frac{\partial U}{\partial x} = \frac{1}{p_x} \cdot \frac{\partial U}{\partial x} = \frac{\frac{\sqrt{y-20}}{\sqrt{x-20}}}{2p_x}.$$

The corresponding term for one dollar of spending on good y is

$$\Delta U = \Delta y \cdot \frac{\partial U}{\partial y} = \frac{\frac{\sqrt{x-20}}{\sqrt{y-20}}}{2p_y}.$$

When the consumer is maximizing utility, he or she must experience equal changes in utility from his or her last dollar of spending on each of the two goods. In other words, when the consumer is maximizing utility, it must be true that

$$\begin{aligned}
\frac{\frac{\sqrt{y-20}}{\sqrt{x-20}}}{2p_x} &= \frac{\frac{\sqrt{x-20}}{\sqrt{y-20}}}{2p_y} \\
\frac{\frac{\sqrt{y-20}}{\sqrt{x-20}}}{\frac{\sqrt{x-20}}{\sqrt{y-20}}} &= \frac{p_x}{p_y} \\
\frac{y-20}{x-20} &= \frac{p_x}{p_y} \\
p_y y &= p_x x - 20p_x + 20p_y.
\end{aligned}$$

[Note that the above formulae can also be derived using a Lagrange-multiplier approach.]

Since

$$I = p_x x + p_y y$$

substituting and rearranging produces

$$\begin{aligned} I &= p_x x + p_x x - 20p_x + 20p_y \\ I + 20p_x - 20p_y &= 2p_x x \\ x &= \frac{I}{2p_x} + 10 - \frac{20p_y}{2p_x}. \end{aligned}$$

Adding 10 to the right-hand side of the equation, while subtracting the same amount in the form $20p_x/2p_x$, yields

$$\begin{aligned} x &= \frac{I}{2p_x} + 20 - \frac{20p_x}{2p_x} - \frac{20p_y}{2p_x} \\ x &= 20 + \frac{I - 20p_x - 20p_y}{2p_x}, \end{aligned}$$

which is the first of equations (4). The second equation is derived similarly.

When utility function (1) is used, the "20"s in the above expressions all disappear, which yields demand curves (2).