Optimal Unemployment Insurance in a Directed Search Model*

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Abstract

Over the last few decades, several economies have experienced changes in the level of aggregate volatility and a rise in wage dispersion. This paper investigates the appropriate labor market policy response to these changes. We introduce unemployment benefits financed by a proportional earnings tax within a model of directed search on the job. We show that there exists a unique positive level of unemployment benefit which maximizes \textit{ex ante} welfare of individuals. The optimal unemployment benefit level is hump-shaped as a function of the level of idiosyncratic risk, and the welfare costs of deviating from the optimal level, plagued by high unemployment rates, are substantial. On the other hand, while the optimal generosity of the unemployment insurance program, which is pro-cyclical, declines monotonically with the amount of aggregate risk in the economy, the welfare costs of deviating from the optimal system are rather small.

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†Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. The research in this paper does not use any confidential Census Bureau information.
1 Introduction

Changes in volatility at the aggregate and individual level over the past few decades have been noted in numerous studies for the US and other economies. Stock and Watson (2005) document a reduction in the volatility of output growth and a moderation of business cycle fluctuations in most G7 economies over the two decades that preceded the last recession. Meanwhile, the great recession sparked a literature that broadly argues that changes in uncertainty or risk are important factors in understanding the evolution of the economy in general and in particular at business cycle frequencies (for example, see Bloom (2009)). Similarly, a vast literature studies the evolution of risk at the individual level. Heathcote et al. (2010) document an increase in wage dispersion in the U.S. over the past three decades. Song et al. (2015) show that the majority of the rise in earnings inequality can be attributed to the rise in inequality of workers across firms rather than within firms. While we remain agnostic about the source of these fundamental changes, the goal of this paper is to investigate their impact on labor market outcomes as well as the appropriate policy response to these changes.

We cast these questions within a directed search model of the labor market with on the job search. Specifically, we introduce a simple government-run unemployment insurance system—an unemployment benefit financed by a proportional earnings tax—within a model of directed search along the lines of Menzio and Shi (2010, 2011). This framework is particularly well suited for our purpose for two main reasons. First, this framework has been shown to generate several important features of the labor market. Second, the model lends itself well to the introduction of both idiosyncratic and aggregate uncertainty, which are at the heart of this paper. While it is possible to introduce both types of uncertainty in a random matching model (see for example Moscarini and Postel-Vinay (2010), Krusell et al. (2010), Postel-Vinay and Turon (2010), Alvarez and Shimer (2011), and Lise and Robin (2017)), directed search models are notoriously more tractable.

1 Other papers in the same issue of the Review of Economic Dynamics present similar facts for other countries.

2 For example, these models have been shown to generate: residual wage inequality; positive returns to tenure and experience; appropriate worker flows between employment, unemployment, and across employers; and volatility of unemployment and vacancies over the business cycle (see Delacroix and Shi (2006), Menzio and Shi (2011), Shi (2009), Gonzalez and Shi (2010), and Herkenhoff (2017)).

3 Technically, this is because equilibria are ‘block recursive,’ so that individuals’ and firms’ decisions are functions of the aggregate state, but not the distribution of workers: see Shi (2009) and Menzio and Shi (2010, 2011).
In our framework, if a firm successfully fills a posted vacancy, the output of the match depends on aggregate and idiosyncratic productivity. The firm offers a sequence of wages (or future utility) to a risk-averse worker to deliver the current promised lifetime utility to the worker. If the aggregate or idiosyncratic component of productivity in a match decreases sufficiently, the firm and the worker end the match endogenously according to the designed contract. An active match may also be destroyed due to successful on-the-job search by the worker, or from an exogenous separation shock. An employed individual’s consumption is equal to her wage net of a proportional tax, while an unemployed worker’s consumption is equal to unemployment benefits provided by the government. Naturally, earnings taxes are used to finance the unemployment insurance system, balancing the budget in long-run equilibrium.

We begin our analysis by studying an environment which abstracts from idiosyncratic and aggregate uncertainty. In this context, we prove existence of an optimal unemployment insurance system which provides positive benefits to unemployed individuals without providing full insurance. The intuition for this result is straightforward: while an increase in unemployment benefits allows (risk-averse) individuals to better smooth consumption between states of employment and unemployment, it also raises the unemployment rate and thus lowers average consumption. The increase in the unemployment rate arises as individuals look for vacancies with higher wages, which on average take more time to find. The budget-balancing tax rate is convex in the level of benefit: as benefits increase, not only is each unemployed individual more costly to insure each period, but unemployment spells are longer as unemployed individuals search for better jobs.

Next, we introduce idiosyncratic uncertainty, which takes the form of a stochastic process governing the idiosyncratic component of productivity. Again, we find a unique optimal level of unemployment benefit. Interestingly, the optimal unemployment benefit is hump-shaped as a function of the level of risk, keeping the mean level of the idiosyncratic component of productivity constant at zero. Initially, an increase in idiosyncratic risk calls for a more generous unemployment insurance system in order to provide additional insurance for individuals. However, as the level of risk increases further, generous unemployment benefits become unsustainable. This is because matches which experience low productivity shocks are endogenously destroyed, shrinking the pool of employed workers who pay for the benefit. As a result, at sufficiently high levels of idiosyncratic risk, the optimal level of unemployment benefit falls sharply as the budget-balancing tax rate increases.
When we introduce aggregate uncertainty, the optimal generosity of the unemployment insurance system declines monotonically with the amount of aggregate risk, keeping the average level of the aggregate component of productivity constant. Intuitively, with a fixed benefit, unemployment becomes relatively more attractive in bad times than it becomes less attractive in good times as risk increases. As a result, the unemployment rate increases and the pool of employed workers who pay for the benefit shrinks. A lower level of unemployment benefit restores the balance between consumption smoothing and job creation, i.e. the incentive for individuals to search for vacancies with relatively high job finding probabilities. For similar reasons, a pro-cyclical benefit is more desirable than a counter-cyclical (or acyclical) one: under a fixed benefit level, the equilibrium allocation features ‘too much’ consumption smoothing in bad times (when wages are relatively low) and ‘too little’ consumption smoothing in good times (when wages are relatively high). It should be pointed out, however, that in the relevant range of aggregate risk, the welfare costs of deviating from the optimal unemployment insurance system are rather small.

The economic mechanism at the center of our analysis is that higher unemployment benefits induce individuals to search for jobs that provide higher wages, which, because these jobs take more time to find, increases unemployment duration. As such, our work is related to Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), who study optimal unemployment insurance in the context of a model where individuals’ search effort is private information. Instead of searching for better jobs with a constant search effort, individuals in their environment search for the same job with a lower intensity when unemployment benefits increase. The idea is nevertheless that optimal unemployment insurance strikes the right balance between risk sharing and unemployment duration.

To extend our analysis and better relate our work to this literature, we investigate an environment in which unemployment benefits comprise two parts: a fixed amount, which can be thought of as a tax-financed welfare payment, and an unemployment benefit which expires stochastically at any time during an individual’s unemployment spell. We find that

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4In the context of our model, one can think of the market (or wage) to which individuals direct their search as private information, creating a moral hazard problem which prevents a private insurer from conditioning benefits on the search strategy that individuals choose.

5Empirical evidence suggests that search effort, though it varies, is small. Krueger and Mueller (2010) find that the average unemployed person searches for 41 minutes on a weekday. Further, the cyclicality of search effort is unclear. Mukoyama et al. (2017) find that search effort is countercyclical, while Gomme and Lkhagvasuren (2015) find support for pro-cyclical search effort.

6The modeling strategy of letting benefits expire stochastically, also used in Mitman and Rabinovich (2015) and Zhang and Faig (2012), greatly simplifies computing equilibria.
welfare is increasing in the expected duration of unemployment benefits. That being said, there is no role in our framework for declining or expiring unemployment benefits. While it is the case that unemployed individuals who lose their benefits do find jobs faster than those who receive benefits—the usual argument for benefits to expire—the gains from a lower tax rate never justify the loss of consumption smoothing.

Unemployment insurance has also been studied in search and matching models of the labor market. In a directed search environment, Acemoglu and Shimer (1999) also find a unique utility maximizing level of unemployment benefit, which in their case is financed by a lump-sum tax. In addition, they prove that a similar result holds for an output-maximizing level of unemployment. While the mechanism for their first result is similar to ours, their latter result is due to the way in which they model the cost of vacancies. They interpret this cost as capital, which firms choose and put in place before the vacancy is filled. Since higher unemployment benefits induce individuals to search for jobs associated with higher wages, which take longer to find, firms have an easier time filling their vacancies. The capital put in place prior to filling the vacancy thus stays idle for a shorter period of time, inducing firms to put more capital in place.

Mitman and Rabinovich (2015) use a Diamond-Mortensen-Pissarides model and optimize the unemployment insurance policy (benefit level and expected duration) using a Ramsey approach where benefits are financed by lump-sum taxes on firm profits. They find the optimal benefit policy to be pro-cyclical in both level and duration, in the sense that both are negatively correlated with unemployment and productivity. As both vacancy creation and search effort are affected by the unemployment benefit in their model, the pro-cyclicality of unemployment benefits arises from a similar tradeoff of insurance and job creation. As in our model, there is no borrowing/saving decision, the government budget constraint holds in expectation, and benefit expiration is stochastic. The welfare cost of deviating from the pro-cyclical optimal policy is high in their framework relative to ours. Search effort in their model produces a high elasticity of the unemployment rate with respect to unemployment benefits in general equilibrium. This is in stark contrast to Landais et al. (2016), who find a small macro elasticity of unemployment to benefits and large gains from counter-cyclical benefits when recessions are characterized by ‘job rationing’ that stems from real wage rigidities. In this context, increasing benefits in recessions improves consumption smoothing without adversely increasing unemployment duration.

Krusell et al. (2010) study unemployment insurance in a Bewley-Huggett-Aiyagari type
model in which the labor market functions as in Diamond-Mortensen-Pissarides. One of their main findings is that unemployment insurance provides little value over and above self-insurance. The optimal replacement ratio in their benchmark economy is lower than the 40 percent they use as a proxy for the U.S. economy. They speculate that the gains from running a cyclical unemployment benefit scheme are rather small, a result which is confirmed by our model. While the generosity of the optimal unemployment insurance system would clearly decline if individuals could self-insure in our model, our focus is on the change in optimal benefits from changes in volatility, and we believe these results would remain similar.\footnote{Evidence from Ganong and Noel (2016) suggests that the extent to which individuals self-insure before or during unemployment spells is very limited. Their consumption path decreases, both in unemployment and when unemployment benefits expire. While a savings decision could be added to our framework along the lines of Chaumont and Shi (Chaumont and Shi), we believe the change in optimal unemployment benefits due to changes in fundamentals would be similar.} Despite the different nature of our modeling choices, the mechanism is nevertheless similar: in their environment, an increase in benefits provides more insurance, but discourages firm entry. This in turn means that it becomes harder for individuals to find a match, much like in our model.

The rest of the paper is organized as follows. The next section presents the economic environment: we introduce a simple unemployment insurance system in a directed search model of on the job search. In section 3, we characterize the optimal unemployment system in an economy which abstracts from (aggregate and idiosyncratic) uncertainty. In section 4 we present numerical results, successively studying the impact of introducing idiosyncratic then aggregate uncertainty. In section 5, we show that our results are robust to an alternative formulation of the unemployment insurance system where unemployment benefits are a function of past employment experience, i.e. has a replacement ratio element. We also study the desirability of expiring benefits and whether unemployment benefits should be extended for longer periods of time in recessions, a policy that was implemented in the U.S. during the great recession. A short conclusion is offered in section 6.

\section{Economic Environment}

Time is discrete and continues forever. The economy is populated by a continuum of risk-averse workers with measure one and a continuum of firms with positive measure.\footnote{Free-entry determines the measure of firms in the economy endogenously.} Workers
maximize the expected sum of per-period utilities discounted at factor \( \beta \in (0, 1) \), with a periodical utility function \( \nu(c) \). Employed workers’ consumption is equal to after-tax labor income, and unemployed workers’ consumption is equal to the unemployment benefit, \( b \), paid by the government.

Each firm produces output through a constant return to scale production function, \((y+z)n\), with \( n \in \{0, 1\} \). When a firm is matched with a worker, its output depends on an aggregate and an idiosyncratic component of productivity. The aggregate component of productivity \( y \) is common across all firms, and its values lie in the set \( y \in Y = \{ y_1, y_2, \ldots, y_{N_Y} \} \), where \( \underline{y} = y_1 \leq y_2 \leq \ldots \leq y_{N_Y} = \bar{y} \) and \( N_Y \geq 1 \) is an integer. The idiosyncratic component of productivity is \( z \in Z = \{ z_1, z_2, \ldots, z_{N_Z} \} \), where \( \underline{z} = z_1 \leq z_2 \leq \ldots \leq z_{N_Z} = \bar{z} \) and \( N_Z \geq 1 \) is an integer. The firm maximizes the expected sum of profits discounted at factor \( \beta \).

The labor market is organized as a continuum of submarkets \( x \in X = [\underline{x}, \bar{x}] \), where \( \underline{x} = \nu(b)/(1-\beta) \) and \( \bar{x} = \nu(\bar{y} + \bar{z})/(1-\beta) \). If a firm meets a worker in submarket \( x \), the firm offers the worker expected lifetime utility \( x \), and we refer to it as the value of that submarket. The probability of finding a job and the probability of filling a vacancy in a submarket depend on the tightness ratio in that submarket. The tightness ratio in submarket \( x \) is defined as the ratio of the number of vacancies created by firms to the number of individuals who search for jobs in that particular submarket, denoted \( \theta(x, \psi) \geq 0 \), where \( \psi \in \Psi \) is the aggregate state of the economy at the beginning of each period.

The aggregate state of the economy \( \psi \) consists of aggregate productivity \( y \) and the distribution of workers among employment positions \( (u, e) \): \( u \) is the measure of unemployed individuals, and \( e \) is the distribution of employed workers. Menzio and Shi (2010) show that the agents’ value and policy functions as well as tightness ratios depend on the aggregate state of the economy \( \psi \) only through aggregate productivity \( y \), and not through the distribution of workers across different employment states, \( (u, e) \). Intuitively, since search is directed, each individual targets his preferred submarket regardless of the distribution of individuals. Similarly, because of free entry (see below), vacancies are created in any submarket until firms make zero profits. Thus, we hereafter write value functions, policy rules, and tightness ratios as function of aggregate productivity, not the aggregate state of economy.

Each period consists of four stages: separation, search, matching, and production. In the separation stage, workers get separated from their match with probability \( d \in [\delta, 1] \), where \( \delta \) denotes the exogenous probability of separation. Matches can also be endogenously
destroyed by setting \( d = 1 \). When a worker loses his job, he must remain unemployed until the start of the searching stage next period.

During the search stage, an individual who has the opportunity to search first decides in which submarket to direct his search. While all individuals who have been unemployed for at least one period have the opportunity to search, employed workers only have the opportunity to search with probability \( \lambda_e \in [0, 1] \). As mentioned above, workers whose unemployment spell started this period cannot search this period. Firms also choose submarkets in which to open vacancies during the search stage.

A firm who meets a worker in submarket \( x \) during the matching stage offers the worker a contract which delivers expected lifetime utility \( x \) to the worker. If the worker accepts the offer, he starts working at the new job in the following production stage. Otherwise, the worker stays in his previous employment position (which could be unemployment). Firms and workers cannot coordinate their actions because of search frictions in the labor market: not all workers succeed in finding a job, and not all firms succeed in hiring a worker. Matches are formed in each submarket via a constant returns to scale matching function. A worker searching in a submarket characterized by tightness ratio \( \theta \) finds a job with probability \( p(\theta) \), while firms in that submarket fill vacancies with probability \( q(\theta) \), with \( q(\theta) = p(\theta)/\theta \).

During the production stage, an employed worker produces \( y+z \), and consumes after-tax wage \( (1-\tau)\omega \). Therefore, the firm’s profits equal \( y+z-\omega \) in that period. Unemployed workers receive and consume fixed unemployment benefit \( b \).\(^9\) At the end of the production stage, nature draws next period’s aggregate productivity \( y' \) and idiosyncratic productivity \( z' \) from the probability distributions \( \Phi_y(y'|y) \) and \( \Phi_z(z'|z) \) respectively.

### 2.1 Worker’s Problem

Consider an individual whose employment status can be summarized by his lifetime utility \( V \). During the search stage, he chooses in which submarket to search in order to maximize his value of search. If the worker searches in submarket \( x \), he finds a job with probability \( p(\theta(x,y)) \) and the job provides lifetime utility \( x \). If he fails to find a job, which occurs with probability \( (1-p(\theta(x,y))) \), he retains his current employment status in the production stage.

\(^9\)In section 5 we explore the possibility that unemployment benefits depend on past earnings (a replacement ratio), as well as the possibility for benefits to expire.
Accordingly, an individual with current lifetime utility $V$ who has the opportunity to search chooses the submarket which maximizes his lifetime utility at the beginning of the search stage, $V + R(V, y)$, where the second term is the value of search at the beginning of the search stage. The worker’s problem at the search stage can thus be written as

$$R(V, y) = \max_{x \in X} p(\theta(x, y))(x - V).$$

(1)

The solution to the above maximization, the optimal submarket in which to search, is denoted $m(V, y)$. To ease notation, let $\tilde{p}(V, y)$ denote the probability of finding a job in the optimal submarket, i.e. $\tilde{p}(V, y) = p(\theta(m(V, y), y))$.

Let $U(y)$ denote the value function of an unemployed worker at the beginning of the production stage. This lifetime utility consists of the current value of consuming unemployment benefit, $b$, and the value of being unemployed and searching tomorrow:

$$U(y) = \nu(b) + \beta E\{U(y') + R(U(y'), y')\}$$

(2)

where the value of a variable next period is denoted by a prime symbol ($'$).

### 2.2 Firm’s Problem

During the matching stage, firms offer contracts $c \in C$ to workers. A contract specifies the current wage $\omega$, the probability that the match will be destroyed in the next separation stage $d'$, and the worker’s lifetime utility $V'$ at the beginning of the next period for each possible state $s' \in S = Y \times Z$. This future utility will be attained by an implicit sequence of future wages and unemployment benefits. The firm chooses the contract to maximize expected lifetime profits $J(V, s)$, where $s = (y, z)$, while delivering the lifetime utility previously contracted (promise-keeping constraint) and remaining compatible with the worker’s option to go to unemployment (individual rationality constraint).\textsuperscript{10} The problem of a firm

\textsuperscript{10}Note that it is in the best interest of the firm to endogenously separate the match when the individual rationality constraint binds. In essence, workers and firms agree on when to endogenously separate.
matched with a worker who was promised lifetime utility $V$ is therefore given by

$$J(V, s) = \max_{\omega, d', V'} \left\{ y + z - \omega + \beta E_{s'} \left[ (1 - d'(s'))(1 - \lambda_e \tilde{p}(V'(s'), y')) J(V'(s'), s') \right] \right\},$$

subject to

$$V = \nu ((1 - \tau) \omega) + \beta E_{s'} \left\{ d'(s') U(y') + (1 - d'(s')) \left[ V'(s') + \lambda_e R(V'(s'), y') \right] \right\},$$

$$d'(s') = \begin{cases} \delta & \text{if } U(y') \leq V'(s') + \lambda_e R(V'(s'), y'); \\ 1 & \text{otherwise}. \end{cases}$$

Let $c = (\omega, d', V')$ denote the optimal contract, with associated policy functions $\omega = \omega(V, s)$ for the wage, $d' = d'(V, s, s')$ for next period’s probability of separation, and $V' = V'(V, s, s')$ for the worker’s lifetime utility next period.

### 2.3 Market Tightness

During the search stage, a measure of firms choose whether to enter the labor market by opening a vacancy. Should they choose to enter, a firm posts how much lifetime utility it offer (i.e. choose a submarket $x$) for all potential applicants to see. New matches all start with initial idiosyncratic productivity of $\tilde{z}$ with certainty, from which $z$ fluctuates according to $\Phi_z(z'|z)$ in subsequent periods. Free entry ensures that vacancies open until the expected value of opening a vacancy is no more than the cost of creating it, denoted $\kappa$. Given the firm’s value of a match in submarket $x, J(x, y, \tilde{z}),$ where $\tilde{z} \in Z$ is the idiosyncratic component of productivity common to all new matches, and the probability of filling a vacancy in that submarket, $q(\theta(x, y))$, free entry implies that

$$\kappa \geq q(\theta(x, y)) J(x, y, \tilde{z}).$$

While the free entry condition must hold with equality for submarkets which are open in equilibrium (i.e. submarkets in which some individuals search), such need not be the case for unvisited submarkets. Following Acemoglu and Shimer (1999) and the subsequent literature, we assume that (4) holds with equality in all submarkets in a relevant range, that is, from the lowest submarket to the submarket where firms would just cover the cost of posting a vacancy with a job filling probability equal to one. Under this assumption, market tightness is a decreasing function of $x$ over the relevant range.
At this point, it may be worth discussing how the generosity of the unemployment insurance system makes its way through the model to affect various aspects of the labor market. Since the unemployment benefit is financed by a proportional tax on the worker’s wage, this tax increases the wage necessary for the firm to provide the same instantaneous utility to the worker, creating a wedge between the firm’s cost of labor relative to what workers receive. In other words, for a given promised utility $V$ in equation (3), the firm’s value $J(V, y)$ decreases as the tax rate increases.

Meanwhile, the free entry condition (4) implies that a firm opens a vacancy in a submarket if the expected benefit of the vacancy posting, $q(\theta(x, y))J(x, y, \tilde{z})$ is equal to the cost $\kappa$. With a higher tax rate and lower value $J$ at any submarket, the firm requires a higher probability $q$ of matching with a worker in order to post a vacancy in that submarket. This implies a lower tightness ratio and job-finding probability for workers in that submarket.

Finally, the workers’ search behavior is directly affected by the generosity of the system, as the value function of unemployed worker (2) is increasing in $b$. In addition, since $\theta$ is decreasing in $x$, $p(\theta(x, y))$ is also decreasing in $x$, and the solution $m(x, y)$ to the worker’s search problem (1) is increasing in $x$.\textsuperscript{11} As the value of unemployment $U$ increases and the submarket where unemployed workers direct their search increases, their realized job-finding probability $\tilde{p}(U, y)$ decreases. Intuitively, workers search for better jobs which have a lower probability of matching with an employer as the unemployment benefit increases.

### 2.4 Laws of Motion

We can compute the probability that a worker transits from one employment state to any other state using the optimal policy functions and the exogenous transition functions of the idiosyncratic productivity shock. Since the shocks $y$ and $z$ take on a finite number of values, a finite number of submarkets will be open in equilibrium. To simplify the exposition of the laws of motion, and slightly abusing notation, we redefine $X$ to be the set of all submarkets that are open in the long run, and let $N_x$ denote the cardinality of that set.\textsuperscript{12} Accordingly, let $e : X \times Y \times Z \rightarrow [0, 1]$ and $u : Y \rightarrow [0, 1]$ respectively denote the probability distribution over employed and unemployed workers at the end of the period. The laws of motion for workers between employment states can most easily be discussed fixing aggregate productivity today.

\textsuperscript{11}These properties are shown in detail in Menzio and Shi (2010).

\textsuperscript{12}Note that not all submarkets need to be open in all periods.
at some state $y$ and tomorrow at some state $y'$.

The measure of unemployed workers at the end of the period tomorrow, $u'(y')$, corresponds to the sum of unemployed workers whose search was unsuccessful plus employed workers who lost their job during the period. An unemployed worker remains unemployed with probability $(1 - \tilde{p}(U(y'),y'))$. As specified by the optimal contract, an employed worker in state $(x, s)$ today will become unemployed tomorrow with probability $d'(x, s, s')$ if state $s' = (y', z')$ occurs tomorrow. Given a mass $e(x, s)$ of workers today, $\Phi_z(z'|z)$ gives the fraction of workers who transit to any state $z'$ tomorrow. The measure of unemployed workers next period can thus be written as

$$u'(y') = (1 - \tilde{p}(U(y'),y'))u + \sum_{x \in X} \sum_{z \in Z} \sum_{z' \in Z} \Phi_z(z'|z) d'(x, s, s') e(x, s).$$

Since all workers in new matches have idiosyncratic productivity $\tilde{z}$, the transition of workers to state $(x', z')$ when $z' \neq \tilde{z}$ is different from when $z' = \tilde{z}$. Let’s start with the case where $z' \neq \tilde{z}$, which is simpler since no one can end up with idiosyncratic shock $z'$ though successful search. For these states, the only way for a worker to end up in state $(x', s')$ with $z' \neq \tilde{z}$ is for his current match to survive $((1 - d'(x, s, s')))$, his search to be unsuccessful $((1 - \lambda e_\tilde{p}(V'(x, s, s'),y'))$, and for today’s contract to specify that tomorrow’s promised utility will be $x'$ if state $s'$ occurs. In other words, the contract must specify that $V'(x, s, s') = x'$. Accordingly, let $\mathbb{I}[V'(x, s, s') = x']$ be equal to 1 if that is the case, and 0 otherwise. Then we have

$$e'(x', s') = \sum_{x \in X} \sum_{z \in Z} \Phi_z(z'|z)(1 - d'(x, s, s'))(1 - \lambda e_\tilde{p}(V'(x, s, s'),y'))\mathbb{I}[V'(x, s, s') = x'] e(x, s).$$

Finally, for state $z' = \tilde{z}$, we have to account for individuals whose search is successful in addition to those who transit to that state though their contract with the firm. First, $x'$ could be the submarket in which unemployed workers search. Let $\mathbb{I}[m(U(y'),y') = x']$ be equal to 1 if that is the case, and 0 otherwise. Similarly, an individual whose contract specifies promised utility $V'$ tomorrow if state $s'$ occurs can search in market $x'$. Let $\mathbb{I}[m(V'(x, s, s'),y') = x']$ be equal to 1 if that is the case, and 0 otherwise. Putting it all together, letting $\tilde{s}' = (\tilde{z}', y')$,
we have
\[ e'(x', s') = \sum_{x \in X} \sum_{z \in Z} \Phi_z(z'|z)(1 - d'(x, s, s'))(1 - \lambda e\bar{p}(V'(x, s, s'), y'))[V'(x, s, s') = x']e(x, s) \]
\[ + \sum_{x \in X} \sum_{z \in Z} \Phi_z(z'|z)(1 - d'(x, s, s'))\lambda e\bar{p}(V'(x, s, s'), y')m(V'(x, s, s'), y') = x'u. \]

The measures defined above take the aggregate states as given. We can use these measures and combine them with the transition matrix of the aggregate state (\( \Phi_y(y'|y) \)) to obtain a transition matrix or Markov chain \( \Phi \), each element of which representing the probability of transiting from some state in \( X \times Y \times Z \) today to any state in \( X \times Y \times Z \) tomorrow. Below we use \( \phi^e: X \times Y \times Z \rightarrow [0, 1] \) and \( \phi^u: Y \rightarrow [0, 1] \) to denote the stationary probability distribution over employed and unemployed workers, respectively, associated with \( \Phi \).

### 2.5 Government

The government follows a fiscal policy which guarantees budget balance in the long run:

\[ b \sum_{y \in Y} \phi^u(y) = \tau \sum_{x \in X} \sum_{y \in Y} \sum_{z \in Z} \omega(x, y, z)e(x, y, z). \] (5)

The left-hand side represents transfers to unemployed workers, and the right-hand side corresponds to revenues from the payroll tax. Without aggregate uncertainty, the above budget constraint reduces to a period-by-period balanced-budget. With aggregate uncertainty, the budget is balanced on average or in the long run.\footnote{Making the program balance on average is useful to avoid having tax rates move with benefits (or the other way around) without explicitly modeling government debt. This abstraction is also used in Krusell et al. (2010) and Mitman and Rabinovich (2015).}

### 2.6 Block Recursive Equilibrium

We have exploited throughout the presentation of the economy the well-known fact that this type of environment admits what is known as a block recursive equilibrium. In a block recursive equilibrium, the functions \{\( \theta, R, m, U, J, c \)\} depend on the aggregate state of the
economy only through the aggregate component of productivity \((y)\) and not through the distribution of workers across employment states \((u,e)\).\(^{14}\)

**Definition 1** A Block Recursive Equilibrium consists of a fiscal policy \((b,\tau)\), a market tightness function \(\theta : X \times Y \to \mathbb{R}_+\), a search value function \(R : X \times Y \to \mathbb{R}_+\), a policy function \(m : X \times Y \to X\), an unemployment value function \(U : Y \to \mathbb{R}\), a value function for firms \(J : X \times Y \times Z \to \mathbb{R}\), a contract policy function \(c : X \times Y \times Z \to C\), a Markov chain \(\Phi : X \times Y \times Z \to X \times Y \times Z\), a stationary probability distribution over employed workers \(\phi^e : X \times Y \times Z \to [0,1]\), and a stationary probability distribution over unemployed workers \(\phi^u : Y \to [0,1]\). These functions satisfy the following requirements:

1. \(R\) satisfies (1) for all \((V,y) \in X \times Y\), and \(m\) is the associated policy function;
2. \(U\) satisfies (2) for all \(y \in Y\);
3. \(J\) satisfies (3) for all \((V,y,z) \in X \times Y \times Z\), and \(c\) is the associated policy function;
4. \(\theta\) satisfies (4) for all \((x,y) \in X \times Y\);
5. \(\Phi\) is derived from the policy functions \((m,c)\) and the exogenous transition functions for \(y\) and \(z\), and \(\phi^e\) and \(\phi^u\) are the associated stationary probability distributions;
6. the unemployment insurance system \((b,\tau)\) satisfies (5).

### 3 Optimal Unemployment Insurance: Analytics

This section establishes the existence of an optimal unemployment insurance system. Although the result holds more generally, we abstract from both idiosyncratic and aggregate uncertainty throughout this section. Furthermore, we assume that workers cannot search on the job, i.e. \(\lambda_e = 0\). Under these assumptions, there is only one open submarket in equilibrium: all unemployed individuals search in that submarket. From the firm’s problem, it is easy to show that (i) the state variable (lifetime utility of the worker) will remain constant throughout the life of the match, (ii) separation will only occur exogenously \((d = \delta)\), and (iii)

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\(^{14}\)See Menzio and Shi (2010) for a proof of existence and uniqueness of a block recursive equilibrium in an environment similar to ours.
the wage will be constant throughout the life of the match. The latter result implies that there is a one to one relationship between \( x \) and \( \omega \), which we loosely use interchangeably below. Accordingly, the value of the firm simplifies to

\[
J(\omega) = \frac{y - \omega}{1 - \beta(1 - \delta)}.
\]

(6)

In turn, the wage in submarket \( x \) must be given by

\[
x = V(\omega) = \frac{\nu((1 - \tau)\omega) + \beta \delta U}{1 - \beta(1 - \delta)},
\]

(7)

where the value of unemployment is

\[
U = \frac{\nu(b) + \beta R(U)}{1 - \beta}.
\]

(8)

The value of search is given by

\[
R(U) = p(\theta)(x - U),
\]

(9)

where \( x = m(U) \) and \( \theta \) satisfies the free entry condition

\[
k = q(\theta)J(\omega).
\]

(10)

Given the properties of the periodic utility function, it is quite trivial to show that there exists an optimal unemployment insurance system which provides positive unemployment benefits without providing full insurance to individuals: while zero benefits can be ruled out by Inada conditions, full insurance can be ruled out because it can only be achieved at zero consumption as all individuals would be unemployed. We nevertheless state that result in the following proposition.

**Proposition 1** Assume that the periodic utility function \( \nu \) is strictly increasing, strictly concave, continuously differentiable, and satisfies Inada conditions. Then there exists an optimal unemployment insurance system which provides positive benefits to unemployed workers without providing full insurance: \( 0 < b < (1 - \tau)\omega \).

**Proof.** By assumption, \( \lim_{c \to 0} \nu'(c) = \infty \). It follows that a small increase in \( b \) away from zero improves welfare. To see that full insurance is not optimal, assume that the unemployment insurance system does provide full insurance, i.e. \( b = (1 - \tau)w \). This implies
that individuals must choose to search in the submarket which provides exactly the same utility as unemployment, that is, \( m(U) = U \) and thus \( R(U) = 0 \). For that to be the case, it must be that \( p(\theta(x)) = 0 \) for all \( x > U \). Since \( J \) in equation (6) is a continuous function of \( x \) (or \( \omega \)), the tightness ratio is also continuous (see equation (10)). By continuity of \( p \), the job finding probability is near zero at \( m(x) = U \), and thus the unemployment rate, \( \phi^u = \delta / (\delta + p(\theta(m(U)))) \approx 1 \), and so consumption is again arbitrarily close to zero. It must therefore be the case that moving away from full risk sharing, i.e. \( b < (1 - \tau)\omega \), improves welfare.

It is worth noting that under a linear periodic utility function, unemployment insurance has no value. To see this, consider the weighted utility of unemployed and employed individuals: \( W = \phi^u b + (1 - \phi^u)(1 - \tau)\omega \). Since the budget constraint of the government imposes that \( \phi^u b = (1 - \phi^u)\tau\omega \), \( W = (1 - \phi^u)w \) is independent of unemployment insurance. This results should not be surprising, as the allocation in directed search environments is typically efficient under a linear utility function (Moen (1997) or Menzio and Shi (2011)).

4 Optimal Unemployment Insurance: Numerical Results

In this section we use a parameterized version of the model to study unemployment insurance in the presence of various types of risk. We first set a benchmark economy without idiosyncratic nor aggregate uncertainty, and successively study the impact of adding each type of risk. We also discuss the desirability for unemployment benefits to be pro- or counter-cyclical. Before doing so, we first parameterize the economy.

4.1 Parameterization

We take a period to be a month and set the discount factor to 0.996, which corresponds to a 5 percent annual discount rate. The worker’s periodic utility function is given by \( (c^{(1 - \sigma)} - 1) / (1 - \sigma) \), where \( \sigma \), the risk aversion coefficient, is equal to 2. The matching technology \( p(\theta) \) has the functional form of \( \theta(1 + \theta^\gamma)^{-1/\gamma} \).\(^{15}\) The elasticity parameter of the

\(^{15}\)The underlying matching function, as first introduced by den Haan et al. (2000), is given by \( (v^{-\gamma} + a^{-\gamma})^{-1/\gamma} \), where \( v \) and \( a \) are the number of vacancies and applicants respectively, and \( \theta = v/a \).
matching function, $\gamma$, is set to 0.5 to target an elasticity of the job-finding probability with respect to the tightness ratio to be 0.415, to be in the midrange of estimates of this elasticity from Brügemann (2008).\footnote{Brügemann (2008) estimates a range of values of 0.37–0.46. Targeting an elasticity of 0.415 gives an equation for $\theta$ and $\gamma$ of $\theta = \left(\frac{1}{1.415} - 1\right)^{\frac{1}{2}}$. Using this value into the equation for $p(\theta)$ yields an equation in terms of $\gamma$, and solving for $\gamma$ at the target job-finding probability of 0.343 yields the value 0.5.}

We set $\delta = 0.02$, $\kappa = 0.11$, and $\lambda_e = 0.31$ in order for our benchmark economy to feature an unemployment rate of 5.37%, a 34.3% job finding probability for unemployed individuals, and job-to-job transition rate of 2.23%, which represent the respective averages for workers aged 20–64 in the Current Population Survey from 1994:1 to 2016:12.\footnote{Since by construction our benchmark economy without uncertainty features very little job-to-job transitions, we use the economy with idiosyncratic uncertainty to set the main parameter which controls job-to-job transitions, i.e. $\lambda_e$.} The aggregate component of productivity follows a two-state Markov chain with $Y = \{0.955, 1.010\}$ and transition matrix $\Phi_y = [0.91 \ 0.09; 0.02 \ 0.98]$. The transition probabilities are chosen such that the expected length of expansions and contractions are 58.4 and 11.1 months respectively, as observed by the NBER dating committee over the post-war period. Given this transition probability, expected output is equal to one, with shocks a little less than 3% of unweighted mean productivity. Finally, the unconditional mean of the idiosyncratic productivity component is set to zero, with $z$ following a three-state Markov chain with $Z = \{-0.2, 0, 0.2\}$ and transition matrix $\Phi_z = [0.85 \ 0.10 \ 0.05; 0.075 \ 0.85 \ 0.075; 0.05 \ 0.10 \ 0.85]$. With the idiosyncratic component of productivity always equal to zero ($\tilde{z} = 0$) in newly formed matches, the firm and worker expect a $\pm$20% shock a little over 6 months after the beginning of the match.

The flow value of consumption in unemployment, $b$, is financed by the government and is set to 0.68. This value is in the middle range of values suggested in Hall and Milgrom (2008), and reflects a value of non-market work equal to 43% of labor productivity and an unemployment benefit which corresponds to 25% of labor productivity. Note that while the average replacement rate of unemployment benefits in the United States is typically higher than 25%, many unemployed workers do not qualify for or do not apply for benefits.
4.2 Benchmark Model: No Uncertainty

We now assume that both aggregate and idiosyncratic productivities are constant at their unconditional means. Figure 1 displays various value and policy functions, the submarket tightness function, as well as the stationary distribution over open submarkets. Since the value of the firm \( J(x) \) is decreasing in promised utility to its worker, firms need to be compensated by a high job filling probability \( q(x) \) to open vacancies in high submarkets. This translates into a tightness ratio \( \theta(x) \) and a job finding probability \( \tilde{p}(x) \) which decrease as promised utility increases. Notice that unemployed individuals search in relatively high submarkets: this is partly due to the generosity of unemployment insurance program. As a result, the optimal submarket in which to search is fairly flat, and the distance between the current lifetime utility \( x \) and the lifetime utility associated with the submarket in which individuals search \( m(x) \) narrows down quickly. It also follows that the value of search \( R(x) \) decreases monotonically with current lifetime utility.

The wage \( \omega(x) \), future utility \( V'(x) \), and the separation probability \( d'(x) \) are all part of the optimal contract chosen by firms. As one would expect, wages and future promised utility both increase with the submarket value. Because of curvature in the periodic utility function, the wage is a convex function of the submarket value over the relevant range: increasingly higher wages are necessary to provide the extra lifetime utility. This convex wage function is also evidence that firms design contracts to backload wage payments: by promising higher future utility, firms can keep current wages relatively low while at the same time lowering the worker’s job finding probability in the future.\(^{18}\) The separation probability is such that no match is ever endogenously destroyed. Finally, note that the model generates some heterogeneity among homogenous workers, as shown in the last panel of Figure 1. The first spike corresponds to the measure of unemployed individuals in the population. As dictated by the policy function \( m(x) \), these individuals search in a higher submarket, which

\(^{18}\)To be more precise, the extent to which firms can backload compensation depends on the current promised utility \( x \). Future promised utility \( V' \) is bounded below by the value of unemployment: below that level, the individual rationality constraint is violated and the match would endogenously separate. As such, the wage in that region increases in a smooth fashion simply to fulfill the current promise-keeping constraint. Beyond that, firms backload compensation by promising increased future utility as a means to postpone wages as well as to retain workers. Since the probability of a successful job-to-job transition is decreasing in the worker’s value function, the amount a firm backloads is decreasing in \( x \). Above the value of the highest open submarket, firms have no incentive to backload as there are no vacancies which provide a higher lifetime utility than the worker’s current job. In that region, the firm’s contract simply specifies \( V' = V \) and offers a wage which satisfies the promise keeping constraint.
Figure 1: Economy without Uncertainty

Notes: In all panels the horizontal axis is the value of submarkets $x$. This figure was generated under unemployment benefit $b = 0.68$ financed by a labor income tax rate $\tau = 0.041$. Other parameter values are described in section 4.1.

corresponds to the second spike, and so on. Of course, a limited number of submarkets can emerge without idiosyncratic nor aggregate uncertainty. Naturally, all open submarkets are located between the value of unemployment and the submarket for which firms would just cover the cost of posting a vacancy with a job filling probability equal to one.

Figure 2 displays how the economy behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government’s budget, in this case on a period-by-period basis. The top left panel, where $CE$ refers to Consumption Equivalence, shows that there exists a unique optimum unemployment insurance configu-
As discussed earlier, the intuition for this result is straightforward: while higher unemployment benefits provide better consumption smoothing, it also reduces production and thus average consumption as the unemployment rate increases.

A more generous unemployment insurance system provides better consumption smoothing not only because benefits are higher, but also because after-tax wages are lower. The lower-left panel of Figure 2 shows that despite lower consumption (the after-tax wage), employed workers work on average in higher submarkets as unemployment benefits increase, at least at relatively low unemployment benefit levels. The extra utility, of course, comes from the possibility of becoming unemployed in the future, which has a higher value. But because unemployed workers search in higher submarkets, for which the job finding probability is lower, individuals spend more time in the unemployment state as unemployment insurance becomes more generous. Eventually, even the value of unemployment falls as the prospect of becoming employed not only becomes less likely due to workers’ job search strategies and less job creation, but also more dire. Finally, we note that the tax rate is convex as a function of benefits, i.e. the tax rate increases faster than benefits. This is due to the fact that as benefits rise, the employment rate falls, and so the tax rate must increase further to compensate for the fall in the tax base.

To better understand the mechanism behind these results, we quantify the sensitivity of the labor market to variations in the unemployment insurance scheme. Changes in unemployment insurance can affect the worker’s search behavior as well as the change in incentives to firms (see section 2.3). The effect of unemployment benefits on workers’ search behavior is best analyzed by looking at the micro-elasticity (in partial equilibrium) of unemployment duration with respect to unemployment benefit. Fixing the firm’s value and the tightness ratio faced by the worker in each submarket, we measure the percentage change in unemployment duration from a percentage change in the unemployment benefit which is only due to changes in the worker’s value of unemployment and optimal search strategy. We find that the elasticity of unemployment duration with respect to unemployment benefit,

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19Consumption Equivalence measures the amount of extra consumption per period in percentage terms that individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one.

20Production need not decrease in an economy in which firms choose the amount of capital to put in place prior to filling vacancies, as shown by Acemoglu and Shimer (1999). In their environment, firms put in place a larger amount of capital when this capital is expected to remain idle for shorter periods of time, as happens when individuals direct their search to higher submarkets.
Figure 2: Optimal Unemployment Insurance without Uncertainty

Notes: In all panels the horizontal axis is the value of unemployment benefits \( b \). For any such value of \( b \), the tax rate is such that the unemployment insurance program is self-financed on a period-by-period basis in the stationary equilibrium. ‘Posted V’ is the average promised utility \( (x) \) of posted vacancies with positive queue length and ‘U’ is the value of being unemployed. Wages refer to average before- and after-tax wages of employed workers. ‘U rate’ is the unemployment rate, and ‘EE rate’ is the job-to-job transition rate. ‘Welfare (CE)’ gives the percent increase in consumption per period which would leave individuals indifferent between any given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

\( \xi_{dur,b} = 0.967. \)

Our value is within the wide range of empirical elasticities reported in Krueger and Meyer (2002), and close to the value of of 0.88 reported in Meyer (1990). Note that while this value is directly targeted in models using search effort (e.g. Mitman and Rabinovich (2015)), it

\(^{21}\)Calculated in the case with no uncertainty, using \( b = 0.68 \), which is our baseline value for the unemployment benefit.
is not explicitly targeted in our framework. The elasticity $\xi_{dur,b}$ in our model is influenced by the change in the value of unemployment (in partial equilibrium through the increased benefit), the curvature of the utility function, and the equilibrium curvature of the matching probability $p(\theta(x))$ across submarkets. In a model of costly search, the shape of the disutility of search effort would influence workers’ search intensity and effective job-finding rate. Both frameworks produce the result that an increase in benefit results in a lower job-finding probability (and longer unemployment duration).

In addition to workers’ search behavior, there is also a general equilibrium effect of benefits through the tax rate’s effect on firms’ incentives to post vacancies (see Section 2.3). To quantify the total effect of unemployment insurance on the labor market, the ‘macro-elasticity’ of unemployment with respect to benefits measures the equilibrium response of the labor market to the increased benefit, which includes changes in the budget-balancing tax rate. The macro-elasticity we find in our model is 0.93, only slightly smaller than the micro-elasticity. Our result is in contrast to a large macro-elasticity of 2.4 found in Mitman and Rabinovich (2015), and a macro-elasticity which is much smaller than the micro-elasticity in Landais et al. (2016).

The small change in the macro-elasticity relative to the micro-elasticity in our model is due to the fact that unemployed workers’ optimal search behavior partially undoes the effects caused by the tax rate. An increased tax rate decreases the firm’s value and tightness ratio, but this also puts downward pressure on the unemployed worker’s value function by decreasing the probability of finding a job in a given submarket. The elasticity of unemployed workers’ value function with respect to unemployment benefit is 0.21 in partial equilibrium relative to 0.18 in general equilibrium. Since a worker’s optimal search strategy is increasing in his current lifetime expected value, he searches in lower-valued submarkets in the general-equilibrium scenario compared to the partial-equilibrium case, slightly dampening the increase in unemployment duration.

4.3 Introducing Idiosyncratic Uncertainty

In this section, we keep the aggregate component of productivity fixed at its unconditional mean ($y = 1$), but introduce idiosyncratic uncertainty in the form of a three-state Markov chain for the idiosyncratic component of productivity $z$ discussed in Section 4.1. Recall that the idiosyncratic component of productivity is assumed to be the same in all new matches
and equal to its unconditional mean ($\tilde{z} = 0$). Thereafter, the idiosyncratic component of productivity evolves according to $\Phi_z$.

Figure 3 illustrates the properties of the economy. Most functions look familiar following the discussion surrounding Figure 1. The contract, however, has some new properties. First note that only the middle (green) value of the firm ($J(x, \tilde{z})$) in the top-left panel is relevant for job creation. No new matches are created for submarkets $x > \hat{x}$, where $\hat{x}$ satisfies $J(\hat{x}, \tilde{z}) = \kappa$—i.e. submarkets for which $\theta(x) = 0$. As we argued before, this limits the degree to which firms can backload compensation since workers cannot find jobs in submarkets higher than $\hat{x}$. Accordingly, firms design contracts such that high productivity individuals have no incentive to search: they search in markets where the job finding probability is zero. Again, because firms cannot offer less than the value of unemployment, this places a lower bound on the firm’s promised future utility $V'$.

Finally, notice that wages are higher in low productivity matches than in high productivity matches for any given submarket $x$. To a large extent, this is due to the persistence of the shock. This feature of the contract is most easily understood by looking at individuals in matches with currently high values of $x$, for which the contract specifies future utilities that are similar for matches currently experiencing neutral and high productivity (see $V'(z'_{i}|z_{m})$ and $V'(z'_{i}|z_{m})$ in Figure 3). While future utilities are fairly similar, the probability to transit to these states are vastly different, with currently high productivity matches being much more likely to remain high. As such, workers in neutral productivity matches need to be compensated with a higher wage today in order to deliver the same current promised utility $x$.

In equilibrium, of course, the probability of being in such a high current $x$ is much lower in neutral productivity matches than in high productivity matches. In other words, individuals in high productivity matches do tend to have higher wages than individuals in neutral or low productivity matches. For example, promised utility conditional on moving to low productivity tends to be fairly low, and the panel for $V'(z_{i}|z_{l})$ shows that there is only so much utility a low productivity individual will ever be offered (the point where the red line crosses the 45° line).

Figure 4 shows that there is much more heterogeneity in the economy with idiosyncratic productivity than in the benchmark economy. Since all unemployed individuals are identical, the first mass point corresponds to the unemployment rate. Thereafter, workers make their

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22 A related fact is that the value of the firm is higher the higher the idiosyncratic component of productivity is.
Figure 3: Economy with Idiosyncratic Uncertainty

Notes: In all panels the horizontal axis is the value of submarkets \( x \). Red lines represent low productivity, green lines represent neutral productivity, and blue lines represent high productivity. This figure was generated under unemployment benefit \( b = 0.68 \) and a labor tax rate \( \tau = 0.04 \). Other parameter values are described in section 4.1.

way up the wage ladder while the idiosyncratic component of productivity moves them up or down. The largest mass point occurs when the blue line in the lower-right plot of Figure 3, i.e. \( V'(z'_h, z_h) \), reaches the 45° line. Similarly, the last large mass point occurs when workers in neutral productivity matches get a good shock, after which they stay at that level until the next time the idiosyncratic component of productivity changes.

Figure 5 displays how the economy with idiosyncratic risk behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government’s budget on a period-by-period basis. As was the case in the economy without
risk, the top left panel indicates that there exists a unique optimum unemployment insurance configuration. Interestingly, the generosity of the system is comparable to that of the benchmark economy. The same mechanism as in the benchmark model is at work: a more generous system provides better risk sharing at the cost of higher unemployment and thus lower average consumption. However, a separate mechanism is responsible for the behavior of the economy as the level of benefits increases past the optimal level, which revolves around endogenous match separation. Unlike in the benchmark economy, where the unemployment rate increased with $b$ only because of the increase in unemployment duration, here the unemployment rate increases also because of the additional flow from employment to unemployment. In other words, both job creation and destruction are at work. Together, these effects translate into very high tax rates, so much so that higher unemployment benefits are simply unsustainable: as the tax rate keeps increasing, the pool of workers and so the tax base keep shrinking, calling for ever increasing tax rates.\footnote{Admittedly, the strength of this mechanism might be diminished if the idiosyncratic component of productivity had a finer support.} For the same reason, the welfare cost of running ‘too generous’ an unemployment insurance system is sizable.

\footnote{Admittedly, the strength of this mechanism might be diminished if the idiosyncratic component of productivity had a finer support.}
Notes: In all panels the horizontal axis is the value of unemployment benefits $b$. For any such value of $b$, the tax rate is such that the unemployment insurance program is self-financed on a period-by-period basis in the stationary equilibrium. ‘Avg U’ is the average value of being unemployed, while ‘Posted V’ is the average promised utility ($x$) of posted vacancies with positive queue length. Wages refer to average before- and after-tax wages of employed workers. ‘U rate’ is the unemployment rate, and ‘EE rate’ is the job-to-job transition rate. ‘Consumption Equivalence’ give the percent increase in consumption per period which would leave individuals indifferent between any given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

Figure 6 illustrates how the optimal unemployment insurance system—as indexed by the level of the benefit—changes with the amount of idiosyncratic risk that individuals face. As idiosyncratic risk rises, the optimal unemployment benefit initially rises smoothly but then falls rather sharply. While the initial rise allows individuals to better smooth consumption, the subsequent fall is due to endogenous separation as discussed above. As the amount of 

\textsuperscript{24}This figure was generated under a very different calibration.
risk rises, more matches with low productivity workers are endogenously destroyed. As a result, the unemployment rate rises faster, and so does the wage tax on the shirking pool of workers.

### 4.4 Introducing Aggregate Uncertainty

We now set the idiosyncratic component of productivity back to its unconditional mean (\( z = 0 \)) but introduce aggregate uncertainty in the form of a two-state Markov chain governing the aggregate component of productivity \( y \) as outlined in section 4.1. Looking at figure 7, the familiar patterns from the economy without uncertainty emerge here as well. Under high productivity, individuals search in higher submarkets. But because the value of the firm is higher, the job finding probability of unemployed workers is nevertheless higher in good times. Aggregate shocks, on their own, aren’t sufficiently large for any match to be destroyed endogenously in equilibrium. The convex shape of wages (outside of very low or very high submarkets) is evidence of back-loading contracts, as discussed in the benchmark economy. The intuition underlying the fact that for any given \( x \) wages are higher under low productivity
Figure 7: Economy with Aggregate Uncertainty

Notes: In all panels the horizontal axis is the value of submarkets $x$. Red lines represent low productivity, and blue lines represent high productivity. This figure was generated under unemployment benefit $b = 0.68$ financed by a labor income tax rate $\tau = 0.041$. Other parameter values are described in section 4.1.

mirrors that with idiosyncratic uncertainty discussed above: in equilibrium, wages are higher in expansions than in recessions. Together with the fact that the unemployment rate is higher in recessions, with a constant unemployment insurance system, the government runs a surplus in good times and a deficit in bad times.

Figure 8 shows the distribution of individuals in the utility space. The first two mass points correspond to the measure of unemployed workers in recessions and expansions, respectively. The fact that the measure is smaller in recessions simply means that on average the economy spends more time in expansions than in recessions. For similar reasons, the largest mass point
corresponds to the highest open submarket in expansions, while the second largest mass point corresponds to the highest open submarket in recessions.

Figure 9 displays how the economy with aggregate uncertainty behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government’s budget on average in the long run equilibrium. Comparing this figure to its counterpart without uncertainty (Figure 2), we conclude that aggregate uncertainty does not significantly alter how unemployment insurance affects the economy in general. Indeed even the optimal level of benefits is similar. Note, however, that the unemployment rate increases faster as unemployment benefits rise with aggregate uncertainty. This fact explains why both the average value of unemployment and the average value of jobs created (Posted V on Figure 9) are rising at the optimal unemployment system occurs: the weight on the value of unemployment is simply rising faster.

In contrast to idiosyncratic uncertainty, the optimal generosity of the unemployment insurance system declines monotonically with the level of aggregate risk. Our interpretation
Figure 9: Optimal Unemployment Insurance with Aggregate Uncertainty

Notes: In all panels the horizontal axis is the value of unemployment benefits $b$. For any such value of $b$, the tax rate is such that the unemployment insurance program is self-financed on a period-by-period basis in the stationary equilibrium. ‘Avg U’ is the average value of being unemployed, while ‘Posted V’ is the average promised utility ($x$) of posted vacancies with positive queue length. Wages refer to average before- and after-tax wages of employed workers. ‘U rate’ is the unemployment rate, and ‘EE rate’ is the job-to-job transition rate. ‘Consumption Equivalence’ give the percent increase in consumption per period which would leave individuals indifferent between any given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

of this finding is that because the unemployment benefit is the same in good and bad times, it has a larger detrimental effect on unemployment duration during bad times, when wages are low, than a beneficial one in good times, when wages are high. On average, then, the unemployment rate is higher, and so the tax rate necessary to finance any level of benefit is higher.

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The same reasoning—that a flat benefit provides ‘too much’ consumption smoothing in bad times, and ‘too little’ in good times—is behind our finding that unemployment benefits should be pro-cyclical, that is, benefits should be reduced in recessions. That being said, the difference between benefits in recessions vs expansions is small: 0.67 vs 0.71. Needless to say, failing to index benefits by the state of the aggregate economy is negligible.

4.5 Aggregate and Idiosyncratic Uncertainty Together

The economy with both aggregate and idiosyncratic uncertainty is dominated by idiosyncratic uncertainty. As such, this economy behaves essentially in the same way as the economy with only idiosyncratic uncertainty. The most interesting aspect of combining the two types of uncertainty is that the amount of endogenous destruction, when present, increases in recessions. For that reason, the highest level of benefit that is sustainable is lower in the model with both types of risk.

5 Extensions

In this section, we first explore how our results would differ if unemployment benefits were specified as a replacement ratio—as is common in practice—as opposed to a fixed benefit. Second, we study a well-known feature of unemployment insurance programs: unemployment benefits eventually expire.

5.1 Replacement Ratio

Unemployment insurance systems typically involve some kind of history dependence: unemployment benefits are a function of past earnings. Accordingly, instead of setting the level of benefits, the government now chooses a replacement ratio: \( b = r \hat{w} \), where \( r \) is the replacement ratio and \( \hat{w} \) is the worker’s last wage prior to separation. The optimal replacement rate turns out to be 69%, which implies an average unemployment benefit of 0.68, which is identical to the optimal benefit level in the benchmark economy. Indeed, while there is more heterogeneity in this economy, the equilibrium is virtually identical to that of the benchmark model.
5.2 Benefit Duration

Unemployment benefits typically expire after a fixed period of time. Accordingly, we now consider a scenario whereby unemployment benefits are not indefinite, but rather expire stochastically with probability $\psi$ each period. We set $\psi$ equal to $1/6$ so that the average duration of unemployment benefits is equal to 6 months, as is the case in normal times in the U.S.

In order to avoid having zero consumption when benefits expire, the flow value of consumption in unemployment, previously equal to $b$, now has two components. The first component, which we call welfare and label $h$, never expires. Slightly abusing notation, $b$ now stands for the expiring component, which we also call unemployment benefits. As such, total consumption while unemployed is equal to $h + b$ if the worker is eligible for unemployment benefits and $h$ if benefits have expired. For comparability purposes with the benchmark model, both $h$ and $b$ are financed by labor income taxes. Following Hall and Milgrom (2008), we set the fixed component $h$ to 0.43, and set $b = 0.25$ such that total benefits are the same as in the benchmark economy when $\psi = 0$.

Since this economy with expiring benefits behaves in a similar fashion as the benchmark economy, we will not dwell on its properties. Rather, Figure 10 displays how the equilibrium changes as the expected duration of benefits changes, from 1 month to infinity, the latter replicating our benchmark economy. As one would expect, the tax rate increases with expected duration, as does the unemployment rate. Nevertheless, all value functions (for both types of unemployed individuals as well as the average posted value of newly created jobs) are increasing in expected duration.

The most salient feature of Figure 10 is that welfare is optimized when benefits do not expire (see Welfare). The intuition for this result is quite transparent. Imagine an economy in which unemployed individuals receive unemployment benefit in perpetuity, and consider implementing a policy whereby benefits expire on average after, say, 30 months. This policy essentially has two effects: (i) it lowers the value of being either employed or unemployed.

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25 We also explore an alternative strategy to set these two parameters. In this second strategy, we set the expiring component $b$ to 0.46, which corresponds to a replacement rate of about 47% of pre-tax earning, as found in Ganong and Noel (2016) using monthly checking account inflow data from the JPMorgan Chase Institute (JPMCI), and set the non-expiring component $h$ equal to 0.22, such that total benefits are again the same as in the benchmark economy when $\psi = 0$. Since results are similar in these two somewhat extreme strategies, we omit results of from the second strategy.

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Figure 10: Optimal Duration of Benefits

Notes: In all panels the horizontal axis is the expected duration of benefits $\psi$, with unemployment benefits fixed at 0.68. For any such value of $\psi$, the tax rate is such that the unemployment insurance program is self-financed on a period-by-period basis in the stationary equilibrium. ‘Avg U’ is the average value of being unemployed (either with or without benefits), while ‘Posted V’ is the average promised utility ($x$) of posted vacancies with positive queue length. Wages refer to average before- and after-tax wages of employed workers. ‘U rate’ is the unemployment rate, and ‘EE rate’ is the job-to-job transition rate. ‘Consumption Equivalence’ give the percent increase in consumption per period which would leave individuals indifferent between any given unemployment insurance duration and the optimal one. The calibration is outlined in Section 4.1.

(even though those whose benefits have expired), but (ii) it lowers the amount of time individuals spend unemployed (the unemployment rate is lower). While very few individuals ever see their benefits expire, the cost in terms of consumption smoothing is very high relative to the cost of insuring against that event, i.e. spending slightly more time unemployed with benefits.
It is worth noting that the above policy does have the desired effect: individuals who lose their benefit do find jobs rather quickly: the average job finding probability is much higher when benefits expire after 6 months than when they last forever. That being said, the (micro) elasticity of unemployment duration with respect to potential benefit duration is fairly modest in our model, at 5%. Allowing the tax rate and tightness ratio in the economy to reach equilibrium, the macro elasticity of unemployment duration with respect to potential benefit duration is 8%. While estimates of this elasticity vary widely in the literature, they are generally well below the elasticity with respect to the level of benefit discussed earlier. For instance, Katz and Meyer (1990) find that a one-week extension in potential duration of benefits increases unemployment duration by 0.16–0.2 weeks, with an implied elasticity around 30%. On the other hand, Card and Levine (2000) find the elasticity of unemployment duration with respect to potential benefit duration to be around 10%.

6 Conclusion

In this paper, we describe the optimal response of unemployment insurance policy to changes in uncertainty at both the idiosyncratic and aggregate level by introducing a tax-financed unemployment insurance system in a directed search model of on the job search. Unemployment insurance introduces a tradeoff in our economy between insurance and job creation: unemployment benefits insure risk-averse unemployed workers at the expense of prolonging job search and reducing job creation through an increased tax rate. This tradeoff is present in most studies of unemployment insurance, but arises naturally from our directed search framework. With directed search, rather than through reductions in costly search effort, unemployment duration increases with benefits as workers change their search behavior to look for higher-valued but tougher to find jobs. We find that the change in unemployed workers’ job search behavior endogenously produces an elasticity of unemployment duration with respect to unemployment benefits of 0.97, which is in line with empirical estimates in the literature.

Characterizing the optimal policy, there exists a unique positive level of unemployment benefit which maximizes \textit{ex ante} welfare of individuals. With idiosyncratic uncertainty, we find the optimal benefit is hump-shaped in idiosyncratic risk. Increased risk calls for more consumption smoothing, but increased benefits eventually induce endogenous separations
into unemployment, reducing the tax base and sharply decreasing welfare. Thus, as risk increases, the sustainable benefit level falls. With aggregate uncertainty, we find that the optimal benefit is mildly pro-cyclical and decreasing monotonically with aggregate risk. Unlike idiosyncratic risk, the welfare cost of deviating from the optimal benefit is quite small. When we consider benefit duration, we find that the optimal duration in our model is infinite. The cost of reduced job creation and increased unemployment duration is small relative to the larger gains from consumption-smoothing in the event of benefit expiration.
References


