Economics 2106H (Spring 2009) — Prof. Greg Trandel
Answers to Question #20 of Homework Assignment # 2

20. When a certain firm produces q units of a product, its total cost of production is given by this formula: $TC = 1600 + 5q + \frac{q^2}{400}$. For any particular unit q, the firm's marginal cost of producing that unit equals this formula: $MC = 5 + \frac{q}{200}$ (or, alternatively: $MC = 5 + .005q$).

(a) *(Optional; only for those of you comfortable with calculus.)* Given the formula for TC, use calculus to explain why the formula for MC is correct.

Taking the derivative of TC with respect to q yields the MC formula.

(b) What is this firm's fixed cost? What is the formula for its variable cost of producing q units?

$FC = 1600$. $VC = 5q + \frac{q^2}{400}$.

(c) What is the formula for this firm's average total cost of producing q units? For this firm's average variable cost of producing q units? For its average fixed cost of producing q units?

$ATC = \frac{1600}{q} + 5 + \frac{q}{400}$. $AVC = 5 + \frac{q}{400}$. $AFC = \frac{1600}{q}$.

(d) Use the total cost formula to find the TC of producing 199 units (i.e., find TC when $q = 199$). Find the TC of producing 200 units. Considering the change in total cost, what is this firm's marginal cost of producing its 200th unit of output?

$TC(q = 199) = 2394.0025$. $TC(q = 200) = 2400$. $\Delta TC = 5.9975$.

(e) Using the marginal cost formula, what is the firm's MC of producing its 200th unit of output?

$MC(q = 200) = 6$.

(f) Do the two approaches to finding marginal cost produce answers that are "close enough" to each other? *[The answer to this question should be "yes".]*

Yes.

(g) Complete a table like the accompanying one.

Unit	Fixed Cost	Variable Cost	Total Cost	Average Variable Cost	Average Total Cost	Marginal Cost
200	1600	1100	2700	5.5	13.5	6
400	1600	2400	4000	6	10	7
600	1600	3900	5500	6.5	9.17	8
800	1600	5600	7200	7	9	9
1000	1600	7500	9100	7.5	9.1	10
1200	1600	9600	11200	8	9.33	11
1400	1600	11900	13500	8.5	9.64	12

(h) Show that ATC hits its lowest level at $q = 800$. This can be done in *either* of two different ways. *(i)* Since we know that $ATC = MC$ when ATC is at its minimum, one method is to solve $ATC = MC$ for q. *(ii)* An alternative method is to use calculus to directly find the q at which the ATC formula is minimized. *[Those who haven't taken calculus should obviously use method (i).]*

$$\begin{aligned} ATC &= MC \\ \frac{1600}{q} + 5 + \frac{q}{400} &= 5 + \frac{q}{200} \\ \frac{1600}{q} &= \frac{q}{400} \\ q^2 &= 640000 \\ q &= 800 \end{aligned}$$

or, take the derivative of ATC with respect to q, set the result equal to zero, and solve for q.

(i) On the attached sheet, use the values in the table to graph the ATC curve, the AVC curve, and the MC curve (some of these are "lines" rather than "curves".) *[Make sure your graphs are based on the actual cost formulae in this question, rather than the shape of "typical" cost curves.]*

(j) Assume that this firm is a price taker. What then must be true about its marginal revenue from selling any unit?

The firm's marginal revenue is equal to the market price of the product it sells (that market price is, for now, an unknown).

(k) Since the market price is currently unknown, let that price be represented by the general term P. Find a general formula for the quantity of output produced and sold by this firm at any possible value of the market price. [In other words: find a formula for this firm's supply curve.] To do so, find a formula of the form $q = a(P - b)$ (the equations used in this problem guarantee the formula will be a straight line), where you've determined what a and b must be. To find the needed formula, start with the rule for profit-maximizing behavior, substitute in the proper terms, and solve for q. *[Note: this is the key step in solving the rest of this problem. If you've spent some time thinking about how to write the needed formula, and find yourself stuck – feel free to contact Prof. Trandel for a hint.]*

A price-taking firm maximizes its profit by selling up to the quantity at which its marginal cost equals the market price. [This is a version of the $MR = MC$ rule that applies only to price takers.] At the moment, the market price is unknown, so refer to it simply as P, then set the firm's marginal cost equal to the market price and solve for q as a function of P.

$$\begin{aligned} MC &= P \\ 5 + \frac{q}{200} &= P \\ \frac{q}{200} &= P - 5 \\ q &= 200(P - 5) \end{aligned}$$

The last formula shows the firm's desired sales as a function of the market price. It is, in other words, the firm's supply curve.

(l) Suppose the market consists of 100 firms identical to this one. Let Q be the market quantity. Find the formula for the quantity supplied by the market at any possible market price (i.e., find a formula of the form $Q^s = -c + dP$). *[Given that a negative value for quantity doesn't really make sense, the $Q^s = -c + dP$ formula holds only for appropriate values of Q and P.]* The formula you just found is the market (short-run) supply curve.

$$\begin{aligned} Q &= 100 \cdot q = 20000(P - 5) \\ Q &= -100,000 + 20,000P \text{ (for } P \geq 5) \end{aligned}$$

(m) Suppose that market demand (writing the quantity demanded as a function of the market price) is $Q^d = 150,000 - 5000P$ (for $0 \leq P \leq 30$). For the purpose of graphing demand, it's more convenient to express demand as $P = 30 - \frac{1}{5000}Q$ (or, alternatively $P = 30 - .0002Q$). Show that these two ways to express demand are equivalent.

$$\begin{aligned} Q &= 150,000 - 5000P \\ 5000P &= 150,000 - Q \\ P &= 150,000/5000 - Q/5000 \\ P &= 30 - .0002Q \end{aligned}$$

(n) Rewrite the formula you found for market supply so that it is the form $P = e + fQ$.

$$\begin{aligned} Q &= -100,000 + 20,000P \\ P &= 100,000/20,000 + Q/20,000 \\ P &= 5 + .00005Q \end{aligned}$$

(o) Also on the attached sheet, illustrate the market demand and market supply curves from parts (m) and (n). Show the equilibrium market price (P^*) and equilibrium market quantity (Q^*).

(p) Use the formulae for market supply and market demand to find numeric values for P^* and Q^*.

$$\begin{aligned} DC &= SC \\ 30 - .0002Q &= 5 + .00005Q \\ 25 &= .00025Q \\ Q^* &= 100,000 \end{aligned}$$

$$P^* = 5 + .00005(100,000) = 5 + 5 = 10$$

(q) Given the market equilibrium price, find a numeric value for the quantity of output (q^*) a single firm wishes to produce and sell.

$$\begin{aligned} P^* &= 10 \\ q^* &= 200(P - 5) = 200(10 - 5) = 1000 \end{aligned}$$

(r) Given your previous answers, find the profit earned by a single firm using the $\Pi = TR - TC$ formula. Find the profit earned by a single firm using the $\Pi = (P - ATC)Q$ formula.

$$\begin{aligned} \Pi &= TR - TC = P \cdot q - TC(q) \\ \Pi &= 10 \cdot 1000 - 9100 = 10000 - 9100 = 900 \end{aligned}$$

$$\Pi = (P - ATC)q = (10 - 9.1)1000 = 900$$

(s) On the graph of the firm's situation you drew for part (i), show the market price (P^*), the profit-maximizing quantity (q^*), and the area that represents profit.

(t) Explain why this firm doesn't stop its production at the quantity at which its average total cost of production reaches its lowest possible level.

A firm that stopped at that level of output ($q = 800$) would miss out on the chance to earn additional profit by selling the units between 800 and 1000, units for which price (marginal revenue) exceeds marginal cost. Put another way, stopping at the quantity at which ATC is minimized would maximize profit *per unit*. This is not equivalent to maximizing (total) profit. In this particular example, a firm that produced 800 units would have $ATC = 9$; if it sold 800 units at $P = 10$, its profit would only equal $\Pi = (P - ATC)q = (10 - 9)800 = 800$.

The homework question stopped at this point, but now that we've covered some additional material, we can talk about how this market will change as time passes.

In the situation described above, the firms currently in the market are earning positive economic profits. That will attract new investment into the market. In this problem, the simplest way to capture that process of entry and expansion is simply allow more firms that look just like the existing firm to enter the market. Let's say the 10 new firms enter the market, so that there are now 110 firms, each with the same supply curve. The market supply curve thus becomes

$$Q = 110 \cdot q = 110 \cdot 200(P - 5) = 22000(P - 5)$$

We can go on to rewrite market supply as

$$\begin{aligned} Q &= 22000(P - 5) \\ Q &= -110,000 + 22,000P \text{ (for } P \geq 5) \end{aligned}$$

We could then set this new market supply curve equal to the market demand curve to find the new equilibrium quantity and price, and the profit earned by each firm. If the individual firms were still earning positive profits, the process of entry would continue.