Demand Shocks and Endogenous Uncertainty∗

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Abstract
Recessions have been documented as periods of heightened aggregate and firm-level uncertainty. To date explanations have either hinged on the notion that second moment shocks have adverse first order effects, or that negative first moments disturbances are responsible for the observed surges in cross sectional dispersion. I explore the symbiotic relationship between uncertainty and aggregate economic activity and propose framework where endogenous uncertainty may exacerbate or abate aggregate shocks hitting the economy. U.S. Compustat and ShopperTrak data are used to discipline an incomplete markets, heterogeneous-firms framework which is able to reproduce the right business cycle co-movements. Results indicate that fluctuations in uncertainty are responsible for about one quarter of aggregate fluctuations in output and employment.

Keywords: Uncertainty, Heterogeneous Firms, Cross sectional firm dynamics.

JEL Classification: E21, E23, E32, G22.

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1 Introduction

Uncertainty fluctuations are large and strongly countercyclical. In the U.S., uncertainty has been systematically documented as having sizable adverse effects on economic activity and inflation. In terms of aggregate output, for example, [Baker and Bloom (2011)] establish that sudden changes in uncertainty may account for GDP declines in the vicinity of two percent. [Gilchrist et al. (2014)] report that uncertainty shocks can explain about one third of the total variation in industrial output and payroll employment; while [Bachmann et al. (2013)] find them responsible for manufacturing losses in excess of one percent. Moreover, [Bloom (2009)] and [Bloom et al. (2012)] argue that increased uncertainty makes it optimal for firms to wait, leading to significant declines in hiring, investment and output; and [Fernández-Villaverde et al. (2013)] establish that time-varying risk shocks may also have negative consequences for price stability.

While it has been well established that uncertainty and aggregate economic activity are negatively related, it is less evident why or how this occurs. To date most research efforts have been devoted to documenting, quantifying and understanding the effects of fluctuations in uncertainty on business conditions. In doing so, studies have often assumed the existence of sharp exogenous changes in the volatility of shocks which, mediated by physical [Bloom (2009)], financial [Gilchrist et al. (2014)] or nominal [Basu and Bundick (2012)] frictions, negatively impact mean economic outcomes. By focusing on the effects of fluctuations in uncertainty, however, almost no attention has been paid to the understanding their probable sources.

Motivated by the above, this study seeks to provide evidence as to the potential origins of fluctuations in uncertainty. In doing so, it delivers a quantitative theory that is consistent with the time-varying cross sectional properties of U.S. macroeconomic aggregates. The paper will focus on the symbiotic relationship between uncertainty and economic activity to explain how first-moment disturbances can abate or exacerbate dispersion, but also to highlight how time-varying uncertainty can affect mean equilibrium outcomes. I argue that while swings in uncertainty appear to be endogenously related to aggregate economic activity, fluctuations in idiosyncratic risk will ultimately affect macroeconomic dynamics. Intuitively recessions are
times of heightened uncertainty, yet greater uncertainty may also exacerbate a recession. In particular, the study will focus on the widely held notion that consumer demand uncertainty experienced by firms could be at the heart of business cycle fluctuations. The analysis is conducted through the lens of an incomplete markets, heterogeneous-agents framework which is able to successfully reproduce the right business cycle co-movements.

The paper has two main goals. The first objective is to further our understanding of the relationship between uncertainty and mean aggregate activity. In doing so I focus on the synchronicity between uncertainty and economic outcomes, and propose an innovative channel through which the former may relate to the business environment. In particular, firms in the model face uncertainty about the number of customers they will need to serve each period (idiosyncratic), as well as the amount of resources these customers may command (aggregate). Being risk-averse firm owners will respond cautiously to changes in macroeconomic conditions, leading to cyclical employment and output fluctuations.

The second goal of this paper is to contribute to the understanding of the cross-sectional dynamics of business cycles. The availability of highly disaggregated, longitudinal microeconomic and sectorial data, has recently shed light over the idiosyncratic responses of economic agents to aggregate shocks. In turn, understanding the cross-sectional behavior of individual firms and households becomes paramount for comprehending aggregate dynamics. In the model endogenous changes in uncertainty further variations in economic activity allowing it to better replicate the observed cyclical patterns of higher moments.

Results indicate that time-varying uncertainty has significant effects on the aggregate economic activity. In the model’s baseline specification, fluctuations in uncertainty accounted for about one-quarter of the overall variation in employment, output and consumption at business cycle frequencies. Moreover, uncertainty swings act as an amplification mechanism reinforcing the original shock to mean level activity. Overall, a one percent negative shock to credit conditions leads to output and employment losses of around 0.8 and 0.6 percent respectively.
The paper makes a few additional contributions to the literature. First, it introduces an innovative way of modeling fluctuations in consumer’s demand. Rather than assuming exogenous changes to a household’s discount factor, the model will keep track of the distribution of customers visiting a firm. Second, the proposed framework sheds light on the relationship between uncertainty and risk averse behavior, in that higher perceived risk might exacerbate the effects of first moment disturbances hitting the economy. Lastly, the study proposes a parsimonious framework capable of capturing fluctuations in uncertainty which requires no nominal rigidities and offers a tractable closed form solution.

1.1 Related Literature

This study is closely related to a fast growing body of literature studying the effects of time-varying uncertainty on economic activity. It follows Bloom (2009), Basu and Bundick (2012), and Leduc and Liu (2012) in that fluctuations in second moments have first order aggregate effects. The overriding idea in this area of research is that spikes in uncertainty, channeled through some adjustment friction, generate the observed fluctuations in economic activity. Moreover, the paper also relates to the scholarly research focusing on uncertainty fluctuations as an endogenous outcome rather than a cause. In this view, Bachmann and Moscarini (2011) propose a model in which recessions tend to incentivize firms’ risk taking behavior and hence lead to higher cross-sectional dispersion. Similarly, Fostel and Geanakoplos (2012) and D’Erasmo and Boedo (2011) suggest alternative mechanisms capable of generating countercyclical uncertainty.

The proposed framework also represents a natural extension to Bewley-type models such as Aiyagari (1994), Huggett (1997) and Krusell and Smith (1998). These models introduce idiosyncratic risk into an incomplete markets neoclassical framework, but focus on labor-income risk, rather than demand uncertainty. Furthermore, the paper closely follows Angeletos (2007) and Quadrini (2014), both of which provide the theoretical underpinnings behind the set-up as well as the chosen solution method.
The study also relates to the literature seeking to understand the idiosyncratic effects of aggregate shocks. [Higson et al. (2002) and Higson et al. (2004)] report that rapidly growing and rapidly declining firms appear to be less sensitive to negative macroeconomic disturbances relative to those firms in the middle range of growth. This appears to be consistent with the fact that the higher moments of the distribution of firm growth rates have significant cyclical patterns. Similarly, [Kehrig (2011)] finds that the cross-sectional dispersion of firm-level total factor productivity in the U.S. tends to be greater in recession than in expansions.

In terms of production, some papers assign a productive role to consumer demand for goods and services. With this in mind, this study follows [Bai et al. (2012) and Petrosky-Nadeau and Wasmer (2011)] in that output will not only be a function of factor inputs (like in any neoclassical framework), but consumer demand will play a paramount role in determining the level of economic activity. Moreover, in line with [Arellano et al. (2010)] the framework also explores the effects of input pre-commitments in optimal firm behavior.

Finally, the study is also related to the literature highlighting the effects of financial frictions on the interaction between uncertainty and economic activity. [Gilchrist et al. (2014)] argue that increases in firm risk lead to bond premia and the cost of capital, which in turn, triggers the prolonged decline in investment activity. It also follows [Jermann and Quadrini (2012)] in that the financial sector may be the source of the business cycle and not solely a propagation channel for shocks that hit other sectors of the economy.

The remainder of this paper is organized as follows: Section 2 presents the empirical motivation and analysis from the Compustat and ShopperTrak data sets. Section 3 explains the model and 4 describes its calibration. Finally, Section 5 presents the main results while Section 6 draws some final conclusions.
2 Empirical Motivation

2.1 Time series facts

The negative association between uncertainty and economic activity finds substantial empirical support in the U.S. economy. The above patterns, however, are not exclusive to it and a plethora of studies have recorded similar realities in countries around the globe. Bachmann et al. (2013) use German data to provide evidence as to the detrimental effects of uncertainty in that country. For the UK, Denis and Kannan (2013) estimate that uncertainty shocks generate industrial production and output losses, while Bloom et al. (2007) finds evidence that supports the claim that higher uncertainty reduces domestic firms’ capital expenditures. Similar conclusions have been reached for developing economies. Arslan et al. (2011) establish that a one standard deviation increase in aggregate uncertainty generates a 4 percent drop in Turkey’s GDP growth rate; while Fernández-Villaverde et al. (2011) compute the negative effects of interest rate volatility for a group of Latin American economies. Globally, Baker et al. (2012) document the effects of uncertainty in slowing down the global recovery.

Given its intrinsically unobservable and yet broad nature, uncertainty can be very hard to measure. It reflects the ambivalence in the minds of consumers, investors, and policymakers about the likelihood of potential future outcomes. It can also reflect skepticism about aggregate events such as the growth rate, credit conditions and exchange rates; or micro phenomena such as industry level legislation or personal ambiguity. Not surprisingly, a plethora of proxies have been developed over the last years in an attempt to capture sudden variations in risk. One of these measures is the Exchange Volatility Index (VIX) which captures the expected thirty days forward implied volatility backed out from option prices. An alternative proxy for uncertainty is the corporate bond spread computed as the difference between the Baa 30 year yield and the U.S. Treasury yield at a comparable maturity. Another measure frequently used is the disagreement amongst professional forecasters. Periods or higher uncertainty usually correlate with greater dispersion in professionals’ opinions. The intuition is that uncertainty makes it harder for agents to make accurate predictions. Finally, Baker et al. (2012) develop an alternative proxy for uncertainty by recording the frequency of newspaper articles reporting
Figure 1 plots a selection of commonly used empirical measures of uncertainty over the business cycle.

![Corporate Bond Spread](image)

![Option Implied Volatility](image)

![Economic Policy Index](image)

Figure 1: Uncertainty indicators over the Business cycle.

Independently on which metric is used, virtually every indicator of uncertainty rises in recessions and subdues during expansions. Conversely, measures of economic activity tend to move in communion with the cycle. Figure 2 shows this graphically, plotting the business cycle evolution of six macroeconomic indicators. Intuitively as economic activity slows down, jobs are lost, consumption falls and capacity utilization rates plummet. Additionally, as aggregate credit conditions deteriorate, sales growth slows down and companies’ net-worth suffer. This is the negative association between uncertainty and economic activity which will be at the core of this study. In particular, by focusing on consumer demand uncertainty the model will successfully reproduce the business cycle dynamics in all six macroeconomic yardsticks.
mentioned above.

Figure 2: Uncertainty and Economic Activity. *Consumption* corresponds to the year-over-year changes in Personal Consumption Expenditures (PCE) as recorded by the BEA, while *Employment* tracks the year over year changes to the level of total non-farm, quarterly employment. *Capacity Utilization* refers the percentage of industrial capacity currently being used by firms domestically to produce the demanded finished products as compiled by the Board of Governors of the Federal Reserve System. *Retail Sales* correspond to the yearly change in the level of retail and food services sales as measured by the U.S. Census Bureau, and *Credit Conditions* refer to the Federal Reserve Bank of Chicago’s National Financial Conditions Index (NFCI), where positive values of the index indicate that financial conditions are tighter than average. Finally, *Firm’s Net Worth* track the evolution of the non-financial corporate business sector’s net worth as a percentage of GDP.
2.2 Firm-level facts

Researchers focusing on the impact of uncertainty on individual firms and households have found that uncertainty at the firm level is also negatively associated with growth and economic activity. Kehrig (2011), for example, shows that for US durable goods manufacturers uncertainty about plant-level TFP rises sharply in recessions affecting firms’ entry and survival rates. Vavra (2013) establishes that uncertainty about prices also surges during recessions, making it harder for the Federal Reserve to conduct monetary policy. Higson et al. (2002) find that risk shocks are negatively correlated with the cycle, but affect firms in an uneven way. Leahy and Whited (1996) find a strong negative relationship between uncertainty and investment for US publicly listed firms.

The primary firm-level data source used in this paper is the US Compustat database. Compustat North America provides the annual and quarterly Income Statement, Balance Sheet, Statement of Cash Flows, and supplemental data items on most publicly held companies in the United States and Canada. Financial data items are collected from a wide variety of sources including news wire services, news releases, shareholder reports, direct company contacts, and quarterly and annual documents filed with the Securities and Exchange Commission. Compustat files also contain information on aggregates, industry segments, banks, market prices, dividends, and earnings. Depending upon the data set, coverage may extend as far back as 1950 through the most recent year-end.

Using Compustat has some advantages versus using census data sets like the Longitudinal Research Dataset (LRD) or the Annual Survey of Manufacturers (ASM), because firm-level data are accessible to all researchers in different countries, and the panel for the US goes as far as the 1950s. Naturally, this data is not not without flaws, the most commonly recognized being the fact that the firm’s recorded in Compustat account by about one-third of US employment (Davis et al. (2006)).

The data set comprises of 32 years of data (1980-2012), with cross-sections that have, on
average over 3,000 firms per year. From the original Compustat data, I select firms that report information on gross and net sales, employment and capital stocks. Following Bloom (2009) I drop firms with missing information as well as remove outliers. To calculate firm-level employment growth rates I use the symmetric adjustment rate definition proposed in Davis et al. (2006):

$$g_{ht} = \frac{h_t^i}{0.5 \ast (h_t^i + h_{t-1}^i)}$$

Firm-level sales growth rates are simple log-differences. To focus on idiosyncratic changes that do not capture differences in industry-specific responses to aggregate shocks, I follow Bachmann et al. (2013) in removing firm effects from employment and sales growth rates. Annual GDP and inflation data come from the Federal Reserve Economic Data (FRED) database. All moments are robust to different inflation indexes specifications. Table 1 summarizes some of the statistical properties of the US Compustat data set.

<table>
<thead>
<tr>
<th>Table 1: U.S. Compustat Moments 1980-2012</th>
<th>ln(sales)</th>
<th>ln(emp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional dispersion</td>
<td>2.227</td>
<td>2.416</td>
</tr>
<tr>
<td>Cross-sectional Skewness</td>
<td>-0.187</td>
<td>-0.125</td>
</tr>
<tr>
<td>Cross-sectional Kurtosis</td>
<td>2.236</td>
<td>2.484</td>
</tr>
<tr>
<td>Dispersion growth rate corr w/ cycle</td>
<td>-0.388**</td>
<td>-0.293**</td>
</tr>
<tr>
<td>Skewness corr w/ cycle</td>
<td>-0.406**</td>
<td>-0.062**</td>
</tr>
<tr>
<td>Kurtosis corr w/ cycle</td>
<td>-0.322*</td>
<td>0.208</td>
</tr>
<tr>
<td># observations (ave. per year)</td>
<td>3,296</td>
<td>1,585</td>
</tr>
<tr>
<td># observations (total)</td>
<td>114,368</td>
<td>53,875</td>
</tr>
</tbody>
</table>

Source: Compustat and U.S. Bureau of Economic Analysis

The table suggests the presence of significant deviations from symmetry and normality in the data. The skewness of both sales and employment is negative implying that the mass of the cross-sectional distribution is concentrated towards the right. This feature is consistent with the fact that most firms surveyed in the Compustat dataset are primarily well established, publicly traded companies. Moreover, both variables experience significant positive kurtosis,
suggesting that a greater proportion of the variance comes infrequent extreme events.

The data also points towards the existence of a considerable amount of cross-sectional heterogeneity in the growth rates of U.S. firms which varies with the aggregate state of the economy. In particular, both the growth rates of the cross-sectional dispersion of employment and sales appear to be negatively correlated with the business cycle. Figure 3 plots the evolution of the growth rate of the cross-sectional dispersion of sales and employment over the course of five recessions as defined by the NBER.

![Cross-sectional dispersion of Sales growth rates](image)

![Cross-sectional dispersion of Employment growth rates](image)

Figure 3: Uncertainty over the business cycle

These observations are in line with what has been documented by other researchers in the field. [Bloom (2009)] and [Bachmann and Bayer (2014)] both report similar findings even when using alternative datasets. Further, [Kehrig (2011)] finds analogous patterns for alternative measures of cross-sectional dispersion. These stylized facts will all be important empirical regularities to be matched by the proposed framework.
3 The Model

The baseline model has two sectors: an entrepreneurial and a household sector. Entrepreneurs are sole owners of firms and will be responsible for producing goods in the economy. Households supply labor and will demand consumption goods. Firms face uncertain demand for their products and hold financial assets to mitigate the effects of adverse idiosyncratic shocks. The full set-up is described below.

3.1 Households

Time is discrete, indexed by \( t \in \{0, 1, \ldots, \infty\} \). There is a continuum of infinitely-lived households whose preferences are separable in consumption, \( c_t \), and labor supply, \( h_t \), as described by:

\[
U_H = E_0 \sum_{t=0}^{\infty} \beta^t \left( c_t - \gamma \frac{h_t^{1+\tau}}{1+\tau} \right)
\]

where \( E_0 \) is the conditional expectation operator, \( \beta \) is the discount factor, \( \gamma > 0 \) measures the relative disutility of labor effort and \( \tau > 0 \) is related to the Frisch elasticity of labor supply. Household supply labor in a competitive market and allocate their labor and financial earnings between consumption goods and risk-free assets. Their budget constraint is:

\[
w_th_t + \frac{b_{t+1}}{R_t} \geq c_t + b_t
\]

where \( w_th_t \) is the period real labor income, \( R_t \) is the gross interest rate and \( b_{t+1} \) is the loan contracted in period \( t \) and due in period \( t+1 \). Balances are settled every period and there is no default. Households may accumulate intertemporal assets, but face the following borrowing constraint:

\[
\Omega \geq \frac{b_{t+1}}{R_t} \forall t
\]
Households will seek to purchase consumption goods before collecting their labor income. Since the goods are acquired before wages are paid and before the opening of financial markets for inter-temporal transactions, all purchases are paid with intra-period credit. This intra-period credit is subject to a limit $\theta_t$, which is stochastic and follows the process:

$$\ln \theta_t = \rho \ln \theta_{t-1} + \epsilon_t$$

$$: \quad \epsilon_t \sim N(\mu_\epsilon, \sigma_\epsilon^2)$$

This time-varying limit is meant to capture the evolution of aggregate consumer credit conditions in the economy.

### 3.2 Entrepreneurs

There is a continuum of entrepreneurs indexed by $i$ with lifetime preferences over consumption streams given by:

$$U_E^i = E_0 \sum_{t=0}^{\infty} \beta^t \ln c_i^t$$

where $E_0$ is the expectation operator conditional on the information available at $t = 0$ and $\beta$ the discount factor.

Entrepreneurs are individual owners of firms and produce a homogeneous, non-storable and competitively traded consumption good. Firms have revenue functions $y_i^t h_i^t$, where the variable $h_i^t$ is the input of labor and $y_i^t$ represents output per worker. A firm’s level of output per unit of labor is an idiosyncratic stochastic variable that will be defined below. For the moment what matters is that the operation of every firm is subject to an idiosyncratic shock $y_i^t$.

Following [Arellano et al., 2010](#) it is assumed that entrepreneurs choose the input of labor before observing the actual realization of $y_i^t$. Moreover, I assume that the wage rate cannot be made contingent on the realization of the idiosyncratic uncertainty. Since labor markets
are competitive, this implies that wage rate will be the same for all firms. Markets are assumed to be incomplete, with only one asset available for entrepreneurs to self-insure against the idiosyncratic risk: a non-contingent bond \( b^i_t \) that pays the gross interest rate \( R_t \). The entrepreneur’s budget constraint is therefore:

\[
y^i_t h^i_t + b^i_t \geq c^i_t + w^i_t h^i_t + \frac{b^i_{t+1}}{R_t}
\]

where \( h^i_t \) is the labor input provided by households to firm \( i \) in period \( t \), and \( y^i_t \) is firm’s \( i \) idiosyncratic output per worker. All in all, these assumptions imply that the firm faces a risk in the choice of labor which cannot be fully insured.

Given the entrepreneur’s preferences over consumption, linear production technology and distributional assumptions on the idiosyncratic uncertainty, its optimal policy is characterized by the following proposition:

**Proposition 1** Define \( \phi_t \) as the value that satisfies

\[
E_y \left[ \frac{y^i_t - w^i_t}{(y^i_t - w^i_t) \phi_t + 1} \right] = 0.
\]

Then the entrepreneur’s policy functions will take the form:

\[
\begin{align*}
    h^i_t & = \phi b^i_t \\
    c^i_t & = (1 - \beta) a^i_t \\
    b^i_{t+1} & = \beta R_t a^i_t
\end{align*}
\]

Especially important is that the employment decision will be linear in \( b^i_t \). The factor of proportionality \( \phi_t \) depends negatively on the wage \( w^i_t \), which is the same for all firms, and on the distribution of \( y^i_t \), which is also the same for all firms. This allows us to derive the aggregate demand for labor as a linear function of the aggregate financial wealth of entrepreneurs.
which is:

\[ H_t = \phi_t \int b_i^t \]

\[ = \phi_t B_t \]

The next step is to describe the determination of the idiosyncratic variable \( y_i^t \) which depends on the uncertainty about the demand of goods produced by an individual firm.

### 3.3 Production and Demand Uncertainty

Every period consumers get randomly distributed among producers. In particular, assume that each household visits \( \chi < 1 \) producers. Even if each household visits the same number of producers, the distribution of consumers over producers is not uniform. This implies that some producers will receive more consumers (per-unit of labor) than others. As such, the demand uncertainty faced by firms derives from the randomness in which households get distributed among entrepreneurs.

Denote by \( n_i^t \) the number of consumers per unit of labor received by producer \( i \) in period \( t \). This variable is stochastic with probability density \( f(n) \). Since each household visits \( \chi \) producers, the distribution must satisfy \( \int n f(n) \, dn = \chi \). That is to say, the average number of consumers per worker received by each producer is \( \chi \).

Given the choice of \( h_i^t \), a firm can produce at most \( \tilde{y} h_i^t \), where \( \tilde{y} > 0 \) is a constant and represents a technological constraint. Since all entrepreneurs utilize the same production technology, \( \tilde{y} \) will be the same for all firms. The quantity \( \tilde{y} h_i^t \) represents the firm’s period \( t \) production capacity after hiring \( h_i^t \) units of labor. The actual production, however, depends on the quantity of goods that the firm can sell, which is unknown to the entrepreneur at the time he or she must make the hiring decision.

Each period can be thought of being divided in three subperiods. In the first subperiod
firms choose employment $h_i^t$ and promise to pay workers the wage $w_i$. In the second subperiod households visit producers shop for consumption goods and engage in production. In the third subperiod households are allowed to re-trade the goods acquired from the entrepreneurs in a Walrasian market and all credit/debit positions, including the promised wages, are settled. Each subperiod is outlined below.

**Subperiod 1: Hiring stage.** Entrepreneurs hire labor $h_i^t$ and set their period productive capacity $\bar{y}_i h_i^t$. The hiring decision takes into account the uncertainty about the goods that the firm will actually be able to sell in the second subperiod.

**Subperiod 2: Decentralized shopping and production.** Since a household has a credit capacity of $\theta_i$ and visits $\chi$ firms, the spending capacity in each producer is $\theta_i / \chi$. Therefore a firm that receives $n_i^t h_i^t$ consumers can sell at most $n_i^t h_i^t \theta_i / \chi$ units of goods, that is the number of consumers multiplied by the credit capacity of each consumer. Assuming that producers have all the bargaining power, the revenue per worker of firm $i$ is:

$$y_i^t = \begin{cases} \bar{y} & \text{if } n_i^t \left( \frac{\theta_i}{\chi} \right) \geq \bar{y} \\ n_i^t \left( \frac{\theta_i}{\chi} \right) & \text{if } n_i^t \left( \frac{\theta_i}{\chi} \right) < \bar{y} \end{cases}$$

Hence production per unit of labor will be determined by the number of customers that a firm receives, as well as by their purchasing capacity (intra-period credit). Last, sales for a firm that hires $h_i^t$ workers is:

$$Y_i^t = y_i^t h_i^t$$

The assumption that the producers hold all the bargaining power guarantees that, when the demand is smaller than the production capacity of the firm, the firm does not sell to customers more goods than their credit capacity. At the same time, the assumption that households are allowed to re-trade the acquired goods in subperiod 3 (as described below)
guarantees that the firm does not charge an interest rate on the intra-period credit when the demand exceeds the production capacity of the firm. Notice that charging an interest rate is equivalent to charging a higher price for the good (units of consumption goods in subperiod 3 per one unit of consumption goods in subperiod 2).

**Subperiod 3: Centralized trading and settlements.** Since during the second subperiod households are randomly matched with producers, the quantity of goods purchased differs across households. By assuming that at this stage the acquired goods can be re-traded in a centralized, anonymous market, all households face the same optimization problem at the end of the period. Specifically, they solve the recursive problem below:

Let \( S_t = \{B_t, \theta_t\} \) represent the aggregate states of the economy at time \( t \), namely the extent of credit conditions in the economy and the aggregate level of wealth.1

Recursively, the household’s optimization problem can be stated as:

\[
V(S, b) = \max_{c, b'} \left\{ c - \frac{h^{1+\tau}}{1 + \tau} + \beta E_t V(S', b') \right\}
\]

subject to:

\[
wh + \frac{b'}{R} \geq c + b
\]

Their optimal policies satisfy the first order conditions:

\[
\alpha_h = w_t
\]

\[
u_c(c_t, h_t) \geq \beta R_t E_t u_c(c_{t+1}, h_{t+1})
\]

where the last condition will satisfied with equality if the inter-temporal borrowing constraint is binding.

Overall, the model’s timing is as follows: each entrepreneur \( i \) enters period \( t \) with risk-free bonds \( b_i^t \) and chooses the labor input \( h_i^t \) knowing \( \theta_t \) but before the realization of the idiosyncratic shocks. I define \( B_t \) as \( B_t = B_t^E + B_t^H \) where \( B_t^E = \int b_i^t dF(i) \) and \( B_t^H = b_t \) in equilibrium.

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1I define \( B_t \) as \( B_t = B_t^E + B_t^H \) where \( B_t^E = \int b_i^t dF(i) \) and \( B_t^H = b_t \) in equilibrium.
ocratic matching \( n'_j \) takes place. Labor markets are competitive and the real wage \( w_t \) fluctuates to equate demand and supply. Once \( n'_j \) is known production takes place, consumers acquire goods on credit, and firms’ profits are realized. Following households collect their wages and balances are settled. In settling their liabilities, households may choose to re-trade some of their purchased goods in an anonymous Walrasian market which opens at the end of every period. Agents who acquired goods on credit beyond their actual possibilities might seek to sell some of their purchases to settle claims. Similarly households who were not able to purchase enough goods from the firms they were matched with, might seek to increase their consumption via this market. Finally, each agent chooses the next period’s bond holding \( b_{t+1}' \).

Figure 4 schematically represents the model’s timing.

Figure 4: Model’s Timing
3.4 Risk and Return trade-off

Every period entrepreneurs must decide on their optimal level of output and employment. They must do so aware of the state of aggregate credit conditions in the economy, but before knowing the actual number of customer that will visit their store. To make their decision, entrepreneurs will take into account their current level of assets $b_t$ as well as the probability distribution of $n_t$ conditional on the realization of $\theta_t$. This conditioning is relevant since the level of aggregate credit will influence the maximum number of customers ($\bar{n}$) that every period could be served by each worker. Effectively this produces a censored distribution of clients per worker as described in Figure 5:

![Figure 5: Distribution of customers per worker](image)

Beyond $\bar{n}$ an entrepreneur knows that those clients will not be served and revenue will be lost. Hiring more employees allows business owners to server more customers, but also increases the level of risk they must bear. Given that workers collect their wages independently of the achieved level of sales, the bigger the wage bill the greater the entrepreneur’s exposure to an adverse realization of $n_t$. In turn, as more households are hired the firm’s expected sales rise, but so does the size of a potential loss. This represents the fundamental trade-off solved by entrepreneurs when confronted with the task of choosing their optimal level of inputs.

In solving this trade-off, entrepreneurs must form expectations about their future level of

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Figure 6: Distribution of customers per firm

sales. These forecasts will depend crucially on two elements: the distribution of customers faced by firms, as well as on the economy’s aggregate credit conditions. The latter will condition the entrepreneurs’ expectations by limiting the maximum number of customers that can be served per worker, while the former will be endogenously influenced by the chosen level of employment. As more workers are recruited, the resulting distribution of customers per firm achieves a higher mean and a higher variance. This implies that the firm’s expected sales will increase, but so will the level of randomness faced by its owner. Figure 6 sketches the mentioned changes endured by the distribution of customers for a firm which increasing its number of employees.

The second element in the trade-off, the wage bill, also increases as more workers are recruited. Crucially, while the wage bill increases monotonically with every new employee, the probability of additional customers does not. Figure 7 simulates the profit distribution for three different firm sizes: 2, 4 and 6 employees. As the number of employees grow the resulting distribution has a higher mean and higher variance, yet more importantly, it begins to increasingly gain mass on the low outcome events. As such, even when the shape of the customer distribution tilts in favor of the entrepreneur, the exposure to a higher wage bill limits the realization of potential profits.

Intuitively as entrepreneurs hire more workers, they increase the scale of their operation. The bigger the size of the firm, the greater its expected sales, but also the greater the en-
entrepreneur’s potential loss. Conditional on their level of safe assets, entrepreneurs will choose a level of employment consistent with their expected sales and potential losses. Since entrepreneurs value consumption and are risk-averse, they will seek to minimize their exposure by hiring conservatively. What follows is a characterization of the model’s equilibrium as well as its steady state dynamics.
3.5 Equilibrium

Households do not face idiosyncratic risk and maximize lifetime utility by choosing $c_t, h_t$ and $b_{t+1}$ for all $t = 0, 1, 2,...$. Let $S_t = \{B_t, \theta_t\}$ represent the aggregate states of the economy at time $t$, namely the extent of credit conditions in the economy and the aggregate level of wealth.

Recursively, the household’s optimization problem can be stated as:

$$
V(S, b) = \max_{c_t, h_t} \left\{ c - \alpha \frac{h_t^{1+\tau}}{1+\tau} + \beta E_\theta V(S', b') \right\}
$$

subject to:

$$
wh + \frac{b'}{R} \geq c + b
$$

where $\Omega \geq \frac{b'}{R}$

Their policies satisfy the first order conditions:

$$
\alpha h_t^\tau = w_t
$$

$$
u_c(c_t, h_t) \geq \beta R_t E_t u_c(c_{t+1}, h_{t+1})
$$

where the last condition will satisfied with equality if the borrowing constraint is binding.

Similarly, the recursive problem for firm $i$ at time $t$ could be written as:

$$
V(S, b_i) = \max_{h_i} \left\{ \max_{b'_i} \left[ \ln \left( y_i h_i + b'_i - wh_i - \frac{b'}{R} \right) + \beta E_\theta V(S', b'_i) \right] \right\}
$$

where $E_\theta$ refers to the expectation of $\theta_{t+1}$ conditional on $\theta_t$ and $E_n$ refers to the unconditional expectation over all potential realizations of $n_i$. This difference resides in that $n_i$ does not exhibit any serial correlation, while $\theta_t$ does. Given the above, we can define a recursive competitive equilibrium as follows:
Definition 1 A Recursive Competitive Equilibrium consists of the following functions:

(a) A value function \( V_E(B, \theta, b') \) and decision rules \( c_i'(B, \theta, b') \), \( h_i'(B, \theta, b') \) and \( b_i'(B, \theta, b') \) for the entrepreneur

(b) A value function \( V_H(B, \theta, b) \) and decision rules \( c(B, \theta, b) \), \( h(B, \theta, b) \), and \( b'(B, \theta, b) \) for the household

(c) Price functions \( w(B, \theta) \) and \( R(B, \theta) \)

(d) A perceived law of motion for the aggregate state \( S' = \Phi(S) = \Phi(B, \theta) \)

such that:

(i) Given c) and d), a) solves the entrepreneur’s optimization problem

(ii) Given c) and d), b) solves the household’s optimization problem

(iii) All markets clear:

\[
\int c_i \, dF(i) + c_i^H = Y_t \quad \text{(Goods market)}
\]

\[
\int \phi_i b_i \, dF(i) = a h_i \quad \text{(Labor market)}
\]

\[
\int (b_i' - \frac{b_{i+1}'}{R_i}) \, dF(i) = b_i + \frac{b_{i+1}}{R_i} \quad \text{(Financial markets)}
\]

(iv) Perceptions about the aggregate states are correct

Implicit in the equilibrium’s definition is the presence of the end of period Walrasian market described above. In turn, agents may seek to maximize lifetime consumption, independently on the number of consumption goods they originally acquired.

Given the equilibrium definition above, the model’s solution is detailed below. Since the choice of labor \( h_i \) is made before the realization of the matching shock \( n_i \), but the saving decision is made after its observation, it will be convenient to define the entrepreneur’s wealth after production has taken place as:

\[
a_i = b_i + (\theta_i n_i - w_i) h_i
\]

Rewriting per-worker sales \( y_i' \) in terms of \( \theta_i \) and \( n_i' \), and Following Angeletos (2007) and Quadrini (2014), I state the following propositions.
Proposition 2 Define $\phi_t$ as the value that satisfies $E_n \left[ \frac{\hat{\theta}_t - w_t}{(\hat{\theta}_t - w_t)\phi_t + 1} \right] = 0$. Then the entrepreneur’s policy functions will take the form:

$$
\begin{align*}
    h_i^t &= \phi b_i^t \\
    c_i^t &= (1 - \beta) a_i^t \\
    b_i^{t+1} &= \beta R_t a_i^t
\end{align*}
$$

Note that the demand for labor will be linear in the entrepreneur’s wealth $b_i^t$. The factor of proportionality is time-varying, but common to all firms. In turn, the aggregate demand for labor can be obtained as:

$$
H_t = \phi_t \int_{i \in N} b_i^t = \phi_t B_t
$$

where $B_t$ denotes the average, per-capita level of wealth. As shown by Quadrini (2014), the factor of proportionality $\phi_t$ will depend negatively on the equilibrium wage rate. In turn, this implies that the aggregate demand of labor will depend negatively on the wage rate (as in any Walrasian model), but positively on the economy’s level of risk-less assets. For individual producers these assets represent a firm’s financial net worth. The corresponding theoretical proof can be found in Appendix 7.1.

The above is a unique feature of the model which sheds some light on the relationship between labor demand and the financial soundness of firms. When businesses’ net worth suffer (as it does during contractions), the demand for labor declines inducing a lower equilibrium output and employment. This happens not as a result of firms lacking the resources to hire employees, or because the value of their collateral has plummeted and access to financing options are scarce. This occurs purely out of risk considerations: with a lower net worth entrepreneurs cannot properly insure against idiosyncratic shocks and seek to reduce their exposure by limiting their hiring. In other words, given the fact that entrepreneurs are risk averse and cannot hedge their hiring bets appropriately, they choose to behave conservatively
and revise their production plans downwards. The opposite will happen in an expansion when a firm’s net worth improves. This a unique and a crucial feature of the model which will greatly affect the equilibrium dynamics as described in the next sections.

Another property worth mentioning is that an entrepreneur’s consumption policy function is linear in wealth. This has two major implications. First, it implies that entrepreneurs will always consume (and save) a constant proportions of their end of period assets. As such, during expansions entrepreneurs will not only seek to consume more but also to increase their stock of savings which will allow them to increase future production. Second, it makes the problem extremely tractable as it allows for linear aggregation. Consequently, even when entrepreneurs might be heterogeneous in asset holdings, in order to understand the aggregate dynamics we only need to keep track of the average level of wealth $B_t$.

**Proposition 3** *In a stationary equilibrium, households will exhaust their credit capacity as long as $\beta R < 1$*

Given that entrepreneurs are risk averse and face uninsurable idiosyncratic risks, they will constantly seek to self-insure. Their desire to smooth consumption would make them save and hold bonds even if $\beta R = 1$. Unfortunately for them, the supply of these assets is constrained by the borrowing limit of households. Being risk neutral and solely exposed to an aggregate shock, households need extra incentives to issue the risk free assets. In turn, in order to induce households to borrow the equilibrium interest must decline. As long as the interest rate is lower than the intertemporal discount rate, households will continue to increase their leverage until their borrowing limit binds setting the steady state interest rate lower than the intertemporal discount rate.
4 Quantitative Analysis

I calibrate parameter values of the model economy to match some relevant statistics from U.S. data. There are two sets of parameters. The first set of parameters is chosen externally without using model-generated data while the second set of parameters is determined jointly by minimizing the distance between the statistics from the model and the data.

The model period is a year, which corresponds to the data frequency obtained from Compustat. I set $\bar{y}$ to match the U.S. long-run capacity utilization measures of approximately eighty percent as reported by the Federal Reserve. Following Reichling and Whalen (2012), I set $\tau = 0.4$, implying a labor elasticity of 2.5. This number is in line with what is used and recommended by the U.S. Congressional Budget Office. The persistence of the aggregate financial shock is estimated as an AR(1) process from the survey of senior loan officers available since the second quarter of 1990. Further, I use customer traffic data to estimate $\sigma$ and set as $\mu = 0$ since the model features constant returns to scale and consequently $\mu$ will only have a scaling effect on the economy. The rest of the parameters ($\beta, \gamma, \Omega$) are calibrated to match the following steady state moments: U.S. long run interest rate of 3%, hours worked = 1/3 and the ratio of unsecured credit to disposable income as reported by Herkenhoff (2013). Table 2 below summarizes this information.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.959</td>
<td>Interest rate $r = 3%$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Disutility of labor</td>
<td>1.11</td>
<td>Hours worked = 1/3</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Borrowing limit</td>
<td>0.122</td>
<td>Unsecured Credit/ Income = 0.4</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Inv. Frisch elasticity</td>
<td>0.40</td>
<td>CBO estimate (2012)</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>Parameter of matching function</td>
<td>$-\frac{1}{2}\sigma^2_n$</td>
<td>Consumer Traffic data</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Parameter of matching function</td>
<td>0.17</td>
<td>Consumer Traffic data</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence of credit shock</td>
<td>0.884</td>
<td>FRB Senior Loan Officer Survey</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Stdev of credit shock</td>
<td>0.008</td>
<td>FRB Senior Loan Officer Survey</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>Maximum output per worker</td>
<td>0.902</td>
<td>FRB U.S. Capacity utilization rate</td>
</tr>
</tbody>
</table>
4.1 ShopperTrak data

Paramount to the study’s analysis is an understanding of the distribution of customers that firms will care for every period. Proprietary ShopperTrak data was used to gain such an insight. ShopperTrak is a multinational corporation specialized in the measurement of consumer traffic flow. The company utilizes electronic traffic counters (ETC) to quantify and monitor customer movements inside as well as in and out stores. ShopperTrak’s proprietary technology allows their clients to better understand consumer patterns and manage their resources more effectively. Currently, the company has some fifty thousands ETC devices installed only in North America and about seventy thousand world wide.

The company has furnished a dataset containing proprietary consumer traffic information for almost one thousand stores all of which are located inside the United States. The data is annual and encompasses a total of four years (2010-2013). The information is geographically diversified with all fifty U.S. states being represented. Because of privacy considerations the actual brands included in the sample were not disclosed, but an anonymous numeric-identifier allows individual stores to be tracked over time. All in all the dataset forms a balanced panel with a total of 3,840 observations. Table 3 lists the key relevant statistics.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer traffic cross-sectional dispersion</td>
<td>1.763</td>
</tr>
<tr>
<td>Consumer traffic cross-sectional skewness</td>
<td>-0.307</td>
</tr>
<tr>
<td>Consumer traffic cross-sectional kurtosis</td>
<td>1.257</td>
</tr>
<tr>
<td>Consumer traffic growth rate corr w/ cycle</td>
<td>0.392**</td>
</tr>
<tr>
<td># observations (ave. per year)</td>
<td>985</td>
</tr>
<tr>
<td># observations (total)</td>
<td>3,840</td>
</tr>
</tbody>
</table>

Source: Own calculations based on ShopperTrak data

As one can see from the table, consumer traffic seems to be pro-cyclical. The data also reveals that the cross-sectional distribution across the U.S. is highly asymmetrical and right skewed, implying that only a handful of stores receive a high volume of customers.

\(^{2}\)See appendix for further details on ShopperTrak and ETCs.
5 Results

In this section I analyze the quantitative implications of the model. First, I showcase the model’s ability to successfully match some broad features of the Compustat data. Second, I describe how a sudden change in aggregate credit conditions may affect the model’s equilibrium values. Third, I decompose and quantify the contribution of endogenous uncertainty to the macroeconomic effects of a first moment disturbance hitting the economy.

5.1 General Results

Table 4 below reports some fundamental simulation results. The basic strategy was to calibrate the model utilizing steady state moments and then validating the framework with non-targeted ones at the business cycle frequency\(^3\). Overall the framework does a good job in matching all three targeted steady state moments.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State interest rate</td>
<td>0.030</td>
<td>0.033</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.333</td>
<td>0.324</td>
</tr>
<tr>
<td>Unsecured debt/ Income</td>
<td>0.400</td>
<td>0.397</td>
</tr>
</tbody>
</table>

In addition, the model can successfully replicate several non-targeted moments. Table 5 summarizes some of these results. In the data, both the growth rates of employment and sales are counter cyclical. This empirical regularity has been often documented by other researchers using different data sets. For example, [Bachmann and Bayer (2013)](Bachmann2013) report similar results for Germany using USTAN data. The model generates the right business cycle co-movement as an improvement in credit conditions induces firms to raise their sales forecasts and consequently

\(^3\)Since the framework has a closed form solution, I’m implicitly assuming that the steady state moments are equal to the model’s ergodic mean; something which in principle is only assured for linearized models. In turn, I perform a consistency check which can be found in the appendix.
increase their hiring. As output rises, a greater share of firms begin producing at their maximum capacity $\bar{g}$, triggering the observed dropped in cross-sectional dispersion. Furthermore, in the data the correlation with the business cycle of sales growth dispersion is stronger than that of employment. This quantitative feature is also correctly matched by the model as sales dispersion tends to evolve faster than employment.

Also interesting is the model’s ability to match higher order moments such as the cross-sectional dispersions and kurtosis. In the data both sales and employment are negatively skewed and simulations of the model are able to reproduce these empirical regularities. In particular, the model’s ergodic distribution of sales has a negative skewness of -0.272 while that of employment of -0.222. While the model does slightly overstate the degree of asymmetry in the data, it does quantitatively match the fact that sales exhibit higher skewness than employment.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional dispersion of employment</td>
<td>2.416</td>
<td>1.675</td>
</tr>
<tr>
<td>Cross-sectional dispersion of sales</td>
<td>2.227</td>
<td>1.231</td>
</tr>
<tr>
<td>Sales growth rate dispersion correl w/ cycle</td>
<td>-0.388</td>
<td>-0.622</td>
</tr>
<tr>
<td>Emp growth rate dispersion correl w/ cycle</td>
<td>-0.248</td>
<td>-0.586</td>
</tr>
<tr>
<td>Employment’s cross-sectional skewness</td>
<td>-0.131</td>
<td>-0.222</td>
</tr>
<tr>
<td>Sale’s cross-sectional skewness</td>
<td>-0.187</td>
<td>-0.272</td>
</tr>
<tr>
<td>Employment’s cross-sectional kurtosis</td>
<td>2.484</td>
<td>1.705</td>
</tr>
<tr>
<td>Sales’s cross-sectional kurtosis</td>
<td>2.236</td>
<td>1.932</td>
</tr>
</tbody>
</table>

In terms of kurtosis, both variables show evidence of heavy tails and peakedness relative to a Gaussian distribution. The model’s baseline specification successfully reproduces this positive kurtosis for the cross-sectional distributions of both employment and sales, although it somewhat understates them both.
5.2 Model Dynamics

In addition to the model’s steady state properties, its dynamic features were also studied. Figure 8 plots the response of output, consumption, employment, wages, interest rates, employment dispersion and capacity utilization to a one percent positive increase in aggregate credit conditions. Upon impact both output and employment rise. Output does so by almost 0.8 percent, while employment’s reaction is slightly weaker at 0.6 percent from its steady state value.

Higher production and higher employment puts upward pressure on the real wage which increases over a quarter of a percent and remains above its steady state value for about fifteen periods. Similarly, capacity utilization rates rise sharply as firms update their production plans to meet the expected growth in demand for consumption goods. The rise in capacity utilization more than doubles that of the original aggregate shock that propitiated it. In turn, this pushes a greater share of firms to produce at their maximum per-worker level \( \bar{y} \), generating a significant drop in employment growth dispersion in excess of six percent.

The increase in firms’ profits propitiates a spike in the demand for safe assets as risk averse entrepreneurs seek to protect themselves from idiosyncratic uncertainty. The supply of these assets is, nonetheless, constrained by the leverage capacity of the representative household. A shift in demand coupled with an inelastic supply induce the price on these assets to rise. Consequently the return on bonds falls as may be seen in the impulse response function below. In particular, the equilibrium interest rate falls close to 0.20 percent from its steady state value.

Lastly, there are the effects on consumption. Improved aggregate credit conditions foster entrepreneur’s profits allowing them to enlarge their demand for consumption goods. Moreover, household’s also increase their equilibrium consumption allocation which depends on their labor income net of debt payments. Given that both wages and hours worked are rising, this increases the household’s revenue. Additionally, interest rates are falling, so their payment liability is decreased. A combination of higher incomes and lower interests allows
the household to enlarge its consumption of goods even beyond what the entrepreneur can. Household’s consumption rises close to 0.8 percent from its original steady state value.

![Graphs showing impulse responses for a 1% shock to $\theta_t$.](image)

**Figure 8:** Impulse responses for a 1% shock to $\theta_t$
5.3 Effect Decomposition

Figure 8 illustrates how changes in an economy’s credit conditions will have a direct impact on the equilibrium values of its macroeconomics aggregates. The dynamic effects discussed above can be thought of having two principal components. The first one is caused by the effects of the change in the mean of the disturbance hitting the economy. There is, however, an additional indirect channel which will condition the magnitude of the final outcome.

Assume for example a positive realization of $\theta_t$. As credit conditions improve, the maximum number of clients that each worker can serve ($\bar{n}$) will decrease. Since each customer is able to command a greater amount of resources, this effectively reduces the probability that the entrepreneur will not receive enough clients to become profitable, thus minimizing the risk of hiring workers for any level of assets $b_i$. Statistically, the censoring point shifts to the left propitiating a redistribution of probability mass which makes it more likely for firms to hit their per worker maximum capacity. Fig 9 helps illustrates these distributional changes.

![Figure 9: Time varying uncertainty](image)

As entrepreneurs’ profitability odds improve, they will revise production plans upwards and seek to expand hiring accordingly. In turn, this channel amplifies the initial expansion in economic activity generated by an increase in $\theta_t$. The opposite happens for a deterioration of credit conditions, which generates an initial economic contraction. As the number of
maximum clients rises, the odds of a potential loss does too making risk averse entrepreneurs reduce their hiring plans. Through the eyes of the model, this is the effect of endogenous time-varying uncertainty. Figures 10 and 11 decompose the overall effect of a one percent rise in $\theta_t$ into a level and a uncertainty effect. On average, about 22 percent of the overall effect generated by a change in credit conditions can be explained by changes in time-varying uncertainty.

Figure 10: Effect decomposition for a 1% shock to $\theta_t$
Figure 11: Effect decomposition for a 1% shock to $\theta_t$
5.4 Sensitivity Analysis

This section explores the sensitivity of the above reported results to variations in some of
the model’s key parameters. Only a few select examples are reported here. The rest of the
analysis can be found in the Appendix 6.

5.4.1 The effects of tau

Intrinsically linked to the response of employment supply to variations in credit condi-
tions, the parameter $\tau$ plays an important role in the overall dynamics of the model. Trade-off
between fluctuations in the real wage and the labor supply will condition the response of
output in the economy. Table 6 below explores the responses of the model to variations in
the parameter governing the labor supply of households. In each case, the model was recal-
ibrated utilizing the same targets specified in section 4, but each time with a particular value
for $\tau$. The tabulated results are the parameter values as well as the response (on impact) of
employment and real wages to a one percent improvement in credit conditions measured as
a percent deviation from their steady state values.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>CR</th>
<th>Employment</th>
<th>Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.111</td>
<td>0.9596593</td>
<td>0.1217439</td>
<td>0.73%</td>
<td>0.21%</td>
</tr>
<tr>
<td>0.4</td>
<td>1.384</td>
<td>0.9596594</td>
<td>0.1217433</td>
<td>0.61%</td>
<td>0.23%</td>
</tr>
<tr>
<td>0.5</td>
<td>1.545</td>
<td>0.9596590</td>
<td>0.1217432</td>
<td>0.53%</td>
<td>0.24%</td>
</tr>
<tr>
<td>0.7</td>
<td>1.925</td>
<td>0.9596593</td>
<td>0.1217436</td>
<td>0.32%</td>
<td>0.37%</td>
</tr>
<tr>
<td>1.0</td>
<td>2.677</td>
<td>0.9596592</td>
<td>0.1217433</td>
<td>0.15%</td>
<td>0.42%</td>
</tr>
</tbody>
</table>

As expected, the response of employment to a positive credit shock becomes stronger as
the Frisch elasticity of labor supply increases. Similarly the lower the value of $\tau$, the weaker
the response of the real wage. In other words, as $\tau$ decreases and the labor supply becomes
more elastic, the equilibrium wage becomes less sensitive to changes in credit conditions.
5.4.2 The effects of capacity utilization

The rate at which productive capacity is being used in the economy will be an important factor governing its cross-sectional dynamics. Interestingly, the lower the steady state capacity utilization rate, the greater the share of firms that will benefit from an improvement in credit conditions and yet the worse the mean entrepreneur might end up being.

This happens because as credit expands and firms seek to increase their production, the increase in labor demand puts pressure on the equilibrium wage. For those firms operating below $\bar{y}$ this increase in cost is still profit maximizing. Yet, for those already operating at their maximum capacity, this increase represents a dent on their profits. Moreover, the greater the share of firms initially operating below full capacity, the greater the increase in labor demand and hence the greater the rise real wages. As firm’s operating costs rise, the mean entrepreneur’s profit falls and so does his equilibrium consumption. Figure [12] plots the model’s dynamic response to a one percent increase in $\theta_t$, starting from a steady capacity utilization rate of sixty percent.

On impact, there is a fifty percent stronger response of output than that described on Figure [8]. In line with this reaction, equilibrium employment also rises more than in the baseline specification. Similarly, employment dispersion drops as capacity utilization rates increase in the economy. Overall most variables’ response seems consistent with the baseline results.

In terms of consumption, however, things change substantially. Since labor costs rise sharply for all firms, the mean entrepreneur’s profit will fall. With less claims on final production their consumption drops slightly, about 0.2 percent from steady state. With less profits, entrepreneurs reduce their appetite for savings, causing bond prices to drop and consequently inducing a rise in its yield. Households, on the other hand, are benefited by the strong increase in wages and hours, although higher interest rates will act like a dent on their available resources. Consequently their consumption rises, but less than in the baseline specification.
Figure 12: Effect decomposition for a 1% shock to $\theta_t$
5.5 Extension: Persistent Demand Shocks

One of the assumptions present in the model’s baseline specification was that the idiosyncratic disturbances faced by entrepreneurs presented no serial correlation. This afforded us a tractable and intuitive closed form solution. However, there are reasons to believe that demand fluctuations may in fact experience certain dependence over time.

![Figure 13: Consumer Traffic and Business Cycle](image)

**Source**: Own calculations based on ICSC data

To gain a deeper understanding of this assumption I utilize the consumer traffic diffusion index from the International Consortium of Shopping Centers (ICSC). The diffusion index is produced monthly by the ICSC from a survey of consumer traffic reported by shopping center’s executives. Readings over 50 imply a general positive momentum in the number of customers visiting shopping centers, while readings below 50 hint of a slowdown. The advantages of this data series over the ShopperTrak data is that it is available for a longer time horizon. The disadvantage, however, is that since it constitutes an aggregated index, individual stores cannot be tracked across time. Figure 13 plots this alternative measure of consumer traffic.
Figure 13 suggests that consumer traffic is pro-cyclical and highly persistent. Even when
there is only enough data to capture two U.S. recessions, both the 2001 and 2008 downturns
appear clearly visible. Not surprisingly the drop in consumer traffic related to the 2008 depression
appears to be deeper and longer-lasting than that of 2001. Furthermore, the process appears to be persistent, with increases and decreases in consumer traffic lasting several months.

With this in mind, this section seeks to extend the baseline specification by including
correlated idiosyncratic disturbances. In turn I relax the original assumption and investigate
the implications and overall performance changes of the framework presented in section 3
when disturbances are serially correlated. In particular, assume now that the distribution of
customers per worker arriving to a store follows:

\[ \ln n_i^t = \rho \ln n_i^{t-1} + \psi_t : \quad \psi_t \sim N(\mu_n, \sigma_n^2) \]

where the persistence parameter for the customer traffic process is estimated utilizing
ICSC data.

Let \( S_t = \{\theta_t, B_t\} \) represent the economy’s aggregate states. The household’s optimization
problem does not change. However, the entrepreneur’s recursive formulation would now be:

\[
V(S, b^i, n^i) = \max_h E_n \left\{ \max_{b'} \left[ \ln \left( y^i h^i + b^i - wh^i - \frac{b'^i}{R} \right) + \beta E_S V(S', b'^i, n'^i) \right] \right\}
\]

where \( E_n \) refers to the expectation of \( n_{i+1}^j \) conditional on the current realization of \( n_i^j \) and \( E_S \)
represents the equivalent conditional expectation for \( S_t \). Since the model loses its closed form
solution I solve it by performing a linear approximation around the agent’s policy function
following Covas (2006). Results are described in the table 7.
As can be seen from table 7, both the sales and employment dispersion growth rates appear countercyclical in all model specifications. This is in line with the data and implies that the model’s results are qualitatively robust. Quantitatively, there is also a slight improvement in the model’s capacity to match the data. The inclusion of serially correlated customer traffic substantially improves the model’s effectiveness at matching the desired moments.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Baseline</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional dispersion of employment</td>
<td>2.416</td>
<td>1.675</td>
<td>1.832</td>
</tr>
<tr>
<td>Cross-sectional dispersion of sales</td>
<td>2.227</td>
<td>1.231</td>
<td>1.515</td>
</tr>
<tr>
<td>Sales growth rate dispersion correl w/ cycle</td>
<td>-0.388</td>
<td>-0.622</td>
<td>-0.533</td>
</tr>
<tr>
<td>Emp growth rate dispersion correl w/ cycle</td>
<td>-0.248</td>
<td>-0.586</td>
<td>-0.397</td>
</tr>
<tr>
<td>Employment’s cross-sectional skewness</td>
<td>-0.131</td>
<td>-0.222</td>
<td>-0.171</td>
</tr>
<tr>
<td>Sale’s cross-sectional skewness</td>
<td>-0.187</td>
<td>-0.272</td>
<td>-0.232</td>
</tr>
<tr>
<td>Employment’s cross-sectional kurtosis</td>
<td>2.484</td>
<td>1.705</td>
<td>1.811</td>
</tr>
<tr>
<td>Sale’s cross-sectional kurtosis</td>
<td>2.236</td>
<td>1.932</td>
<td>2.052</td>
</tr>
</tbody>
</table>
6 Conclusion

In this study I have investigated the effects of fluctuations in uncertainty on aggregate economic activity. In particular, I have done so contemplating the hypothesis that changes in uncertainty are endogenous to the current state of the economy. The paper develops a general equilibrium incomplete markets framework with heterogeneous firms that account for the asymmetric fluctuations of the U.S. labor market and output. The fundamental property of the model is that expansions and contractions in the economy are initiated by shifts in aggregate credit conditions and these, in turn, may induce changes in uncertainty.

The model generates realistic volatility in aggregate employment and output. Moreover, I have found that endogenous fluctuations in uncertainty may significantly amplify the real effects of first moment shocks. The uncertainty channel is shown to be able to propagate approximately thirty percent of a level’s shock initial effect. The model also predicts that the level of uncertainty varies with the business cycle. This is in line with what has been documented for the U.S. where every measure of uncertainty systematically falls in expansions and rises during recessions.

I have also found that aggregate fluctuations will have effects on the cross-sectional dispersion of output and employment. This highlights the importance of taking into account the risk tolerance of individual producers which is often washed away in aggregate figures. Results confirm that the proper understanding of business cycles requires knowledge of the cross-sectional distributions as well as the aggregate time-series. There is need for theories that can explain not just the mean variation of consumption, output, and employment, but also why the distribution of firm behavior changes considerably over the cycle and how this may (or may not) matter in determining the amplitude of the cycle and the process of job creation and destruction.

There are several extensions that might be useful to consider. The first one would be to add capital to the framework. This would allow the model to provide insights into fluctuations in
investment, which is usually a more fundamental contributor to business cycle dynamics than employment. Moreover, it could also shed light to the relationship between uncertainty and asset allocation. Under a set-up with capital, the entrepreneur would now have two instruments in which to save one yielding a safe but low return, and another one yielding a more risky yet potentially more rewarding alternative.

Further, there are two potentially interesting extensions regarding the effects of uncertainty on nominal variables. First, since the model is written in real terms, there is no explicit role for monetary assets. Adding money to the study’s framework would allow for the exploration of the effects of fluctuations in uncertainty on nominal shocks, as well as its effect on the role of monetary policy. Additionally, the model could provide insights into the effectiveness of monetary policy at different levels of economic uncertainty throughout the business cycle. These extensions are left for future research efforts.
Appendix

7.1 Omitted Theoretical Proofs

Proposition 1. Individual labor demand is linear in financial wealth \((b^i_t)\), while consumption and savings are linear in total assets \((a^i_t)\):

\[
\begin{align*}
    h^i_t &= \phi^i b^i_t \\
    b^i_{t+1} &= R^i \beta a^i_t \\
    c^i_t &= (1 - \beta) a^i_t
\end{align*}
\]

Proof Proposition 1.

The recursive formulation of the entrepreneur’s problem presented in section 3.5 can also be written in terms of the information available to the agent at the time of making a decision.

In turn, I define the following two stages or sub-problems:

Stage I:

\[
V_t(\theta_t, B_t, b^i_t) = \max_{h^i_t} \hat{V}_t(\theta_t, B_t, a^i_t)
\]

\[
\text{s.t. : } a^i_t = (\theta_t n^i_t - w_t) h^i_t + b^i_t
\]

Stage II:

\[
\hat{V}_t(\theta_t, B_t, a^i_t) = \max_{c^i_t} \left[ \ln c^i_t + \beta E_{\theta_{t+1}} V_{t+1}(\theta_{t+1}, B_{t+1}, b^i_{t+1}) \right]
\]

\[
\text{s.t. : } a^i_t \geq c^i_t + \frac{b^i_{t+1}}{R_t}
\]

where \(E_{\theta_{t+1}}\) stands for the expectation of \(\theta_{t+1}\) conditional on the realization of \(\theta_t\).

In stage I the entrepreneur chooses its labor inputs aware of the extent of credit conditions, yet uncertain about the level of demand that he will receive that period. In stage II, the entrepreneur observes the realization of \(n^i_t\) and allocates the end of period wealth between consumption and savings. The stage I first order condition is:
\[ \frac{\delta V_i}{\delta h_t^i} \iff E_{n_t} \left[ \frac{\delta \hat{V}_t}{\delta a_t^i} \frac{\delta a_t^i}{\delta h_t^i} \right] = 0 \]

The envelope condition \( \delta \hat{V}_t / \delta a_t^i = 1 / c_t^i \) is derived and then used in the expression above to yield:

\[ \frac{\delta V_i}{\delta h_t^i} \iff E_{n_t} \left[ \frac{\theta_t n_t^i - w_t}{c_t^i} \right] = 0 \]

The stage II first order condition is:

\[ \frac{\delta \hat{V}_t}{\delta c_t^i} = 0 \iff \frac{1}{c_t^i} + \beta E_{n_{t+1}} \left[ \frac{\delta V_{t+1}}{\delta b_{t+1}^i} (-R_t) \right] = 0 \]

Substituting the relevant envelope condition, and denoting \( E_t \) as conditional expectation given the information set at time \( t \) yields the following Euler equation:

\[ \frac{1}{c_t^i} = \beta E_t R_t \left( \frac{1}{c_{t+1}^i} \right) \]

Next I prove Proposition 1 following a guess-and-verify approach. Begin by guessing the following policy functions:

\[ h_t^i = \phi_t b_t^i \quad (1) \]

\[ b_{t+1}^i = R_t \beta a_t^i \quad (2) \]

Replacing (2) in the stage II budget constraint

\[ \dot{c}_t^i = a_t^i - \frac{b_{t+1}^i}{R_t} \quad (3) \]

yields the policy function for consumption:

\[ \dot{c}_t^i = (1 - \beta) a_t^i \quad (4) \]

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From the Euler equation (FOC of stage II) we have that

\[
\frac{1}{c_i^t} = \beta R_t E_t \left( \frac{1}{c_{i+1}^t} \right)
\]

\[
\implies \frac{1}{a_i^t} = \beta R_t E_t \left( \frac{1}{a_{i+1}^t} \right)
\]

(5)

Combining the definition of \( a_i^t \) and (1) yields

\[
a_{i+1}^t = \left[ (\theta_{t+1} n_{t+1} - w_{t+1}) \phi_{t+1} + 1 \right] b_{i+1}^t
\]

(6)

which implies that (5) can be written as:

\[
\frac{1}{a_i^t} = \left( \frac{\beta R_t}{b_{i+1}^t} \right) E_t \left( \frac{1}{1 + (\theta_{t+1} n_{t+1} - w_{t+1}) \phi_{t+1}} \right)
\]

\[
\implies 1 = E_t \left( \frac{1}{1 + (\theta_{t+1} n_{t+1} - w_{t+1}) \phi_{t+1}} \right)
\]

(7)

For the proof to be complete I need to verify that (7) satisfies the problem’s FOCs:

\[
E_t \left[ \frac{\theta_t n_t^i - w_t}{(\theta_t n_t^i - w_t) \phi_t + 1} \right] = 0
\]

(8)

In turn, from (7)

\[
E_t \left[ \frac{1}{1 + (\theta_t n_t^i - w_t) \phi_t} \right] - 1 = 0
\]

(9)

\[
\implies E_t \left[ \frac{1 - 1 - (\theta_t n_t^i - w_t) \phi}{1 + (\theta_t n_t^i - w_t) \phi_t} \right] = 0
\]

\[
\implies (-\phi) E_t \left[ \frac{\theta_t n_t^i - w_t}{1 + (\theta_t n_t^i - w_t) \phi_t} \right] = 0
\]

\[
\implies E_t \left[ \frac{\theta_t n_t^i - w_t}{(\theta_t n_t^i - w_t) \phi_t + 1} \right] = 0
\]

(10)

which satisfies (8).
7.2 Aggregate Measures

For this economy, aggregate real income will equal the profits of the entrepreneurs and the labor income of the representative household. In turn:

\[ Y_t = \int (y_i^t - w_t) h_i^t \ dF(i) + w_t h_t \]

\[ = \int y_i^t h_i^t \ dF(i) - \int w_i h_i^t \ dF(i) + w_t h_t \]

\[ = \int y_i^t h_i^t \ dF(i) \]  \hspace{1cm} (11)

In terms of real consumption:

\[ c_t^E dF(i) = (y_i^t - w_t) h_i^t + b_t^e - \frac{b_{t+1}^e}{R_t} \]

\[ \Rightarrow \int c_t^E dF(i) = \int (y_i^t - w_t) h_i^t dF(i) + \int b_t^e dF(i) - \int \frac{b_{t+1}^e}{R_t} dF(i) \]

This implies that the aggregate consumption of entrepreneurs can be written as:

\[ C_t^E = Y_t - w_t h_t + b_t^e - \frac{b_{t+1}^e}{R_t} \]

and aggregate consumption of the households as

\[ C_t^H = w_t h_t + \frac{b_{t+1}^H}{R_t} - b_t^H \]

Hence total consumption in the economy would be equal to:

\[ C_t^E + C_t^H = Y_t - w_t h_t + b_t^e - \frac{b_{t+1}^e}{R_t} + w_t h_t + \frac{b_{t+1}^H}{R_t} - b_t^H \]

\[ = Y_t + b_t^e - \frac{b_{t+1}^e}{R_t} + \frac{b_{t+1}^H}{R_t} - b_t^H \]

\[ = Y_t \]

which is the total income/production described by expression [11].
Figure 14: Disagreement amongst professional forecasters. The figure above plots the cross sectional dispersion in private sector forecasts over the business cycle. The data comes from the Federal Reserve Bank of Philadelphia’s survey of professional forecasters from 1968Q4 - 2014Q3 for the first four variables and 1981Q3 - 2014Q3 for the remaining two. Beginning from top left we have the forecasts for Real GDP, the Price Deflator, Industrial Production, the Unemployment rate, Real Consumption and Non-residential fixed investment. In times of higher uncertainty forecasts become less precise and dispersion amongst predictions increases. Not surprisingly, recessions tend to be periods of greatest disagreement amongst forecasters.
7.4 Evidence of Corporate Lending

In the framework introduced in Section 3, resources would, in equilibrium, flow from the entrepreneurs to the households sector. At first, this result might seem like an odd feature of the model. However, in the U.S., the private corporate sector has been a net lender since the beginning of the 2000s as seen in figure 15. The only exception to date has been the year 2008 at the height of the Big Recession, when the financial assets held by most corporations dropped in value.

![Graph showing net financial assets in the nonfinancial business sector as a percentage of total nonfinancial assets. Source: Federal Reserve Flow of Funds Report.](image)

Figure 15: Net Financial Assets in the nonfinancial business sector as a percentage of total nonfinancial assets. **Source:** Federal Reserve Flow of Funds Report.

Interestingly, the reversal from net borrower to lender has so far only occurred in the U.S. Corporate sector, and not in the Noncorporate one. The evidence reported on the figure above shows that a large fraction of the business sector is self-financing and no longer dependent on outside sources. And even when the aggregate figures may mask some firm level heterogeneity, they do paint a general picture of the evolution of the overall trend across time.
7.5 About ShopperTrak data

Founded in 1989 and headquartered in Chicago, Illinois; ShopperTrak Corporation is the world’s largest retail traffic counter. The company provides shopper insights and analytics solutions to improve retail profitability and effectiveness. ShopperTrak helps companies identify, understand, and maximize their total shopper conversion rate (the percentage of shoppers who actually purchase something) and improves store performance through shopper behavior insights. It also helps retailers with solutions for store traffic counting, interior analytics, and industry benchmarking; and provides unique data benchmark tools that help retailers understand their performance in context of the market. ShopperTrak is the leader in its industry and the only one to provide an end-to-end service: from device installation to data analysis. The company serves major brands, retailers, mall owners, and financial institutions. Table 8 provides a small selection of its customer base.

ShopperTrak utilizes proprietary technology to analyze and monitor customer traffic. Its fifth-generation device, called the Orbit, is smart enough to detect shoppers who enter side-by-side or in groups, distinguish children from adults and ignore shopping carts or strollers. Figure 16 provides a few examples of this technology. All images were taken from local stores in Santa Monica, CA.

<table>
<thead>
<tr>
<th>Apparel</th>
<th>Home Improvement</th>
<th>Technology</th>
<th>Food</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAP Inc. (Old Navy, GAP, BR)</td>
<td>Home Depot Lowe’s</td>
<td>Apple Inc.</td>
<td>Godiva</td>
</tr>
<tr>
<td>Crocs</td>
<td></td>
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<tr>
<td>Victoria’s Secret</td>
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<td>Payless Shoes</td>
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<tr>
<td>American Eagle Outfitters</td>
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<tr>
<td>J. Crew</td>
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<tr>
<td>Thomas Sabo</td>
<td></td>
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</tbody>
</table>

Source: ShopperTrak’s website and specialized press
Figure 16: ShopperTrak’s technology
References


