Shopping for Information: Consumer Learning with Optimal Pricing and Product Design

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Abstract

I study a monopolistic pricing problem in which the consumer performs product research to determine whether or not to purchase the good. The consumer receives a signal of quality via a Brownian motion process with a type-dependent drift. I fully characterize the consumer’s optimal strategy; she buys the product when she is sufficiently optimistic about the quality and ceases to pay for the signal when she is sufficiently pessimistic. I examine the implications of this behavior for the seller’s optimal pricing decision. I find that the seller prefers to encourage product research when quality is likely to be high and prefers to discourage research when quality is likely to be low. I show that lowering search costs or increasing the quality of information can either raise or lower equilibrium price. I also extend the model so that the seller chooses both price and the level of quality dispersion and demonstrate that the optimal level of dispersion need not be extremal.

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1 Introduction

Consumers often acquire product information in order to make a purchase decision. This can include everything from reading product reviews on websites like Amazon, to looking at quality reviews such as Consumer Reports, to getting advice from friends, to trying the product out in

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the store. Often, this search for information is an effort to buy a high-quality product and avoid purchasing a low-quality product. The internet has increased access to information, which makes product research even more prominent and important to understand. According to “The 2011 Social Shopping Survey” by PowerReviews and the e-tailing group, 50% of consumers spend at least 75% of their total shopping time performing online product research, as opposed to just 21% of consumers in 2010. In fact, 15% of people spend 90% or more of their shopping time on research. The incentive to gather information in order to avoid purchasing a low-quality good grows stronger the more important or expensive a good is. A 2013 survey by GE Capital Retail Bank considers items valued at $500 or more, and finds that not only do 81% of consumers do online research, but that they spend an average of 79 days gathering information before making a purchase. A 2016 survey from Ipsos and Zillow indicates how this may translate into actual time spent by asking how many hours of research consumers do before buying. On average, people spend 26 hours for a home, 11 for a car, 5 for a computer, and 4 for a cell phone or tablet.

I study an optimal pricing problem in the presence of consumer product research. A monopolist has a single good for sale which is of either low or high quality. He can choose price and possibly some element of product design before the consumer acts. The consumer begins with a belief that the product is of high quality, which is influenced by brand reputation or advertising. She can choose to buy the product at any time for the posted price, walk away without purchasing the good, or pay to receive some (additional) signal of quality. She receives information via an arithmetic Brownian motion process, where a high-quality good sends better signals, on average. The consumer can collect as much information as she desires before making a purchase decision.

I fully characterize the consumer’s optimal information-acquisition strategy, which is an optimal stopping rule in the belief space. More specifically, the buyer sets upper and lower belief boundaries, both of which are absorbing and time invariant. If good signals buoy her belief to the upper boundary, she will buy the good at the posted price, and if bad signals reduce her belief to the lower boundary, she will discontinue search without purchasing the good. The consumer’s optimal strategy is therefore to have an interval of beliefs over which she does additional product research.

I investigate the implications of the consumer’s product research strategy for the seller’s optimal pricing decision. It is clear that if the seller posts a sufficiently high price, the consumer will never buy his product, and if he posts a sufficiently low price, the consumer will buy the product immediately without search. It is unclear, however, under which circumstances the monopolist would prefer the consumer to gather additional information and under which ones he would prefer immediate sale. This is due to his tradeoff between price and probability of sale. With a low enough price, the monopolist can ensure purchase, but if the price is high enough for the consumer to choose to gather information, a series of bad signals may result in no sale.

I show that if the consumer is sufficiently optimistic about product quality, the seller induces
product research by posting a high price, while if the consumer is pessimistic, he is more likely to post a low price and sell the good immediately. When belief about quality is high, the seller is optimistic that the signal will reveal that the good is truly of high quality, raising the buyer’s willingness to pay, without running too high of a risk that she will walk away without purchasing. When belief is low, however, the seller expects that if the consumer receives the signal, it will reveal his product to be of low quality. He therefore wishes to sell right away to avoid the risk of the consumer not buying at all. I also show a similar result regarding the cost of search and the quality of information. If search cost is below some threshold, or information quality is above some threshold, then inducing search is always the optimal strategy for the seller.

The above result is in direct contrast to the existing one in the literature. Branco et al. (2012) analyze a model in which consumers search for information about a product with many attributes. They can pay a search cost to discover if each one of the unknown attributes has a positive or negative effect on their values, and use an optimal stopping rule to decide when to purchase. In this case, new information affects value directly, rather than being a signal or underlying quality as in my model. Branco et al. find that the seller prefers to encourage search when the initial value is low and sell immediately when initial value is high. This drastic difference in outcomes is due to the fact that in my model, the seller has an indication of what the signal will reveal, whereas with private values, new information is equally likely to increase or decrease value. Therefore, in their model, the monopolist prefers to capitalize on high current values by selling right away.

Additionally, I analyze the effects of decreasing search costs (raising information quality) on the seller’s optimal pricing decision. Intuitively, as search becomes less expensive, the consumer will do more of it, which will increase the information rent she receives from the seller, and decrease price. This reasoning holds when it is optimal for the seller to post a low price and sell immediately. If the seller prefers search, however, I show that it is more likely that a decrease in cost will cause the seller to raise price. When the monopolist desires the consumer to acquire information, if search becomes easier or more appealing, he must raise the price in order to keep incentives balanced and continue to induce product research.

I also study the implications of the consumer’s product research strategy on the seller’s optimal choice of product design. Following the literature, I assume that the seller can choose the dispersion of product quality as well as price. This aspect of product design is another tool that the seller can use to encourage or discourage search by making it more or less appealing. I show that if the firm wishes to sell the good immediately, it posts a low price and prefers no dispersion of product

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1This baseline model is extended by Ke et al. (2015) and Branco et al. (2015) to incorporate two products and the choice of information quality, respectively.

2I also consider the effects of an increase in prior belief on equilibrium price, and find that price may be discontinuous in belief. When belief becomes large enough for the seller to prefer product research, there is a discrete jump upwards.
quality. In contrast, if the seller prefers consumer search, he posts a relatively high price and desires an intermediate level of dispersion.

The seller’s potential desire to choose an interior level of quality dispersion sharply contrasts a common result in the literature that an extremal level of dispersion is always optimal. Previous research (e.g. Johnson and Myatt (2006) and Bar-Isaac et al. (2012)) finds that the seller’s optimal level of dispersion is always either as much or as little as possible. Both Johnson and Myatt (2006) and Bar-Isaac et al. (2012) consider consumers whose willingnesses to pay are drawn from a distribution, and the monopolist can control the dispersion of that distribution as well as price. In my model, a firm wishing to sell immediately still desires extreme dispersion. If the seller desires the consumer to gather additional information, however, the optimal level of dispersion is interior. How product research drives this difference will be discussed in more detail in Section 6.

The analysis of the consumer’s problem draws on the literature concerning experimentation with an underlying state. Making use of the real options frameworks of McDonald and Siegel (1986) and Dixit and Pindyck (1994), the literature largely follows Chernoff (1972), who was the first to study the problem of learning about an unknown but constant drift of a Brownian motion process, using the current belief as a sufficient statistic for the history of accumulated information. Others extend this method, including Bernardo and Chowdhry (2002) and Felli and Harris (1996) who study investment and productivity problems, and Bergemann and Välimäki (2000) and Bolton and Harris (1999) who consider social learning of a group of agents, each receiving different signals.

The most related paper from the real options literature is that of Décamps et al. (2005). They also analyze a framework in which the drift of the signal process can have a high or low value. However, in their model, the signal and value of the product the same. This means that the person exercising the option is concerned not only with the belief about the drift of the process but also with the path of the process, or the value itself. Therefore, belief is no longer a sufficient statistic to summarize knowledge, and there is path dependency. To overcome this, they use filtering and martingale techniques rather than dynamic programming. The current model does not suffer from the same path dependency because the Brownian motion process is only a signal of the constant value to the consumer, rather than the value changing with the signal process.

The consumer’s problem is also related to the recent literature concerning games of seller reputation with a public signal (e.g. Daley and Green (2012), Gul and Pesendorfer (2012), Bar-Isaac (2003), Kolb (2015a), Kolb (2015b), and Dilmé (2016)). All of these papers analyze the case in which the seller of a good (or political party in Gul and Pesendorfer (2012)) is informed about the quality of the good. He can then choose either how much effort to exert or whether or not to remain in the market as a signal of quality. Except for Kolb (2015b), they also consider a market of short-lived, competitive buyers. This creates an equilibrium structure with either a reflecting
or resetting barrier in the belief space. My model differs in that I assume that the seller has no private information, the consumer is strategic and long-lived, and the seller commits to staying in the game.

My analysis of the consumer’s optimal product research strategy is particularly close to that of Gul and Pesendorfer (2012), who study the case in which all agents are symmetrically informed about quality. The major difference is that they assume that there are two players that compete and choose to buy information, whereas in my model, only the consumer can receive signals. This disparity creates different optimal strategies for when to stop receiving the signal and therefore end the game. In addition, the competition between the two players constitutes the entire game; there is no player like the monopolist making decisions before the other players act. They therefore do not examine when and how instruments like price might be used to influence agents’ information gathering strategies.

The rest of the paper is organized as follows: Section 2 presents the underlying environment and considers the consumer’s problem. Section 3 examines the pricing strategy of the seller. Section 4 analyzes the planner’s problem. Section 5 studies how the optimal strategy is affected by changes in prior belief, cost, and information quality. Section 6 extends the model to also incorporate a product design choice. Section 7 concludes by presenting a discussion of extensions and applications.

2 Consumer’s Problem

2.1 Environment

Consider a consumer who is considering purchasing a good of unknown quality and can do product research. The good is of either high quality or low quality, and the consumer believes that the good is of high quality with probability $\hat{q}$. If the product is of high quality, it is worth $V_H > 0$ to the consumer, and if it is of low quality, it is worth $V_L < V_H$ to her. Note that no assumption is made about the sign of $V_L$. The consumer can buy the good for the posted price of $P$. I assume that both the buyer is risk neutral, so that the consumer attains a utility of $V_a - P$ for $a \in \{H, L\}$ at the time of purchase.

At each point in time, the consumer has three choices: she can buy the product, walk away without purchasing the good, or search for more information about the quality, incurring flow cost $c > 0$. If the consumer chooses to walk away, then both she receives nothing. If she decides to do research, however, then she obtains more information about the quality of the good through a type-dependent signal. She will use this signal to update her belief about product quality and to...
determine when she has acquired a sufficient amount of information to buy the good.\footnote{The consumer’s problem is related to the theory of real options. The consumer is holding a call option to irreversibly invest in, or buy, the product for a fixed price. If and when she decides to buy the good, she is giving up the possibility of waiting for new information that might affect her belief about quality.}

If the consumer does research, then information about the good arrives according to the following exogenous process:

\[ dX_t = \alpha dt + \sigma dZ_t, \]

where \( Z = (Z_t)_{t \geq 0} \) is a standard one-dimensional Brownian motion, with an expected value of 0 for each \( t \). In addition, the process has a positive variance \((\sigma > 0)\), and \( \alpha \) is a type-dependent drift. If the good is of high quality, \( \alpha \) is equal to \( \mu > 0 \), and if it is of low quality, \( \alpha \) is equal to \(-\mu\). Therefore, if the good is of high quality, the cumulative signal is expected to increase over time, while if the good is of low quality, it is expected to decrease. This indicates that the cumulative signal \( X_t \) is normally distributed with mean \( \alpha t \) and variance \( \sigma^2 t \).

### 2.2 Consumer’s Strategy

The consumer’s optimal strategy is based on her belief about product quality. All relevant information for the belief is contained in the cumulative signal, \( X_t \).\footnote{See, for example, Chernoff (1972), Chapter 17.} Denoting \( \mathcal{F}_t \) as the history, or filtration generated by past observations, the consumer’s posterior belief that product quality is high can be obtained by combining Bayes’ rule with the distribution of the signal as follows

\[
q_t = \Pr(\alpha = \mu | \mathcal{F}_t) = \frac{\hat{q} \exp \left( -\frac{(X_t-\mu)^2}{2\sigma^2 t} \right)}{\hat{q} \exp \left( -\frac{(X_t-\mu)^2}{2\sigma^2 t} \right) + (1 - \hat{q}) \exp \left( -\frac{(X_t+\mu)^2}{2\sigma^2 t} \right)} = \frac{\hat{q}}{\hat{q} + (1 - \hat{q}) \exp \left( \frac{-2\mu X_t}{\sigma^2} \right)}. \tag{1}
\]

After applying Itô’s lemma, which is the stochastic calculus counterpart to the chain rule, to equation (1), the belief process satisfies the following filtering equation:

\[
dq_t = q_t(1 - q_t) \frac{2\mu}{\sigma} dZ_t. \tag{2}
\]

A few things are worth noting. First, because the expected value of \( Z_t \) is zero, belief about product quality is a martingale, which is necessary for consistency. Second, how much the consumer adjusts her belief due to new information depends on the current belief. This new information will matter the most when the consumer is most unsure about the type (i.e. when \( q_t \) is close to 1/2). Finally, it is useful to define the quality of news by \( \gamma \equiv 4\mu^2 / \sigma^2 \) (meaning that equation (2) can be...
rewritten as \( dq_t = q_t(1 - q_t)\sqrt{\gamma dZ_t} \). Intuitively, the consumer will gain more from search if either goods of different qualities send very different signals (large \( \mu \)) or there is little noise (small \( \sigma \)). Therefore, equation (2) indicates that belief will adjust more to new information when the quality of news is high.

The consumer uses this belief evolution to determine her continuation value \( V(q) \). For the remainder of the analysis, drop time subscripts and assume that neither the consumer nor the firm discount future consumption. If the consumer is acquiring information, then her continuation payoff can be written in the following way:

\[
V(q) = -cdt + \mathbb{E}V(q + dq) \\
= -cdt + V(q) + V'(q)\mathbb{E}[dq] + \frac{1}{2}V''(q)\mathbb{E}[(dq)^2].
\]

For any belief, her value is the flow cost of paying to see the signal, plus how much the value is expected to change due to any additional information. The second term can be rewritten with a Taylor expansion, and simplified by noting that \( \mathbb{E}[dq] = 0 \) (because belief is a martingale) and \( \mathbb{E}[(dq)^2] = (q(1 - q)2\mu/\sigma)^2 \). Therefore, the relevant stochastic Bellman, or HJB, equation reduces to

\[
0 = -c + \frac{\gamma}{2} q^2 (1 - q)^2 V''.
\]

It is straightforward to show that the solution to the HJB equation is given by

\[
V(q) = \frac{2c}{\gamma} (1 - 2q) \ln \left( \frac{1 - q}{q} \right) + k_2 q + k_1,
\]

where \( k_1 \) and \( k_2 \) are constants of integration.

The following proposition is a complete characterization of the consumer’s optimal information acquisition strategy.

**Proposition 1** There exist beliefs \( \bar{q} \) and \( q \leq \bar{q} \) such that the consumer gathers more information when current belief \( q \) is in the interval \((q, \bar{q})\). She walks away without purchase if her belief is less than or equal to \( \bar{q} \), and buys the good if her belief is greater than or equal to \( \bar{q} \). These boundaries are identified by the solution to the system of equations:

\[
\bar{q}V_H + (1 - \bar{q})V_L - P = \frac{2c}{\gamma} \left( (2\bar{q} - 1)(\ln - \overline{\ln}) + \frac{(1 - 2\bar{q})(\bar{q} - q)}{q(1 - q)} \right),
\]

\[
\bar{q}V_H + (1 - \bar{q})V_L - P = \frac{2c}{\gamma} \left( (2\bar{q} - 1)(\ln - \overline{\ln}) + \frac{(1 - 2\bar{q})(\bar{q} - q)}{q(1 - q)} \right)
\]

where \( \overline{\ln} \equiv \ln \left( \frac{1 - q}{\bar{q}} \right) \) and \( \ln \equiv \ln \left( \frac{1 - q}{\bar{q}} \right) \).
The consumer’s optimal strategy is an optimal stopping rule.\footnote{Note that the consumer takes the price as given because the seller commits to it before she acts. Also note that while \( \hat{q} \) does not affect the belief bounds directly (there is path independence), it does affect them indirectly through the equilibrium price, which is endogenized in Section 3.} If her belief ever drops to \( q \), the consumer walks away, and the game is over. If her belief ever rises to \( \bar{q} \), she buys. Two simulations of this strategy are shown in Figure 1. The upper line is a possible belief path for a good that is eventually sold, while the lower line represents a possible belief evolution for a product that is not purchased by the consumer.

Four boundary conditions on the value function are required to identify the two constants of integration in equation (3) and belief boundaries \( \bar{q} \) and \( q \). The first two are value matching conditions:

\[
\begin{align*}
V(q) &= 0 \\
V(\bar{q}) &= V_H \bar{q} + V_L (1 - \bar{q}) - P.
\end{align*}
\]  

They say that the consumer’s continuation value must be equal to her outside option of 0 when she walks away, and equal to the expected value less the price upon purchase. In other words, \( V(q) \) must be continuous. The second two boundary conditions are smooth-pasting conditions:

\[
\begin{align*}
V'(q) &= 0 \\
V'(\bar{q}) &= V_H - V_L.
\end{align*}
\]  

They guarantee the optimality of the consumer’s research strategy by ensuring the value function
is globally differentiable.\footnote{To demonstrate how this guarantees optimality, consider what would happen if \( V(q) \) approached \( q \) with a slope greater than 0, creating a kink. If the consumer chose to continue searching for a short interval of time, \( \Delta t \), she would observe another signal, and her belief would either be \( q' < q \) or \( q'' > q \). The average of these two points would yield a higher continuation value than the kink point itself, indicating that the chosen lower boundary is not the optimal belief at which to stop product research. Therefore, it must be that \( V'(q) = 0 \). A similar argument can be applied to the upper boundary \( q_0 \).}

Even though explicit solutions for \( q \) and \( q_0 \) are not available, it is possible to characterize how they adjust in response to changes in exogenous parameters and price.\footnote{It is common not to obtain explicit solutions for boundaries in problems with learning. For example, Chernoff (1972), Bernardo and Chowdhry (2002), Felli and Harris (1996), Bolton and Harris (1999), and Gul and Pesendorfer (2012) all have models with learning and no explicit solutions for cutoffs.} The following proposition uses the implicit function theorem to summarize potential shifts in \( q \) and \( q_0 \).

**Proposition 2**

(i) An increase in \( c \) causes \( q_0 \) to decrease and \( q \) to increase, so that \([q, q_0]\) contracts,

(ii) an increase in \( \gamma \) causes \( q_0 \) to increase and \( q \) to decrease, so that \([q, q_0]\) expands,

(iii) an increase in \( P \) causes both \( q_0 \) and \( q \) to increase, so that \([q, q_0]\) shifts up, and

(iv) an increase in \( V_H \) or \( V_L \) causes both \( q_0 \) and \( q \) to decrease, so that \([q, q_0]\) shifts down.

**Proof.** See Appendix. \( \blacksquare \)

The intuition behind the results in Proposition 2 is as follows. If the value of information decreases (\( c \) increases or \( \gamma \) decreases), then the interval \([q, q_0]\) contracts. In other words, as search becomes more expensive, the buyer will do it less; she will buy the good when she is less confident about the good’s quality and walk away when she is more confident about quality. Additionally, if the net value of the product decreases (\( P \) increases or \( V_H \) or \( V_L \) decreases), then \([q, q_0]\) shifts up. The reward for buying a good object is now smaller, so the consumer needs to be more confident in order to purchase the product and is less willing to continue learning for a smaller payoff. Understanding how the consumer responds to changes in parameter values will be especially useful in analyzing the seller’s decisions in subsequent sections.

## 3 Optimal Pricing

### 3.1 Seller’s Problem

The monopolist set the price \( P \geq 0 \) at the beginning of the game and has no more information than the consumer about the quality of the good. The marginal cost of production is normalized to
0. This can be interpreted as a situation in which the product is new and untested. In addition, the
seller is risk neutral so that his utility is $P$ if the good is purchased and 0 otherwise.

The seller faces the usual tradeoff between price and sale. In typical problems, the quantity
the monopolist can sell decreases as he increases price. In the current model, the seller’s expected
profit is the price multiplied by the ex-ante probability of sale. I show below that the probability of
sale is decreasing in the price. Therefore, the seller’s decision to increase the posted price is based
on how much the probability of sale will fall as a result.

Despite the intuitive nature of the tradeoff between price and probability of sale, the seller’s
optimal pricing decision is complicated by the fact that the probability of sale is affected by the
consumer’s optimal strategy. Raising the price will decrease the chance of sale precisely because
the consumer adjusts her behavior by choosing to be more confident in the good’s quality both
when she buys and walks away.

Taking all of this into account, firm profit is defined by the following Lemma.

**Lemma 1** The seller’s expected profit is

\[
\Pi = \frac{\hat{q} - q(P)}{\bar{q}(P) - q(P)} P, \tag{7}
\]

where $\hat{q}$ is the initial belief that the product is of high quality, and the consumer’s behavior is a
function of price.

**Proof.** See Appendix. \(\blacksquare\)

The first term is the likelihood of sale, or the chance that belief increases $\bar{q} - \hat{q}$ before it decreases
$\hat{q} - q$. In other words, it is the chance that the upper bound, $\bar{q}$, is hit before the lower bound, $q$.
Therefore, increasing price (shifting $[q, \bar{q}]$ up) lowers the probability of sale by moving $\hat{q}$ relatively
closer to the lower bound.

If the seller posts an interior price, it will be characterized by the first-order condition. This
price ($P^*$) will dictate a consumer strategy ($q^*, \bar{q}^*$). The equilibrium price and belief bounds are
the solution to the three-equation system of equation (4) and the following first-order condition of
profit:\(^8\)

\[
0 = \frac{\hat{q} - q^*}{\bar{q}^* - q^*} - P^* \frac{(\bar{q}^* - \hat{q})(\bar{q}^*)^2(1 - q^*)^2 + (\hat{q} - q^*)(\bar{q}^*)^2(1 - \bar{q}^*)^2}{(\bar{q}^* - q^*)^3}. \tag{8}
\]

\(^8\)Note that the existence or uniqueness of a solution to this system is not guaranteed because as discussed later in
this section, the profit function may not be single-peaked. Throughout the paper, $P^*$ refers to the interior solution, if
it exists, that yields the most profit.
3.2 Boundary Pricing

The range of prices that might maximize the seller’s profit is affected by the consumer’s behavior. If the price is very high, the consumer will walk away immediately, and if the price is very low, she may buy the good immediately. She will only search for additional information when price is neither too high nor too low and \( \hat{q} \in [q, \overline{q}] \).

Define \( P^W \) as the price at which the consumer is indifferent between walking away and searching and \( P^B \) as the price at which she is indifferent between doing product research and buying immediately. At a price of \( P^B \), it must be that \( \overline{q} = \hat{q} \). Plugging this information into system (4) yields

\[
P^B = \hat{q}V_H + (1 - \hat{q})V_L - \frac{2c}{\gamma} \left( - (1 - 2\hat{q})(\ln B - \hat{\ln}) + \frac{(\hat{q} - q^B)(1 - 2q^B)}{q^B(1 - q^B)} \right),
\]

where \( q^B \) is the lower bound associated with \( \overline{q} = \hat{q} \), \( \ln B = \ln \left( \frac{1 - q^B}{q^B} \right) \), and \( q^B \) is uniquely defined by

\[
\frac{2c}{\gamma} \left( \frac{1 - 2q^B}{q^B(1 - q^B)} + 2\ln B \right) = \frac{2c}{\gamma} \left( \frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})} + 2\hat{\ln} \right) + V_H - V_L.
\]

Notice that the second term of equation (9) is always negative. Therefore, the price can be interpreted as the expected value the consumer attains upon purchase, less the option value of product research, which she is giving up.

It is worth noting that one or both of these prices can be negative. If \( P^B < 0 \), the consumer is unwilling to purchase the good immediately at any feasible price, and if \( P^W < 0 \), no price is low enough to induce the consumer to ever buy. Overall, the shopper will search for more information if \( P \in (\max \{0, P^B\}, P^W) \).

It is clear that it is never optimal for the monopolist to charge \( P^W \), but it may be optimal for him to charge a price of \( P^B \). \( P^W \) is ruled out because as \( P \to P^W \), the probability of sale and profit approach 0. A price of \( P^B \) will be ideal when the firm wishes to avoid consumer research altogether and sell the good immediately.

For subsequent analysis, it is useful to understand the comparative statics of \( P^B \) with respect to cost, information quality, and prior belief, as summarized below.

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9 Similarly, \( P^W \) is defined where \( \hat{q} = \overline{q} \) and system (4), but is omitted because it is absent in later analysis.

10 Note that the right-hand side of the equation is constant, while the left-hand side is strictly decreasing, yielding a unique \( q^B \), and therefore a unique \( P^B \).

11 If \( P^W < 0 \), the analysis is trivial, so I only consider cases where \( P^W \geq 0 \).

12 Note that each price is associated with a strategy \( \{q, \overline{q}\} \) (from system (4)). This relationship between price and strategy is not dependent on the prior \( \hat{q} \). Which prices (and therefore which \( \{q, \overline{q}\} \)'s) are relevant for the firm’s problem, however, depend on \( \hat{q} \).
Proposition 3 The price at which the consumer is indifferent between purchasing the good immediately and gathering more information, \( P^B \), is increasing in prior belief (\( \hat{q} \)) and cost (\( c \)), while it is decreasing in information quality (\( \gamma \)).

Proof. See Appendix.

As noted above, \( P^B \) is the initial expected value minus the option value of gathering additional information. A high option value lowers price because the consumer must be compensated more for choosing not to search. Two opposing factors influence how \( P^B \) changes when \( c (\gamma) \) increases. On the one hand, if the consumer were to search for additional information, she would search less (\( \hat{q} \) and \( q^B \) are closer together) and therefore learn less, decreasing the option value and putting upward pressure on price. On the other hand, it is more expensive for the seller to compensate the consumer for not doing product research, which puts downward pressure on the price. Overall, however, the first motive dominates, so that the option value decreases and the price increases. When \( \hat{q} \) increases, the value of buying immediately also increases, but the option value could increase or decrease, depending on how much the consumer’s strategy changes. In the end, however, the rise in initial expected value outweighs any effect the option value has, and price increases.

3.3 Pricing for Product Research

It is important to understand under what circumstances the seller finds it optimal to incentivize consumer research by posting a relatively high price of \( P^* \).

In general, the seller prefers to post a low price and sell immediately if prior belief is low and prefers to post a high price, encouraging search, if prior belief is high. Intuitively, if prior belief about product quality is low, the good is likely to be of low quality and send negative signals. It is therefore risky for the seller to allow product research because it is relatively more likely that negative signals depress belief, causing the consumer to walk away without purchase. The seller avoids this risk by posting a low price and selling the product immediately. If prior belief about quality is high, however, then the seller expects the signals to be positive, and search is relatively less risky. Even though it is still possible that negative signals cause the consumer to walk away without purchase, it is less likely. In this case, the seller feels confident enough to post a high price and induce search.

To understand when the seller prefers information acquisition, consider the following sufficient condition for product research,

\[
1 - P^B \frac{\hat{q}^2 (1 - \hat{q})^2}{(\hat{q} - q^B)^2} > 0,
\]

\( (10) \)
that profit is increasing at $\max\{0, P^B\}$.\(^{13}\) Equation (10) exemplifies the monopolist’s tradeoff between the price and the probability of sale. Charging a higher price benefits the monopolist, but also causes the probability of purchase to fall. The second term on the left-hand side of equation (10) is the price multiplied by the derivative of the probability of sale with respect to price at $P^B$. Therefore, for the monopolist to desire product research, either $P^B$ must be low enough, or the probability of sale must fall slowly enough when price is raised from $P^B$.

Proposition 4 characterizes when the sufficient condition holds as prior belief changes.

**Proposition 4** As $\hat{q} \to 1$, the sufficient condition holds, and the seller prefers the consumer to acquire information. As $\hat{q} \to 0$, the sufficient condition holds $\Leftrightarrow V_L < 0$.

**Proof.** See Appendix.

To understand Proposition 4, consider how the seller’s two motives are affected by a change in $\hat{q}$, or reputation. First, $P^B$ increases with $\hat{q}$, so that the highest buy-immediately price is around $\hat{q} = 1$. This gives the seller the strongest incentive to induce search when he cannot sell the good for a high price initially, or near $\hat{q} = 0$. Second, how much the probability of sale drops due to product research may change nonmonotonically in prior belief. It is certain, however, that it drops a negligible amount as $\hat{q} \to 1$, but drastically as $\hat{q} \to 0$. This gives the seller the strongest incentive to induce search near $\hat{q} = 1$ and the least incentive near $\hat{q} = 0$, where product research is the riskiest. Therefore, at least at the extreme values, the seller’s two motives work against one another.

To solidify intuition, consider Figures 2a and 2b depicting the probability of sale for different values of $\hat{q}$. Recall that buying immediately indicates that $\hat{q} = \bar{q}$ and that the probability of sale is

\[P^B \leq 0 \text{ or } P^B > 0\]

Note that profit is always increasing at 0 so the sufficient condition reduces to $P^B \leq 0$ or $P^B > 0$ and equation (10).  

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one. In the figures, this is represented by the top-left side of each panel. Increasing price from $P_B$ results in a shift up of $[q, \tilde{q}]$, or a movement toward the top right corner of each figure. It is clear that the likelihood of sale will decrease much less for a high reputation like $\tilde{q} = .8$ than for a low reputation like $\tilde{q} = .2$ when price is raised to induce search.

Proposition 4 demonstrates that when $\tilde{q}$ is high, the seller’s dominant motive is the probability of sale. When raising price results in only a small drop in the probability of sale, inciting product research is “cheap” enough. In other words, even though the seller can guarantee himself a high price by charging $P_B$, he prefers to induce search. For low values of $\tilde{q}$, the probability of sale motive is again dominant if $V_L > 0$; even though the monopolist cannot guarantee himself a high price, it may still be better to sell immediately because the alternative involves too much risk that the consumer becomes discouraged and walks away without purchasing. If $V_L$ is less than the cost of production, however, it is not possible to sell immediately for very low priors because $P_B$ is negative. In this case, product research is the only way to attain a positive expected profit.

Similar intuition can be used to understand the following proposition, which characterizes when the sufficient condition holds as the cost of search and quality of information change.

**Proposition 5** If $V_L \leq 0$, then $\exists c'$ ($\gamma'$) such that the seller prefers the consumer to gather information $\forall c \leq c'$ ($\gamma \geq \gamma'$). If $V_L > 0$ and $\tilde{q} > 1 - V_L^{-1/2}$, then $\exists c''$ ($\gamma''$) such that the seller prefers the consumer to gather information $\forall c \leq c''$ ($\gamma \geq \gamma''$).

**Proof.** See Appendix.

Consider the seller’s two incentives as cost increases (quality of information decreases). First, as search costs increase, so does $P_B$, which makes immediate sale more attractive to the seller. Second, the probability of sale drops more quickly for higher costs. To see this graphically, consider Figure 2a. Again, a price of $P_B$ creates bounds somewhere along the top-left side of the plot. The lower bound increases in $c$, which moves the equilibrium closer to the front-left corner, where the probability of sale drops more sharply in price. Overall, the seller’s motives work in the same direction and make the corner solution more appealing as cost rises. This yields clear cutoffs under which search is preferable.

To better understand the result, consider the extreme cases. As $c \to \infty$, search will never happen in equilibrium. As $c \to 0$, $P_B \to V_L$. This is not a feasible price if $V_L < 0$, making product research preferable to the seller. If $V_L > 0$, however, the sufficient condition holds only if reputation is sufficiently high. The intuition is the same as that of Proposition 4; a low enough $\tilde{q}$ could cause the seller to post $P_B$ and sell immediately because inducing product research is risky.

The above analysis can be summarized by the following intuition. If the firm is not very optimistic about the quality of the good, it has an incentive to sell the good immediately because if the consumer is allowed to search, she is likely to get bad signals and walk away without purchasing.
If the firm is more confident that the product is good, however, it has an incentive to allow the consumer to search. If she does search, she will likely get good signals and want to buy the good, allowing the firm to charge a higher price upon purchase.

This intuition remains relevant even when the sufficient condition does not hold, as in the first three panels of Figure 3. In this case, the profit function may not be concave or even single-peaked. The relative strengths of the monopolist’s two incentives can be seen in the sizes of profit at $P_B$ and at the interior maximum, $P^*$. For low prior beliefs, it is better for the firm to charge $P_B$. As $\hat{q}$ grows, expected profit from the interior maximum grows as well, and eventually, allowing the consumer to search becomes the profit-maximizing strategy for the firm.

As noted earlier, the result that product research is generally more desirable for higher prior beliefs stands in direct contrast to the result in Branco et al. (2012). In their model, the monopolist chooses to sell right away when value is high because he does not know if the consumer’s value will decrease or increase if she is allowed to search. In my model with underlying values, however, the firm has an indication of what the signal will reveal. The firm therefore wishes to capitalize on high beliefs by inducing product research and wishes avoid the signal for low beliefs by posting a low price.

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14 Note that the top right and bottom left panels of Figure 3 clearly demonstrate why equation (10) is sufficient but not necessary for an interior solution.
4 Planner’s Problem

Consider a social planner who takes consumer behavior as given and sets the price at time 0. Denote the socially efficient price as $P^S$. If $V_L$ is larger than its production cost of 0, then $P^S \leq P^B$ so that the good is transferred to the consumer immediately, avoiding socially wasteful search. Therefore, any amount of search is inefficiently large when $V_L > 0$. If $V_L < 0$, however, information acquisition may be efficient because if $P^B < 0$, the good cannot be sold immediately. In order to understand when product research is efficient, consider the social surplus,

$$S = \Pr(\text{sale})\mathbb{E}[\text{value}] - c\mathbb{E}[\tau]$$

$$= \hat{q} - q (qV_H + (1 - q)V_L)$$

$$- \frac{2c}{\gamma(\hat{q} - q)} \left( (\hat{q} - q)(2\hat{q} - 1)\ln - (\hat{q} - q)(2q - 1)\ln - (\hat{q} - q)(2q - 1)\ln \right),$$

(11)

where $\hat{\ln} = \ln \left((1 - \hat{q})/\hat{q}\right)$ and $\tau$ is the time at which either the upper or the lower bound is hit, and the game ends. The planner, like the seller, anticipates the consumer’s strategic response to any posted price.

The following is the first-order condition of the planner’s problem and shows that the socially efficient price is the lowest feasible price.

$$\frac{dS}{dP} = - \frac{P}{(\hat{q} - q)^2} \left( \frac{dq}{dP} (\hat{q} - q) + \frac{d\hat{q}}{dP} (\hat{q} - \hat{q}) \right)$$

(12)

Equation (12) is simply the derivative of (11) with information from the consumer’s optimal behavior substituted in. Recall that the interval $[\hat{q}, \hat{q}]$ shifts up as price increases, indicating that (12) is weakly negative. This is because given consumer behavior, surplus is maximized when the probability of sale is maximized. The chance of a sale is decreasing in price, so that the socially efficient price will be the lowest feasible price, or $\max\{0, P^B\}$. The planner would like to sell the good immediately if $P^B > 0$ to avoid socially wasteful search.\(^{15}\) Barring that, he prefers the good to be sold at cost (0), and the consumer to acquire additional information.

If $V_L < 0$, search is efficient only if it is cheap enough or the prior belief is high enough, as summarized below:\(^{16}\)

**Proposition 6** If $V_L < 0$

\(^{15}\)Note that $P^B$ is bounded below by $V_L$ so that if $V_L$ is positive, then $P^B$ is as well, regardless of other parameter values.

\(^{16}\)Recall that $P^B$ moves monotonically in all parameters. That guarantees the existence of cutoff values for cost, information quality, and prior belief.
Figure 4: Monopolist (solid) vs. Planner (dashed) $E[\tau]$. $c = 2$, $\gamma = 7.11$, $V_H = 5$, and $V_L = -5$.

(i) and $\hat{q}V_H + (1 - \hat{q})V_L \leq 0$, then $P^S = 0 \forall c$,

(ii) and $\hat{q}V_H + (1 - \hat{q})V_L > 0$, then there exists $c^*$ ($\gamma^*$) such that $P^S = 0 \forall c \leq c^*$ ($\gamma \geq \gamma^*$) and $P^S = P^B \forall c > c^*$ ($\gamma < \gamma^*$),

(iii) then there exists $\hat{q}^*$ such that $P^S = 0 \forall \hat{q} \leq \hat{q}^*$ and $P^S = P^B \forall \hat{q} > \hat{q}^*$.

Proposition 6 states that the planner prefers the consumer to gather information only when the search cost or the prior belief is too low for the good to be sold immediately.

On the extensive margin, there is an inefficiently large amount of search in equilibrium. To see this, recall that the monopolist also has a cost cutoff under which search is optimal, $c'$. Comparing them, $c^* < c'$, indicating that the monopolist never prefers to induce product research when the planner does not. Therefore, there is an inefficiently large amount of search on the extensive margin. In addition, because charging a price of 0 is never profit-maximizing, the monopolist charges a weakly higher price than the planner.

When both the planner and the monopolist prefer the consumer to acquire information, however, it is difficult to say which solution yields “more” search. A natural measure of the quantity of search is the ex-ante expected time that the consumer will spend gathering information. This, however, may be higher for the monopolist or the planner, depending on the parameter values, as seen in Figure 4. In this particular case, if $\hat{q}$ is small, the planner’s solution yields more search in expectation, while if $\hat{q}$ is relatively high, the opposite is true.
5 Changes in Reputation and Cost of Search

5.1 Effects of a Higher Prior Belief

Given how crucial the reputation of the firm is to its pricing decision, it is of interest how changes in reputation affect the firm’s optimal price and profit. From the above discussion, we already know that \( P^B \) increases in \( \hat{q} \).

The following proposition shows that interior price and profit increase in reputation as well.

**Proposition 7** Both optimal profit, \( \Pi \equiv \max \left\{ P^B, \Pi^* \equiv P^* \frac{\hat{q} - q^*}{\hat{q} - q} \right\} \), and interior price, \( P^* \), are increasing in \( \hat{q} \).

**Proof.** See Appendix.

These results are intuitive. A higher \( \hat{q} \) allows the seller to capitalize on the fact that the signal will likely indicate that the product is of high quality by charging a higher price. Therefore, regardless of whether the optimal solution involves search or not, the price charged by the seller will be increasing in belief or reputation.

It need not be the case, however, that price be a continuous function of \( \hat{q} \). For the same parameter values as used in Figure 3, the optimal price as a function of reputation is pictured in the top panel of Figure 5. For low enough beliefs, the firm prefers to set a low price so that the consumer purchases immediately. For high beliefs, the firm does prefers to induce search. The top panel shows that for these parameters, the switch from the corner solution to the interior solution occurs.
around $\hat{q} = .3$, while the bottom panel illustrates how at that belief, $P^B$ and $P^*$ yield roughly the same profit.

Profit increases in $\hat{q}$ because as the consumer becomes more confident about product quality, she is willing to pay more. We have already seen that $P^B$ is increasing in reputation. To see why $\Pi^*$ is also increasing in $\hat{q}$, apply the envelope theorem, which yields $d\Pi^*/d\hat{q} = P^*/(\bar{q}^* - \hat{q}^*) > 0$. In other words, higher expected values result in higher profit.

### 5.2 Effects of Higher Search Costs (Less Informative Search)

It is natural to consider how changes in search costs affect price and profit, especially since the internet has made information acquisition significantly easier. Above, I showed that $P^B$ increases in $c(-\gamma)$ because as costs rise, the option to search becomes less valuable. Analysis of how $P^*$ reacts to a change in $c$ or $\gamma$, however, will be significantly more complicated. Before turning to the specifics, first consider the intuition.

At first glance, one might be tempted to assume that equilibrium price should increase with cost. When costs go down, consumers are better informed about product quality and therefore require more information rent from the producer. This pushes price down. When the optimal price is $P^B$, this is surely the case. When the optimal price is $P^*$, however, a change in cost or information quality changes the consumer’s search behavior, which in turn affects the seller’s incentives. It will be easiest to form intuition if we disentangle the effects of the movement of $q$ and the movement of $\hat{q}$.

Examining how a movement in $q$ affects the seller’s motives shows that a change in $q$ always pushes optimal price in the opposite direction. If the consumer’s belief hits the lower bound, she walks away, and the seller receives nothing. Therefore, a movement of the lower bound only affects the seller’s incentives through the probability of sale. Think about a marginal increase in the lower bound from $q'$ to $q''$, so that the solution remains interior ($\hat{q} \geq q''$). This change means that the consumer is more likely to walk away without buying, leaving the seller with nothing. The seller therefore has an incentive to lower the price (and therefore $q'$) in order to induce search and increase the probability of sale. The opposite is true if the lower bounds decreases. In this case, the seller takes advantage of the increased probability of sale by increasing the price. Either way, a change in $q$ will put pressure on the optimal price in the opposite direction ($q \uparrow \Rightarrow$ price $\downarrow$).\(^{17}\)

Next, consider how a change in the upper bound will affect pricing decisions. This effect is more complicated because the seller wants belief to hit the upper bound. He now faces a tradeoff between the price and the probability of trade. Think about a marginal decrease in the upper bound from $\bar{q}'$ to $\bar{q}''$, where the solution remains interior ($\hat{q} \leq \bar{q}''$). The seller has two conflicting motives.\(^{17}\)

\footnote{Note that a larger change to $\bar{q}'' \geq \hat{q}$ also puts downward pressure on price.}
On the one hand, he would like to take advantage of the higher probability of sale by raising the
price. On the other hand, he has the option to ensure sale by lowering the price (and therefore \( \overline{q} \))
even further. Whether the change in the upper bound will put upward or downward pressure on the
price will depend on which of these motives is stronger.\(^{18}\)

This intuition is formalized for interior solutions in the following proposition.

**Proposition 8** The impact on \( P^* \) of a marginal change in \( X \in \{c, \gamma \} \) will have the same sign as

\[
- \frac{dq}{dX} \left( 2 \frac{d\overline{q}}{dP}(\hat{q} - q)^2 + \frac{dq}{dP}(\overline{q} - q)(q + 2\overline{q} - 3q) + 2(\overline{q} - q)(\overline{q} - \hat{q})q(1 - q)(1 - 2q) \right) \\
- \frac{d\overline{q}}{dX}(\hat{q} - q) \left( \frac{dq}{dP}(2\hat{q} - q - \overline{q}) - 2 \frac{d\overline{q}}{dP}(\overline{q} - q) + 2(\overline{q} - q)\overline{q}(1 - \overline{q})(1 - 2\overline{q}) \right). 
\]

Equation (13) helps decompose the effects of the upper and lower bounds from the consumer’s
strategy. To see how the lower bound affects the seller’s pricing incentive, first recognize that term
\( A \) is always positive. This means that a change in \( X \) will put pressure in opposite directions on
\( q \) and \( P^* \), as suggested by the intuition above. Term \( B \) is not as clear. At low values of \( \hat{q} \), it is
certainly negative, but at high values of \( \hat{q} \), it is nearly always positive. This indicates that when the
seller’s reputation is low, the motive to ensure purchase dominates, while when reputation is high,
the motive to take advantage of increases in the probability of sale by raising price is stronger.

Proposition 8 indicates that optimal price is decreasing in \( c(\gamma) \) when the seller’s primary mo-
tive is to ensure sale. Intuitively, if the monopolist prefers the consumer to acquire information (i.e.
the optimal price is \( P^* \)), then as search becomes less appealing (more costly or less informative),
his must lower the price in order to compensate and induce product research.

Combining these results indicates that overall, the effect on price of an decrease in search
costs is ambiguous. When the good is sold immediately, price decreases with cost, but when the
consumer gets additional information, price can increase when cost decreases. The fact that the
consumer can choose whether to acquire additional information is the driving force for this result.
If the monopolist desires the consumer to search, he must raise the price in order to keep incentives
balanced when product research becomes cheaper or more appealing.

Price changing ambiguously with \( c(\gamma) \) signals that equilibrium profit may as well. While we
know that \( P^B \) increases in \( c(\gamma) \), the effect of a change in \( c \) or \( \gamma \) on \( \Pi^* \) is unclear. As with \( \hat{q} \), apply

\(^{18}\)Note that a larger change to \( \overline{q}'''' \leq \hat{q} \) means the consumer buys immediately so that the seller raises price for sure.
the envelope theorem to attain

$$\frac{d\Pi}{dX} = -\frac{P^*}{(\hat{q}^* - q^*)^2} \left( (\hat{q} - q^*) \frac{\partial \hat{q}}{\partial X} + (q^* - \hat{q}) \frac{\partial q}{\partial X} \right)$$

for $X \in \{c, \gamma\}$. Recall that as $c$ increases, the interval $[q, \bar{q}]$ contracts. How much each boundary moves, however, depends on their initial levels, cost, and price. Therefore, as the interval contracts, the sign of the above equation is ambiguous because the probability of sale could increase if $\hat{q}$ becomes relatively closer to $\bar{q}$ than $q$.

The ambiguity of profit with respect to changes in search cost can also be seen as $c \to \infty$. As cost approaches infinity, the consumer never searches and price will tend towards $P^B$. As this happens, the option value of search approaches 0, so that $P^B \to \bar{q}V_H + (1 - \bar{q})V_L$. If $\bar{q}V_H + (1 - \bar{q})V_L < 0$, then price and profit approach 0. If, however, $\bar{q}V_H + (1 - \bar{q})V_L > 0$, then whether this increase in cost is good or bad for the firm depends on how this price compares to the profit the firm was initially earning. Consider Figure 6. The only difference between the top and bottom panels is the value of $\hat{q}$. The horizontal lines represent the initial expected value, or what profit converges to as $c \to \infty$. It is unclear whether increasing $c$ will increase or decrease the firm’s profits in the limit; for $\hat{q} = .6$, the firm is better off with low costs, and for $\hat{q} = .8$, the firm makes a higher profit with increased costs.

Figure 6: Profit compared to the limit. $c = .5$, $\gamma = 4$, $V_H = 5$, and $V_L = -5$. 
6 Product Design

Now consider a richer model in which the seller has control, not only over price, but also over some element of product design. Specifically, he is able to choose the dispersion of product quality. For example, consider a firm that is choosing how innovative to be with a new product, such as a smart phone. If it chooses to be conservative and innovate very little relative to the last version of the phone, quality dispersion is low. In other words, the firm may not know if the new phone is of high or low quality, but the high-quality and low-quality products are very similar. If instead, the firm chooses to be very innovative and introduce many new features to the phone, the new product could be a great success or a great failure, and product quality dispersion is high.

To model quality dispersion, I assume that the value of the low-quality object and initial expected value, denoted by $\hat{V}$, are fixed, while the seller chooses the high-quality value $V_H$. The choice of $V_H$ determines the belief such that $\hat{V} = \hat{q}V_H + (1 - \hat{q})V_L$. In other words, the seller chooses a mean-preserving spread of product quality. This product design choice is another tool the seller can use to encourage or discourage information acquisition by making it more or less appealing.

To solidify intuition, suppose that $V_L = 0$, so that the low-quality good has no value to the consumer. Adjusting $V_H$ (and therefore $\hat{q}$) can be seen as changing the likelihood that the consumer will match well with the good and obtain any utility from purchase. If the seller chooses a $V_H$ very near $V_L = 0$, the consumer will gain little from search because she will receive a very small payoff regardless of whether the good is of low or high quality. In contrast, if the seller chooses a very large $V_H$, the consumer finds it worthwhile to search in order to determine if the good is worth buying.

The seller’s profit is characterized by a tradeoff between $V_H$ and $\hat{q}$. Note that prior belief is determined by $\hat{q} = (\hat{V} - V_L)/(V_H - V_L)$ so that high choices of $V_H$ are associated with low values of $\hat{q}$ and visa versa. In other words, the seller can either choose to be very confident that the good is of high quality if $V_H$ is very close to initial expected value, or he can choose a much more unfavorable belief in exchange for the high-quality good being very valuable.

Profit for the seller under these conditions is

$$\Pi(P, V_H) = P \frac{\hat{V} - V_L}{\hat{q}(P, V_H) - \hat{q}(P, V_H)} - q(P, V_H),$$

which takes the reaction of the consumer to the joint choice of $P$ and $V_H$ into account.

In previous work, Johnson and Myatt (2006) find that an extremal level of dispersion is always optimal. They consider consumers whose willingnesses to pay are drawn from a distribution. The monopolist has control over price and the dispersion of the distribution. They show that the firm
desires either as little or as much dispersion as is allowed. In order to obtain this result, Johnson and Myatt first make a local argument; they fix the level of dispersion and show that if the optimal price is low (below some cutoff), then profit is decreasing in dispersion, while if optimal price is high, it is increasing in dispersion. They then make an additional assumption to make the local argument global.

I take a similar approach as Johnson and Myatt (2006) by analyzing how dispersion affects profits, given the pricing decision of the seller.

**Proposition 9** When it is optimal for the seller to discourage search, profit is decreasing in dispersion.

**Proof.** See Appendix.

Proposition 9 says that if the optimal price for the seller to post is \( P^B \), profit is decreasing in \( V_H \). Intuitively, higher dispersion increases the consumer’s option value of search, lowering profit. To minimize this option value, the seller prefers to lower the level of dispersion by decreasing \( V_H \). Therefore, if the seller’s profit is maximized by selling the good immediately, he prefers the least amount of dispersion possible.

Next, consider the case in which it is optimal for the seller to post a high price (\( P^* \)) and induce search. Invoking the envelope theorem shows how a change in \( V_H \) affects profit through both \( \hat{q} \) and the consumer’s search strategy. Overall,

\[
\frac{d\Pi}{dV_H} = \frac{P^*}{(\bar{q} - \hat{q})^2} \left( \frac{(\bar{q} - \hat{q})\hat{q} \hat{q}^2}{\bar{V} - V_L} + \frac{1}{\hat{q} - \bar{q}} \left( \frac{(\bar{q} - \hat{q})\hat{q} \hat{q}^2 (1 - \hat{q})}{\hat{q}} + \frac{(\bar{q} - \hat{q})\hat{q} \hat{q}^2 (1 - \hat{q})^2}{\hat{q}} \right) \right). \tag{14}
\]

Equation (14) shows that there are conflicting effects of increasing dispersion on equilibrium profit. On the one hand, \( V_H \) rising causes the range of beliefs for which a consumer will search, \([\bar{q}, \hat{q}]\), to shift down, increasing the probability of sale. This positively impacts profit, as seen in the last two terms of equation (14). On the other hand, the resulting fall in \( \hat{q} \) has a negative effect on profit by moving the belief relatively closer to the lower bound, as indicated by the first term in parentheses in equation (14).

While the overall effect of an increase in dispersion is ambiguous, we can say the following.

**Proposition 10** When the seller prefers to encourage consumer search, an interior level of dispersion is optimal (i.e. the optimal \( V_H \) is strictly less than \( \infty \)).

**Proof.** See Appendix.
Some amount of dispersion is necessary for the consumer to do research, but Proposition 10 shows that this level of dispersion will not be extremal. Intuitively, even though the positive effect in equation (14) \( [\overline{q}, \overline{\overline{q}}] \) shifting down) dominates for small amounts of dispersion, as \( V_H \) becomes large and these bounds get closer to 0, they move less and less in response to additional changes. Belief, however, continues to fall at the same rate. At some point, the negative effect dominates, and profit begins to decrease in dispersion.

An example of the seller’s interior choice of dispersion is shown in Figure 7. It is clear that the seller prefers to choose a high level of dispersion in order to raise the option value of search and induce the consumer to gather information. This dispersion comes at a cost, however, and too much of it deteriorates the probability of sale to such an extent that profit falls.

A sufficient condition for product research to be optimal is that selling immediately is infeasible \( (P_B \leq 0) \), even for the lowest level of dispersion. This will be true if \( \widehat{V} \leq 0 \) because \( P_B \rightarrow \widehat{V} \) as \( V_H \rightarrow \widehat{V} \). In other words, if the initial expected value is too low, some dispersion is necessary to convince the consumer to gather information and give the firm a chance of selling the good. This is the case in which without consumer search, potentially beneficial trades are not made because they are initially too risky. Information acquisition allows the trade of these goods.

If this sufficient condition fails and \( \widehat{V} > 0 \), however, it may be better to sell the good immediately. Consider Figure 8. In this case, the seller maximizes profit by reducing dispersion as much as possible \( (\widehat{q} = 1, V_H = \widehat{V}) \) and posting a low price in order to sell the product immediately.

Overall, we see that an extreme choice of dispersion is optimal if the firm’s objective is to sell
the good right away but not if consumer search is optimal. This is due to the tradeoff between dispersion and the probability of sale. If search is optimal, too much dispersion eventually decreases the probability of sale, lowering profit. In contrast, if immediate sale is best, the probability of sale is always 1, and the lack of this tradeoff creates an extreme optimal choice.

Consider the differences between these results and those in Johnson and Myatt. In their model, charging a high price means that the firm sells only to the small number of consumers who have very high valuations of the good. Therefore, increasing dispersion increases profit because the benefit of making the top portion of the distribution even more enthusiastic about the product always outweighs the cost of selling a slightly smaller quantity. The difference in tradeoff drives the extremal choice of dispersion.

The analysis of consumer search with product design can also be compared to Bar-Isaac et al. (2012), who find that firms with low initial values desire high levels of dispersion and those with high initial values prefer low levels of dispersion. Their model, which builds off of Johnson and Myatt’s, considers many firms whose products each have an innate value and a consumer-specific component. Firms cannot alter their innate values, but they can choose the dispersion of the idiosyncratic quality. Just as in Johnson and Myatt’s model, firms choose extremal levels of dispersion. Firms with low initial values use dispersion to compensate for their low values by selling only to buyers who are well-matched with their products. This is comparable to the sufficient condition for product research in my model because dispersion helps a firm with an initially low value to sell its product. In Bar-Isaac et al., firms use dispersion to sell to only the
most well-matched consumers, and in the current model, the seller uses dispersion to incentivize search.

7 Conclusion

I conclude by discussing potential empirical implications and possible topics for future research.

7.1 Implications

It is always necessary in theoretical models to abstract away from reality to some extent for reasons of tractability and clarity. This does not mean, however, that the results obtained have no bearing on or insight into the real-world behavior of market participants. For example, in the current model, one assumption that might be problematic for testing predictions of the model is that the seller is a monopolist.

The assumption that the firm is a monopolist is not as restrictive as it might initially appear, however, for two reasons. First, and most obviously, some markets may indeed be relatively close to monopolies if their products have very poor substitutes. Second, consider the case where there is some “standard” in the market, and the consumer wishes to find out about a new or less well-known good. In this case, as she learns about the new product, she always has the outside option to buy the “standard” good. This would be the same as walking away in my model where the value of the outside option is normalized to 0.

Adjusting our interpretation in this way yields three testable implications for online marketplaces, such as Amazon. First, consumers spend little or no time looking through comments and reviews, and purchase soon after arriving at the webpage if the posted price is low relative to the outside option. In other words, they will not find product research worthwhile if it is obvious they are being charged a low price.

Second, firms are more likely to post lower prices if their reputations are poor. While this may seem straightforward, the way firms transition from low to high reputation might not be. If the value of the low-quality good is less than the marginal cost of production, the model predicts a smooth price increase as reputation climbs because selling immediately is not an option for low beliefs. If the value is larger than marginal cost, however, which is arguably the more common case, my model predicts a jump up in price as firms transition to encouraging research in equilibrium.

To see why and how there might be a discontinuity in price, consider websites like Product Elf and AMZ Review Trader. They are a channel through which sellers contact interested Amazon consumers. In exchange for severely discounted (sometimes free) products, consumers agree to write honest reviews about the good. Sellers agree to this in hopes of getting good reviews and
improving their ratings or reputations on Amazon. Once a seller’s reputation has increased enough, he will stop using the discount website, creating a jump in the price of the good.

The third implication of my model is that the recent decline in search costs due to the availability of the internet will have had an overall ambiguous effect on posted prices. More specifically, goods that are priced to sell quickly will have seen price decreases, while goods with higher prices that induce consumer search may have seen price increases. This is because if the seller allows the consumer to gather additional information, it means he is confident that the signal the consumer receives will be positive on average. In order to keep incentives balanced after the cost of search falls, he may need to raise price to compensate.

7.2 Future Work

The model proposed in this paper constructs a very natural environment, so there are many more interesting questions that could be asked in a similar framework. For example, we have seen that whether the value of a good is underlying or idiosyncratic is crucial to the seller’s pricing decision. If the value is purely underlying, selling immediately is optimal when initial value is low, while the opposite is true if the value is purely idiosyncratic. But the reality for many products lies somewhere in the middle. For example, some laptops are objectively better than others, breaking down less on average, while different features of a particular laptop (screen resolution, memory, etc.) may appeal more to one person than another. A model with both underlying and idiosyncratic components could further identify optimal pricing strategies, depending on where a particular good falls on the spectrum.

It is also possible to consider multiple sellers, where the consumer can search across sellers and acquire information about only one good at a time. She has to form a strategy that decides not only when to purchase and walk away, but also when to switch from acquiring information about one good to gathering signals about the other. Ke et al. (2015) examine this problem for the idiosyncratic value case based off of Branco et al. (2012)’s model. Given how different the results in the current paper are from those in their baseline model, however, one could expect important differences in the equilibrium with multiple sellers as well.

Relatedly, one could consider multiple buyers where the first buyer gets a lower price than any subsequent consumers. Under these assumptions, the product is sold sooner than in the model with only one consumer, as buyers compete to receive the low price. How the seller’s pricing strategy will be affected is not obvious. On the one hand, he has an incentive to raise prices relative to the current model to take advantage of the fact that consumers will be willing to buy the first good when less confident about quality in order to receive the low price. On the other hand, the seller does not want to raise prices too much, as it would deteriorate the probability of sale of the second
item sold.

Finally, one could examine a different choice variable than price for the seller. As discussed above, firms can choose to sell their products for severely discounted prices in exchange for online reviews. It is also possible for them to pay companies to write favorable reviews of their products to increase their ratings. Either way, the seller is attempting to manipulate the signal consumers receive. This could be modeled as the seller choosing the variance of the signal process. It could also be seen as an upwards modification of the drift parameter, either once at the beginning of the game, or continuously, as a function of the current belief.

Appendix: Omitted Proofs

Proof of Proposition 2. We will use the implicit function theorem. Rearrange (4) to get

\[ f_1(V_H, V_L, c, \gamma, q, q, P) = \frac{1}{q - q} \left( \bar{q} V_H + (1 - \bar{q}) V_L - P - \frac{2c}{\gamma} (1 - 2\bar{q}) \ln \frac{q}{q - q} + \frac{2c}{\gamma} (1 - 2\bar{q}) \ln \right) - \frac{2c}{\gamma} \left( \frac{1 - 2q}{q(1 - q)} + 2\ln \right) \]

(15)

\[ f_2(V_H, V_L, c, \gamma, q, q, P) = \frac{1}{q - q} \left( \bar{q} V_H + (1 - \bar{q}) V_L - P - \frac{2c}{\gamma} (1 - 2\bar{q}) \ln \frac{q}{q - q} + \frac{2c}{\gamma} (1 - 2\bar{q}) \ln \right) - (V_H - V_L) - \frac{2c}{\gamma} \left( \frac{1 - 2\bar{q}}{q(1 - q)} + 2\ln \right) . \]

(16)

In order to apply the implicit function theorem, we must first show that the Jacobian determinant is nonzero, or that

\[ \left| \begin{array}{cc} \frac{\partial f_1}{\partial q} & \frac{\partial f_2}{\partial q} \\ \frac{\partial f_1}{\partial \bar{q}} & \frac{\partial f_2}{\partial \bar{q}} \end{array} \right| = \frac{\partial f_1}{\partial q} \frac{\partial f_2}{\partial \bar{q}} - \frac{\partial f_1}{\partial \bar{q}} \frac{\partial f_2}{\partial q} \neq 0. \]

We find that

\[ \frac{\partial f_1}{\partial q} \frac{\partial f_2}{\partial \bar{q}} - \frac{\partial f_1}{\partial \bar{q}} \frac{\partial f_2}{\partial q} = \frac{2c}{\gamma} \left( \frac{\partial f_1}{\partial q} \frac{1}{q^2 (1 - q)^2} - \frac{\partial f_1}{\partial q} q^2 \frac{1}{q^2 (1 - q)^2} \right) \]

which will be nonzero as long as \(2c/\gamma\) is nonzero, which is true by assumption. Then to apply the implicit function theorem, define the following:

\[ \mu = (c, \gamma, V_H, V_L, P) \]
\[ q = (q, \bar{q}) \]
\[ f = (f_1(q; \mu), f_2(q : \mu)) \]
\[ D_{\mu}f = \begin{bmatrix} \frac{\partial f_1}{\partial c} & \frac{\partial f_1}{\partial \gamma} & \frac{\partial f_1}{\partial V_H} & \frac{\partial f_1}{\partial V_L} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial c} & \frac{\partial f_2}{\partial \gamma} & \frac{\partial f_2}{\partial V_H} & \frac{\partial f_2}{\partial V_L} & \frac{\partial f_2}{\partial P} \end{bmatrix} \]

\[ D_{\mu}\eta = \begin{bmatrix} \frac{\partial \eta}{\partial c} & \frac{\partial \eta}{\partial \gamma} & \frac{\partial \eta}{\partial V_H} & \frac{\partial \eta}{\partial V_L} & \frac{\partial \eta}{\partial P} \end{bmatrix} \]

\[-[D_{q}f]^{-1} = \begin{bmatrix} -\frac{\partial f_1}{\partial q} & -\frac{\partial f_1}{\partial q} \\ -\frac{\partial f_2}{\partial q} & -\frac{\partial f_2}{\partial q} \end{bmatrix}^{-1} = \frac{1}{2c - \frac{\partial f_1}{\partial q}(1-q)^2} \begin{bmatrix} -\frac{\partial f_2}{\partial q} & -\frac{\partial f_1}{\partial q} \\ -\frac{\partial f_2}{\partial q} & -\frac{\partial f_1}{\partial q} \end{bmatrix} \]

Then the comparative statics can be characterized by

\[ D_{\mu}\eta = -[D_{q}f]^{-1} D_{\mu}f. \]

Substituting in \( f_1 = f_2 = 0 \) to the above yields:

\[ \frac{\partial q}{\partial c} = -\frac{2q^2(1-q)^2}{\gamma(q-q)} \left( (1-2q)(\ln - \bar{m}) + \frac{(2q-1)(q-q)}{q(1-q)} \right) \]

\[ = -\frac{q^2(1-q)^2}{q-q} \frac{1}{c} (P - qV_H - (1-q)V_L) < 0 \]

\[ \frac{\partial q}{\partial c} = \frac{2q^2(1-q)^2}{\gamma(q-q)} \left( (2q-1)(\ln - \bar{m}) + \frac{(1-2q)(q-q)}{q(1-q)} \right) \]

\[ = \frac{q^2(1-q)^2}{q-q} \frac{1}{c} (qV_H + (1-q)V_L - P) > 0 \]

\[ \frac{\partial q}{\partial \gamma} = \frac{q^2(1-q)^2}{\gamma(q-q)} \frac{2c}{(1-2q)(\ln - \bar{m}) + \frac{(2q-1)(q-q)}{q(1-q)}} \]

\[ = \frac{q^2(1-q)^2}{q-q} \frac{1}{\gamma} (P - qV_H - (1-q)V_L) > 0 \]

\[ \frac{\partial q}{\partial \gamma} = -\frac{q^2(1-q)^2}{\gamma(q-q)} \frac{2c}{(2q-1)(\ln - \bar{m}) + \frac{(1-2q)(q-q)}{q(1-q)}} \]

\[ = -\frac{q^2(1-q)^2}{q-q} \frac{1}{\gamma} (qV_H + (1-q)V_L - P) < 0 \]
\[ \frac{\partial \bar{q}}{\partial V_L} = \frac{(1-q)q^2(1-q)^2}{\bar{q} - q} < 0 \]
\[ \frac{\partial q}{\partial V_L} = \frac{(1-\bar{q})q^2(1-q)^2}{\bar{q} - q} < 0 \]

\[ \frac{\partial \bar{q}}{\partial V_H} = \frac{q\bar{q}^2(1-\bar{q})^2}{\bar{q} - q} < 0 \]
\[ \frac{\partial q}{\partial V_H} = \frac{q\bar{q}^2(1-q)^2}{\bar{q} - q} < 0 \]

\[ \frac{\partial \bar{q}}{\partial P} = \frac{\bar{q}^2(1-\bar{q})^2}{\bar{q} - q} > 0 \]
\[ \frac{\partial q}{\partial P} = \frac{q^2(1-q)^2}{\bar{q} - q} > 0 \]

**Proof of Lemma 1.** We will calculate the probability of sale directly from the signal, \( X_t \), then convert it to the belief space. This is possible because there is a one to one relationship between the signal and beliefs. Recall that

\[
dX_t = \alpha dt + \sigma dZ_t
\]

\[ q = \frac{1}{1 + (1-\hat{q})/\hat{q} \exp \left( \frac{-2\mu X}{\sigma^2} \right)} \]

\[ \exp \left( -\frac{2\mu \bar{X}}{\sigma^2} \right) = \frac{\hat{q}}{1-\hat{q}} \frac{1-\bar{q}}{\bar{q}} \]

\[ \exp \left( -\frac{2\mu X}{\sigma^2} \right) = \frac{\hat{q}}{1-\hat{q}} \frac{1-q}{\hat{q}} \]

where \( \bar{X} \) is the cumulative signal at which the consumer purchases, and \( X \) is the cumulative signal at which the consumer walks away. For \( a > 0, b > 0 \), the probability that a process with a drift \( \alpha \)
and variance $\sigma^2$ increases $a$ before decreasing $b$ is

$$1 - e^{(2\alpha/\sigma^2)b} \over e^{(-2\alpha/\sigma^2)a} - e^{(2\alpha/\sigma^2)b},$$

Therefore,

$$\Pr(\text{sale}) = \Pr(X \uparrow X \text{ before } X \downarrow X)$$

$$= \frac{1 - \exp \left( -\frac{2\alpha}{\sigma^2}X \right)}{\exp \left( -\frac{2\alpha}{\sigma^2}X \right) - \exp \left( -\frac{2\alpha}{\sigma^2}X \right)}$$

$$= \hat{q} \left( \frac{1 - \exp \left( -\frac{2\mu}{\sigma^2}X \right)}{\exp \left( -\frac{2\mu}{\sigma^2}X \right) - \exp \left( -\frac{2\mu}{\sigma^2}X \right)} \right) + (1 - \hat{q}) \left( \frac{1 - \exp \left( \frac{2\mu}{\sigma^2}X \right)}{\exp \left( \frac{2\mu}{\sigma^2}X \right) - \exp \left( \frac{2\mu}{\sigma^2}X \right)} \right)$$

$$= \hat{q} \left( \frac{1 - \hat{q}}{1 - \hat{q}} \frac{1 - q}{1 - q} \frac{1 - q}{1 - q} \right) + (1 - \hat{q}) \left( \frac{1 - \hat{q}}{1 - \hat{q}} \frac{1 - q}{1 - q} \frac{1 - q}{1 - q} \right)$$

$$= \frac{\hat{q} - q(P)}{q(P) - q(P)}$$

**Proof of Proposition 3.** In order to do comparative statics on $P^B$, we must first do them on $q^B$ via the implicit function theorem. Define

$$f = \frac{1 - 2q^B}{q^B(1 - q^B)} + 2\ln^B - \frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})} - 2\ln - \frac{\gamma}{2c} (V_H - V_L)$$

$$D_\mu f = \begin{bmatrix} \frac{\partial f}{\partial c} & \frac{\partial f}{\partial \gamma} & \frac{\partial f}{\partial \hat{q}} \end{bmatrix}$$

$$D_\mu \eta = \begin{bmatrix} \frac{dq^B}{dc} & \frac{dq^B}{d\gamma} & \frac{dq^B}{d\hat{q}} \end{bmatrix}$$

$$-[D_q f]^{-1} = -\frac{1}{\frac{\partial f}{\partial q^B}}, \quad D_\mu \eta = -[D_q f]^{-1} D_\mu f.$$
Then we have
\[
\frac{\partial f}{\partial q^B} = -\frac{1}{(q^B)^2(1 - q^B)^2},
\]
so that
\[
\frac{dq^B}{dc} = \frac{\gamma}{2c^2} (V_H - V_L)(q^B)^2(1 - q^B)^2 > 0
\]
\[
\frac{dq^B}{d\gamma} = -\frac{1}{2c} (V_H - V_L)(q^B)^2(1 - q^B)^2 < 0
\]
\[
\frac{dq^B}{d\hat{q}} = \frac{(q^B)^2(1 - q^B)^2}{\hat{q}^2(1 - \hat{q})^2} > 0.
\]

That easily gives us the marginal changes in \( P_B \).
\[
\frac{dP_B}{dc} = \frac{2}{\gamma} \left( (\ln^B - \ln)(1 - 2\hat{q} + 2(\hat{q} - q^B)(2(q^B)^2 - 2q^B + 1)) \right)
\]
\[
+ \frac{2}{\gamma} \left( -\frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})}(\hat{q} - q^B)(2(q^B)^2 - 2q^B + 1) - 2(1 - 2q^B)(\hat{q} - q^B) \right) > 0
\]
\[
\frac{dP_B}{d\gamma} = -\frac{c}{\gamma} \frac{dP_B}{dc} < 0
\]
\[
\frac{dP_B}{d\hat{q}} = \frac{2c}{\gamma} \frac{\hat{q} - q^B}{\hat{q}^2(1 - \hat{q})^2} (3 - 2q_B^2(1 - q^B)) > 0
\]

**Proof of Proposition 4.** We will start with \( \hat{q} \to 0 \) (so \( q^B \to 0 \)) and consider what happens to \( P_B \) in the limit. First,
\[
\lim_{\hat{q} \to 0} (1 - 2\hat{q})(\ln^B - \ln) = \ln \left( \lim_{\hat{q} \to 0} \frac{\hat{q}}{q^B} \right) = \ln \left( \lim_{\hat{q} \to 0} \frac{1}{\frac{dq^B}{dq}} \right) = \ln \left( \lim_{\hat{q} \to 0} \left( \frac{\hat{q}}{q^B} \right)^2 \right) \in \{\infty, 0\},
\]
depending on if \( \lim_{\hat{q} \to 0} \frac{\hat{q}}{q^B} \in \{0, 1\} \) respectively. In addition,
\[
\lim_{\hat{q} \to 0} \frac{(\hat{q} - q^B)(1 - 2q^B)}{q^B(1 - q^B)} = \lim_{\hat{q} \to 0} \frac{\hat{q} - q^B}{q^B} = \lim_{\hat{q} \to 0} \frac{1 - \frac{dq^B}{dq}}{\frac{dq^B}{dq}} = \lim_{\hat{q} \to 0} \left( \left( \frac{\hat{q}}{q^B} \right)^2 - 1 \right) \in \{-1, 0\},
\]
meaning \( \lim_{\hat{q} \to 0} P_B \in \{\infty, V_L\} \). We can rule out a price of \( \infty \) as the consumer and monopolist become certain the good is of low quality, so that \( \lim_{\hat{q} \to 0} \frac{\hat{q}}{q^B} = 1 \) and \( \lim_{\hat{q} \to 0} P_B = V_L \). Finally,
the limit of the sufficient condition will be

\[
\lim_{\hat{q} \to 0} 1 - P_B \left( \frac{\hat{q}(1 - \hat{q})}{\hat{q} - q^B} \right)^2 = 1 - V_L \lim_{\hat{q} \to 0} \left( \frac{\hat{q}}{\hat{q} - q^B} \right)^2 = 1 - V_L \lim_{\hat{q} \to 0} \left( \frac{1}{1 - \frac{dq^B}{dq}} \right)^2
\]

\[
= 1 - V_L \lim_{\hat{q} \to 0} \left( \frac{1}{1 - (\frac{q^B}{\hat{q}})^2} \right)^2 = \begin{cases} -\infty \text{ if } V_L > 0 \\ \infty \text{ if } V_L < 0. \end{cases}
\]

Next, we will examine \( \hat{q} \to 1 \) (so \( q^B \to 1 \)) and consider

\[
\lim_{\hat{q} \to 1} (1 - 2\hat{q})(\ln^B - \hat{\ln}) = -\lim_{\hat{q} \to 1} \ln^B - \hat{\ln} = -\ln \left( \lim_{\hat{q} \to 1} \frac{1 - q^B}{1 - \hat{q}} \right) = -\ln \left( \lim_{\hat{q} \to 1} \left( 1 - \frac{q^B}{1 - \hat{q}} \right)^2 \right) \in \{\infty, 0\},
\]

depending if \( \lim_{\hat{q} \to 1} \frac{1 - q^B}{1 - \hat{q}} \in \{0, 1\} \) respectively. In addition,

\[
\lim_{\hat{q} \to 1} \frac{(\hat{q} - q^B)(1 - 2\hat{q})}{q^B(1 - q^B)} = \lim_{\hat{q} \to 1} \frac{\hat{q} - q^B}{1 - q^B} = \lim_{\hat{q} \to 1} \frac{1 - \frac{dq^B}{dq}}{1 - \frac{dq^B}{dq}} = \lim_{\hat{q} \to 1} \left( 1 - \frac{q^B}{1 - \hat{q}} \right)^2 + 1
\]

Then note that

\[
\lim_{\hat{q} \to 1} \frac{1 - \hat{q}}{1 - q^B} = \lim_{\hat{q} \to 1} \frac{-\frac{1}{dq^B}}{\frac{1}{dq^B}} = \left( \lim_{\hat{q} \to 1} \frac{1 - \hat{q}}{1 - q^B} \right)^2
\]

so that \( \lim_{\hat{q} \to 1} \frac{1 - \hat{q}}{1 - q^B} \in \{0, 1\} \). This means \( \lim_{\hat{q} \to 1} \frac{(\hat{q} - q^B)(1 - 2\hat{q})}{q^B(1 - q^B)} \in \{1, 0\} \). Therefore, \( \lim_{\hat{q} \to 1} P_B \in \{\infty, V_H\} \). We can again rule out a price of \( \infty \), so \( \lim_{\hat{q} \to 1} \frac{1 - \hat{q}}{1 - q^B} = \lim_{\hat{q} \to 1} \frac{1 - q^B}{1 - \hat{q}} = 1 \). Finally, the limit of the sufficient condition is

\[
\lim_{\hat{q} \to 1} 1 - P_B \left( \frac{\hat{q}(1 - \hat{q})}{\hat{q} - q^B} \right)^2 = 1 - V_H \lim_{\hat{q} \to 1} \left( \frac{\hat{q}(1 - \hat{q})}{\hat{q} - q^B} \right)^2 = 1 - V_H \left( \lim_{\hat{q} \to 1} \frac{-1}{1 - \frac{dq^B}{dq}} \right)^2 = \infty,
\]

and the sufficient condition holds for all parameter values.

\[\text{Proof of Proposition 5.}\] We can easily see that the left-hand side of (10) is decreasing in \( c \):

\[
\frac{dFOC(P_B)}{dc} = -\frac{dP_B}{dc} \left( \frac{\hat{q}(1 - \hat{q})}{\hat{q} - q^B} \right)^2 - 2P_B(\hat{q}(1 - \hat{q}))^2(\hat{q} - q^B)^{-3} \frac{dq^B}{dc} < 0.
\]
We must then examine the limits as \( c \) approaches 0 and \( \infty \). As \( c \to 0 \), \( q^B \to 0 \). Then to see what happens to \( P^B \), consider

\[
\lim_{c \to 0} (1 - 2\hat{q})(\ln B - \hat{\ln}) \frac{2c}{\gamma} = (1 - 2\hat{q}) \frac{2}{\gamma} \lim_{c \to 0} c \ln B
\]

\[
= (1 - 2\hat{q}) \frac{2}{\gamma} \lim_{c \to 0} \frac{\ln B}{1/c}
\]

\[
= (1 - 2\hat{q}) \frac{2}{\gamma} \lim_{c \to 0} \frac{-\frac{1}{2} q^B (1 - q^B) \frac{dq^B}{dc}}{-1/c^2}
\]

\[
= (1 - 2\hat{q})(V_H - V_L) \lim_{c \to 0} q^B (1 - q^B)
\]

\[
= 0.
\]

\[
\lim_{c \to 0} -\frac{2c}{\gamma} \frac{q^B - q^B(1 - q^B)}{q^B(1 - q^B)} = -\frac{2}{\gamma} \hat{q} \lim_{c \to 0} \frac{1}{q^B(1 - q^B) \frac{dq^B}{dc}}
\]

\[
= -\left( \frac{2}{\gamma} \right)^2 \hat{q} V_H - V_L \lim_{c \to 0} \frac{1}{q^B(1 - q^B)}
\]

\[
= (1 - 2\hat{q})(V_H - V_L) \hat{q} (1 - \hat{q})
\]

This means \( \lim_{c \to 0} \frac{c}{q^B(1 - q^B)} = \frac{\gamma}{2}(V_H - V_L) \), so that \( \lim_{c \to 0} -\frac{2c}{\gamma} \frac{q^B - q^B(1 - q^B)}{q^B(1 - q^B)} = -\hat{q}(V_H - V_L) \).

Overall, \( \lim_{c \to 0} P^B = V_L \), so that \( \lim_{c \to 0} FOC(P^B) = 1 - V_L(1 - \hat{q})^2 \). The sufficient condition will always hold for \( V_L \leq 0 \), and for \( V_L > 0 \) if \( \hat{q} > 1 - V_L^{-1/2} \).

Next, as \( c \to \infty \), \( q^B \to \hat{q} \). To see what happens to \( P^B \), consider

\[
\lim_{c \to \infty} \frac{2c}{\gamma} (1 - 2\hat{q})(\ln B - \hat{\ln}) = \frac{2}{\gamma} (1 - 2\hat{q}) \lim_{c \to \infty} \frac{\ln \left( \frac{1 - q^B}{1 - \hat{q}} \right)}{1/c}
\]

\[
= (1 - 2\hat{q}) \frac{2}{\gamma} \lim_{c \to \infty} \frac{-\frac{1}{2} q^B (1 - q^B) \frac{dq^B}{dc}}{-1/c^2}
\]

\[
= (1 - 2\hat{q})(V_H - V_L) \lim_{c \to \infty} q^B (1 - q^B)
\]

\[
= (1 - 2\hat{q})(V_H - V_L) \hat{q} (1 - \hat{q}).
\]
\[
\lim_{c \to \infty} -\frac{2c}{\gamma} (\hat{q} - q^B)(1 - 2q^B) = -\frac{2}{\gamma} \frac{1 - 2\hat{q}}{q(1 - \hat{q})} \lim_{c \to \infty} \frac{\hat{q} - q^B}{1/c} = -\frac{2}{\gamma} \frac{1 - 2\hat{q}}{q(1 - \hat{q})} \lim_{c \to \infty} \frac{-dq^B}{-\frac{1}{c^2}} = -(1 - 2\hat{q})(V_H - V_L)\hat{q}(1 - \hat{q})
\]

Therefore, \(\lim_{c \to \infty} P^B = \hat{q}V_H + (1 - \hat{q})V_L\). Then \(\lim_{c \to \infty} FOC(P^B) = -\infty\). Then (10) never holds, regardless of other parameters. Because \(FOC(P^B)\) is decreasing in \(c\), this implies the existence of \(c'\) and \(c''\). A symmetric argument can be made for \(\gamma\). ■

**Proof of Proposition 7.** The first- and second-order conditions (FOC and SOC) for \(\Pi\) are as follows:

\[
\frac{\partial \Pi}{\partial P} = \frac{\hat{q} - q(P)}{\bar{q}(P) - q(P)} - P\left[\frac{(\bar{q}(P) - \hat{q})q(P)^2(1 - q(P))^2 + (\hat{q} - q(P))\bar{q}(P)^2(1 - \bar{q}(P))^2}{(\bar{q}(P) - q(P))^3}\right]
\]

\[
\frac{\partial^2 \Pi}{\partial P^2} = -2\frac{(\bar{q} - \hat{q})q^2(1 - q)^2 + (\hat{q} - \bar{q})\bar{q}^2(1 - \bar{q})^2}{(\bar{q} - \hat{q})^3}
\]

\[
- P\left(\frac{2(\bar{q} - \hat{q})q^3(1 - q)^3(\bar{q} - \hat{q})(1 - 2q) - \hat{q}^4(1 - \bar{q})^3(\hat{q} - \bar{q})(2\hat{q} - 1))}{(\bar{q} - \hat{q})^5}\right)
\]

\[
- P\left(\frac{3q^4(1 - q)^4(\bar{q} - \hat{q}) - \bar{q}^4(1 - \bar{q})^4(\bar{q} - \hat{q}) + q^2\bar{q}^2(1 - q)^2(1 - \bar{q})^2(2\hat{q} - \bar{q} - q))}{(\bar{q} - \hat{q})^5}\right).
\]

If the solution is interior, it will be characterized by the FOC, and the SOC will be negative. Applying the implicit function theorem,

\[
\frac{dP}{dq} = -\frac{1}{SOC} \frac{\partial FOC}{\partial \hat{q}}.
\]

Therefore, the effect on price will have the same sign as the marginal effect on the FOC, given by

\[
\frac{\partial FOC}{\partial \hat{q}} = \frac{1}{\bar{q} - \hat{q}} - \frac{P}{(\bar{q} - \hat{q})^2} \left(\frac{-dq}{dP} + \frac{d\hat{q}}{dP}\right)
\]

\[
= \frac{1}{\bar{q} - \hat{q}} \left[1 - \frac{(\bar{q} - \hat{q})(\bar{q}^2(1 - \bar{q})^2 - \hat{q}^2(1 - q)^2)}{(\bar{q} - \hat{q})\bar{q}^2(1 - \bar{q})^2 + (\bar{q} - \hat{q})q^2(1 - \bar{q}^2)}\right] > 0,
\]

so at an interior maximum, an increase in prior belief always increases price.

For profit, if \(\Pi^* = P^*\Pr(P^*)\), the solution is interior. Apply the envelope theorem to get
\[ \frac{d\Pi}{dX} = \frac{\partial\Pi}{\partial X}. \] For \( \hat{q} \), this takes a particularly simple form:

\[ \frac{d\Pi}{dq} = \frac{P^*}{q - q} > 0. \]

**Proof of Proposition 8.** Using the same strategy as in Proposition 7, the marginal effects of the parameters are given by

\[
\begin{bmatrix}
\frac{dP}{dc} & \frac{dP}{d\gamma}
\end{bmatrix} = -\frac{1}{SOC} \begin{bmatrix}
\frac{\partial FOC}{\partial c} & \frac{\partial FOC}{\partial \gamma}
\end{bmatrix}.
\]

Therefore, the effect on price will have the same sign as the marginal effect on the FOC. For \( X \in \{c, \gamma\} \) the change in FOC will be given by

\[
\frac{\partial FOC}{\partial X} = \frac{-\hat{q} - q}{(q - q)^2} d\hat{q} d\hat{X} - \frac{P}{(q - q)^3} d\hat{q} \left( \frac{dq}{dP} (3\hat{q} - q - 2\hat{q}) - 3 \frac{d\hat{q}}{dP} (\hat{q} - q) + 2\hat{q}(1 - \hat{q})(1 - 2\hat{q}) \right) + \frac{P}{(q - q)^3} d\hat{q} \left( \frac{dq}{dP} (3\hat{q} - q - 2\hat{q}) + 3 \frac{d\hat{q}}{dP} (\hat{q} - q) + 2\hat{q}(1 - q)(1 - 2\hat{q}) \right).
\]

This can be simplified by plugging in \( P^* \) from the FOC.

\[
\frac{\partial FOC}{\partial X} = \frac{1}{(q - q)[(q - q)q^2(1 - \hat{q})^2 + (q - q)q^2(1 - q)^2]} \left( -\frac{d\hat{q}}{dX} (\hat{q} - q) \left( \frac{dq}{dP} (2\hat{q} - q - \hat{q}) - 2 \frac{d\hat{q}}{dP} (\hat{q} - q) + 2(\hat{q} - q) \hat{q}(1 - \hat{q})(1 - 2\hat{q}) \right) \right) \left( 2 \frac{d\hat{q}}{dP} (\hat{q} - q)^2 + \frac{dq}{dP} (\hat{q} - q)(2\hat{q} - 3q) + 2(\hat{q} - q)(\hat{q} - q) q(1 - q)(1 - 2\hat{q}) \right).
\]

The first term does not help determine sign, as it is always positive, and is dropped in the statement of the proposition in the main text.

**Proof of Proposition 9.** It is equivalent to consider a change in \( \hat{q} \) where \( V_H = \frac{1}{q}(\hat{V} - (1 - \hat{q})V_L) \). Then

\[
P^B = \hat{V} - \frac{2c}{\gamma} \left( -(1 - 2\hat{q})(\ln^B - \hat{ln}) + \frac{(\hat{q} - q^B)(1 - 2q^B)}{q^B (1 - q^B)} \right),
\]

\[ 36 \]
where $q^B$ is defined by

$$\frac{2c}{\gamma} \left( \frac{1 - 2q^B}{q^B(1 - q^B)} + 2\ln B \right) = \frac{2c}{\gamma} \left( \frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})} + 2\ln \frac{B}{\hat{q}} \right) + \frac{1}{\hat{q}} (\hat{V} - V_L).$$

First consider how $q^B$ changes in $\hat{q}$. We have

$$f = \frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})} + 2\ln \frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})} - 2\ln \frac{\gamma}{2c \hat{q}} (\hat{V} - V_L)$$

$$\frac{\partial f}{\partial q^B} = - \frac{1}{(q^B)^2(1 - q^B)^2}$$

$$\frac{\partial f}{\partial \hat{q}} = \frac{1}{\hat{q}^2(1 - \hat{q})^2} \left( 4\hat{q}(1 - \hat{q}) + (1 - 2\hat{q})^2 + \frac{\gamma}{2c} (\hat{V} - V_L)(1 - \hat{q})^2 \right),$$

so that

$$\frac{dq^B}{dq} = - \frac{\partial f}{\partial \hat{q}} \frac{\partial f}{\partial q^B} = \left( \frac{q^B(1 - q^B)}{\hat{q}(1 - \hat{q})} \right)^2 \left( 1 + \frac{\gamma}{2c} (\hat{V} - V_L)(1 - \hat{q})^2 \right).$$

Plugging this and the fact that $f = 0$ into $P^B$ yields

$$\frac{dP^B}{dq} = - \frac{2c}{\gamma} \left( \frac{q^B}{\hat{q}} \left( 2(\ln B - \hat{\ln}) - \frac{1 - 2\hat{q}}{\hat{q}(1 - \hat{q})} + \frac{1 - 2q^B}{q^B(1 - q^B)} \right) - \frac{\hat{q} - q^B}{q^2(1 - q)^2} \right) \geq 0$$

**Proof of Proposition 10.** It is equivalent to consider a change in $\hat{q}$ where $V_H = \frac{1}{\hat{q}} (\hat{V} - (1 - \hat{q})V_L)$. Some dispersion is necessary for search. We need only rule out the extremal solution, or $\hat{q} = 0$. We will do this by showing the first-order condition of profit is positive as $\hat{q} \to 0$. With the change in choice variable, the first-order condition becomes

$$\frac{d\Pi}{d(\hat{q})} = \frac{P^*}{(\hat{q} - q)^2} \left( \frac{\hat{q} - q}{\hat{q}^4} - \frac{\hat{V} - V_L}{\hat{q}^2(\hat{q} - q)} (\hat{q} - q)^2 (1 - \hat{q})^2 + (\hat{q} - q)\hat{q}^2 (1 - \hat{q})^2 \right).$$

Then, because $\hat{q}$ and $q \to 0$ as $\hat{q} \to 0$,

$$\lim_{\hat{q} \to 0} \frac{d\Pi}{dq} = \lim_{\hat{q} \to 0} P^* \left( \frac{1}{q - \hat{q}} - (\hat{q} - V_L) \frac{\hat{q}q}{q(\hat{q} - q)^2} \right).$$
To continue, use the comparative statics result from Proposition 2

\[
\frac{dq}{dq} = \frac{dq}{dV_H} \frac{\partial V_H}{\partial q} = \frac{\hat{V} - V_L}{q^2(q - \bar{q})} \bar{q} q^2 (1 - \bar{q})^2 \]

\[
\frac{d\bar{q}}{dq} = \frac{d\bar{q}}{dV_H} \frac{\partial V_H}{\partial \bar{q}} = \frac{\hat{V} - V_L}{q^2(q - \bar{q})} \bar{q} q^2 (1 - \bar{q})^2 .
\]

Then,

\[
\lim_{\bar{q} \to 0} \frac{\bar{q} - \bar{q}}{\bar{q}(\bar{q} - q)^2} = \lim_{\bar{q} \to 0} \frac{\hat{V} - V_L}{q^2(q - \bar{q})} \bar{q} q^2 (1 - \bar{q})^2 + q^2 \bar{q}^2 (1 - \bar{q})^2 
\]

\[
= \lim_{\bar{q} \to 0} \frac{\bar{q} q}{\bar{q}(\bar{q} - q)^2} \frac{2\bar{q} q \hat{V} - V_L}{q^2(q - \bar{q})^2 + 2(\hat{V} - V_L)\bar{q} q} .
\]

So either the limit of the first term is 0, or the limit of the second term is 1. Consider

\[
\lim_{\bar{q} \to 0} \frac{\bar{q} - q}{q} = \lim_{\bar{q} \to 0} \frac{\hat{V} - V_L}{q^2(q - \bar{q})} \bar{q} q^2 (1 - \bar{q})^2 - \bar{q} q^2 (1 - \bar{q})^2 = (\hat{V} - V_L) \lim_{\bar{q} \to 0} \frac{\bar{q} q}{q^2} .
\]

Then

\[
\lim_{\bar{q} \to 0} \frac{2\bar{q} q \hat{V} - V_L}{q^2(q - \bar{q})^2 + 2(\hat{V} - V_L)\bar{q} q} = (\hat{V} - V_L) \lim_{\bar{q} \to 0} \frac{2\bar{q} q}{q^2} + 2\bar{q} q = (\hat{V} - V_L) \lim_{\bar{q} \to 0} \frac{2}{1 + 2\bar{q}^2} = 2(\hat{V} - V_L) \neq 1 .
\]

Finally,

\[
\lim_{\bar{q} \to 0} \frac{\bar{q} q}{\bar{q}(\bar{q} - q)^2} = 0 ,
\]

so that

\[
\lim_{\bar{q} \to 0} \frac{d\Pi^*}{dq} = \infty .
\]

\[
\square
\]

References


