Abstract

In recent years, the increased concentration of activity in the banking system has received much attention. In addition, numerous central banks have expanded their range of policy tools to include paying interest on reserves. The objective of this paper is to study the implications of concentration in the banking sector and the effects of various reserve policies. Changes in the competitive structure affect investment, risk-sharing, and social welfare. A key aspect of our analysis is that banks in more concentrated systems allocate a lot of resources towards cash reserves rather than investing in productive assets. However, perfect competition should not be a regulatory goal for the banking system. The model also demonstrates that the efficacy of monetary policy varies with the degree of concentration. Such observations are important as regulators and monetary policy authorities confront the challenges of an evolving competitive landscape out of the recent financial crisis.

1 Introduction

There have been considerable changes in the concentration of the banking sector across the globe. In particular, during the recent financial crisis, a number of institutions grew so large that they were deemed “too big to fail.” Furthermore, it is widely acknowledged that such institutions generally became even larger after the crisis. Yet, such trends have been part of a long-term pattern of consolidation activity. In particular, the Bank for International Settlements (2001) provides a thorough review of consolidation in the banking sector across countries. In the United States, for example, there were nearly 19,000 financial institutions in 1989. Just a decade later, only 10,000 were active in the sector. Moreover, Janicki and Prescott (2006) provide evidence indicating that the largest banks in the United States (which consist of less than 1% of active institutions) held over 75% of assets in the banking system prior to the crisis. With this large of a role in economic activity, it is hard to believe that these

1See Gongloff et al. (2013) and Gandel (2013).
institutions would not take advantage of their market power in the financial system.

Further, the tools of central banks used to conduct monetary policy have expanded in recent years. Notably, the Federal Reserve began paying interest on bank reserves in 2008 and it continues to do so. Around the same time, the Bank of England adopted a similar policy. By comparison, the European Central Bank (ECB) has had this authority since the ECB was established, but lowered the rate down to zero in 2012.

Such dramatic observations raise some very important questions for regulatory and monetary policy authorities to address. What is the optimal competitive structure of the financial system? Should perfect competition in the banking system be a regulatory goal? How does the degree of concentration affect investment and capital accumulation? Does the efficacy of monetary policy depend on the degree of concentration? What is the optimal interest rate policy? How does it depend on the competitive structure of the banking system?

The objective of this paper is to study the implications of concentration in the banking sector for economic activity. In particular, we demonstrate that changes in the competitive structure affect the level of investment, risk-sharing, and social welfare. A key aspect of our analysis is that banks in more concentrated systems allocate a lot of resources towards money balances. That is, they have a tendency to hoard cash reserves. The implications of the competitive structure in our model lie at the core of the role of financial intermediation for economic activity. Notably, Bencivenga and Smith (1991) stress that an active intermediary sector promotes risk-pooling services and “eliminates excessive investment in unproductive liquid assets...[the absence] leads towards unfavorable levels of capital accumulation.” Yet, in stark contrast, we demonstrate that more concentrated banking systems considerably deviate from this function.

In order to study the role of financial sector competition for investment and capital accumulation, we follow Bencivenga and Smith (1991) by studying an overlapping generations version of Diamond and Dybvig (1983) with production. As in Schreft and Smith (1997, 1998), limited communication and restrictions on asset portability generate a transactions role for fiat money. Physical capital and money balances are the only two assets in the economy. In contrast to Schreft and Smith, the banking sector is imperfectly competitive with fixed entry. In particular, intermediaries engage in Cournot competition in the market for capital. As a result, differing degrees of concentration in the banking sector affect the provision of risk-pooling and investment.

Notably, banks engage in strategic behavior and exploit their market power in the capital sector. That is, in contrast to perfectly competitive banks lacking market power, banks take into account that they face a downward-sloping demand for capital by firms. As a result, the industrial organization of the banking sector has serious consequences for real activity. In particular, intermediaries exploit their market power by holding back resources available to firms and devote more funding to unproductive money balances. Consequently, highly concentrated sectors provide a large amount of insurance against liquidity risk. Thus, the model presents a trade-off between the provision of productive re-
sources to firms and risk-pooling to depositors as the competitive structure of the banking system varies.

We turn to the questions posed above. For example, does the efficacy of monetary policy depend on the degree of concentration? Notably, changes in the degree of concentration can render policy less effective in stimulating investment and capital accumulation. In particular, as concentration increases (and institutions become very large in size) it leads to reduced efficacy because banks would respond more to the desire to exploit their market power and would acquire even more money balances. In this manner, increased concentration "clogs" the transmission channels of monetary policy. Therefore, it should not be difficult to understand why central banks have needed to resort to aggressive methods of policy intervention in recent years.

Next, should policymakers unambiguously strive for increased competition in the banking sector? Should perfect competition be a regulatory goal? No. Moreover, the optimal competitive structure depends on the amount of liquidity risk that individuals face. If the degree of liquidity risk is high, the amount of capital accumulation is unfavorably low and distortions from market power in the capital market are relatively high. Thus, an increase in competition would lead to a welfare gain as the capital stock would be significantly higher. However, there are limits to the gains from promoting competition. It is possible that the degree of competition can be too high because excessive amounts of competition would lead to less income derived from the capital sector as institutions less aggressively exploit their market power. Therefore, policymakers should carefully consider recent policy initiatives in many countries to promote competition. For example, in the U.K. there have been changes in banking requirements to lower barriers to entry in order to encourage credit funding.\textsuperscript{2,3}

By comparison, if liquidity risk is low, a lot of resources are devoted to investment. As a result, the returns from the capital are too low and the banking sector would benefit from greater concentration than under perfect competition. In fact, in some environments, a highly concentrated, monopolistic sector would produce the best social outcome.

Finally, we address the implications of the competitive structure for optimal monetary policy. Interestingly, irrespective of agents' degree of exposure to liquidity risk, the welfare-maximizing interest rate on excess reserves is (weakly) increasing in the degree of banking competition. In other words, in response to the recent trend towards concentration of firms in the banking sector, the model would call for lower rates on excess reserves. Simply put, concentrated banks with market power tend to hold excessively large amounts of money balances – the optimal policy would discourage such behavior.

The remainder of the paper is as follows. Section 2 provides a brief overview

\textsuperscript{2}Financial Services Authority (2013) describes the changes in banking requirements. One of them is a reduction in capital requirements for new entrants.

\textsuperscript{3}Both Hannan (1991) and Corvoisier and Gropp (2002) contend that interest rates on loans are higher in markets with higher concentration ratios. Furthermore, Beck, Demirgüç-Kunt and Maksimovic (2003) document that credit rationing occurs more often in concentrated banking systems.
of the related literature. Section 3 outlines the physical environment of the model. Section 4 studies a monetary steady-state equilibrium. Section 5 addresses the welfare implications of the model, including the optimal competitive structure and the optimal monetary policy rule under different degrees of concentration of the banking sector. Finally, Section 6 concludes.

2 Related Literature

Our paper contributes to recent work which examines the implications of the industrial organization of the financial system for economic activity. In particular, Paal, Smith, and Wang (2005) construct an endogenous growth framework to study the effects of different competitive structures of the banking system for economic growth. Their model demonstrates that a monopolistic banking system may produce a higher rate of growth than a competitive banking system. In addition, Ghossoub (2012) studies the implications of the competitive structure for prices in capital markets.

In a model of credit market activity, Ghossoub, Laosuthi, and Reed (2012) investigate the effects of monetary policy and find that there are important qualitative differences in the effects of policy across competitive structures. Under perfect competition, the transmission of monetary policy is straightforward in which higher rates of money growth lead to an increase in credit market activity. However, in a distorted monopolistic setting, policy produces the opposite effect. Building on the structure of Ghossoub, Laosuthi, and Reed (2012), Matsuoka (2011) examines optimal policy across competitive structures.

All of these papers focus exclusively on financial markets under price competition. Consequently, they are unable to address the implications of changes in the degree of concentration. By comparison, Ghossoub and Reed (2013) develop a model of banks with Cournot competition where banks differ in size to consider the optimal size distribution of the banking system and the strength of the impact of monetary policy. However, in contrast to our framework, they study an endowment economy and focus on the role of credit markets to promote consumption smoothing. Laosuthi and Reed (2013a) develop a model of equilibrium entry into the banking system to show that inflation distorts the ability of institutions to promote risk-sharing and thereby deters firm entry. In this manner, inflation exacerbates distortions in the credit market due to barriers to entry. Alternatively, Laosuthi and Reed (2013b) develop a model of imperfectly competitive behavior in the market for transactions-demand financial services. In addition, Ghossoub and Reed (2017) study the effects of concentration in economies with production externalities. However, the rate of money growth is the primary policy tool used by the central bank in their model. By comparison, the current structure is the first to examine the implications of

\[^\text{4}\text{In a similar vein, Williamson (1986) studies a model of Cournot competition in credit markets. However, financial intermediaries do not perform risk-pooling services as in our setup.}\]
Interest on reserves in a model with an imperfectly competitive banking system where depositors are subject to liquidity risk as in Diamond and Dybvig (1983).

There are also other noteworthy papers that introduce imperfect competition in real economies. For example, Cetorelli and Peretto (2010) construct a general equilibrium model with capital accumulation in which financial intermediaries engage in Cournot competition. However, as money does not circulate in their framework, they do not consider the connections between monetary policy and the competitive structure of the banking system.

In addition to theoretical work, Cechetti (1999) and Kashyap and Stein (1997) look at how the effectiveness of policy depends on the industrial organization of the financial system. In particular, Cechetti shows that the impact of monetary policy shocks on output varies across countries. As our model shows at relatively high degrees of concentration, output responds more to monetary policy in countries with banking systems that are more competitive. Peersmans’s (2004) evidence focusing on countries in Europe is also consistent with the work of Cechetti and Kashyap and Stein (1997). Kashyap and Stein (2000) find that the impact of monetary policy is stronger among smaller banks in the financial system.

3 Environment

Consider a discrete-time economy with two geographically separated locations or islands. Let $t = 1, 2, \ldots, \infty$, index the time period. On each location, there are two types of agents that live for two periods: workers (potential depositors) and bankers. At the beginning of each time period, a unit mass of ex-ante identical workers and $N$ financial intermediaries (or bankers) are born on each island. Each bank is indexed by $j$, where $j = 1, 2, \ldots, N$.

Workers are born with one unit of labor effort which they supply inelastically when young and are retired when old. In comparison to workers, bankers have no endowments. Furthermore, all agents derive utility from consuming the economy’s single consumption good when old ($c_{t+1}$). As a benchmark, the preferences of a typical worker are expressed by $u(c_{t+1}) = \ln c_{t+1}$. On the other hand, bankers are risk-neutral agents.

There are two types of assets in this economy: money (fiat currency) and physical capital. Denote the aggregate nominal monetary base and capital stock available in period $t$ by $M_t$ and $K_t$, respectively. One unit of goods invested by a young worker in period $t$ yields one unit of capital in $t + 1$. Moreover, at the initial date 0, the generation of old workers at each location is endowed with the initial aggregate stocks of capital and money ($K_0$ and $M_0$). Since the population of workers is equal to one, these variables also represent their values per worker. Assuming that the price level is common across locations, we refer to $P_t$ as the number of units of currency per unit of goods at time $t$.

The consumption good is produced by a representative firm using capital and labor as inputs. The production function is of the Cobb-Douglas form, $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where $Y_t$ and $L_t$ are period $t$ aggregate output and labor,
respectively. In addition, $A$ is a technology parameter and $\alpha \in (0, 1)$ reflects the capital intensity. Further, the capital stock depreciates completely in the production process.

Following previous work such as Schreft and Smith (1997, 1998), private information and limited communication between locations require workers to use cash if they move to a different location. The communication friction provides a rationale for the circulation of fiat money – due to incomplete information across islands, individuals will not accept privately issued liabilities. Thus, in some trades between agents, only fiat money will be accepted. Moreover, workers in the economy are subject to relocation shocks. After exchange occurs in the first period, a fraction $\pi \in (0, 1)$ of agents is randomly chosen to relocate. These agents are called “movers.” By comparison, individuals who remain on the same location are “non-movers.” While $\pi$ is known at the beginning of the period, agents are privately informed about their types at the end of the period.\footnote{There are also numerous papers in which the relocation probability is stochastic. Models generally introduce aggregate uncertainty in order to study banking crises and lender of last resort policies. For example, see Antinolfi, Huybens and Keister (2001) who show that introducing a discount window may achieve complete risk sharing but also subjects the economy to a continuum of inflationary equilibria. In addition, Boyd, de Nicoló, and Smith (2004) study the probabilities of banking crises across monopolistic and perfectly competitive banking systems.}

Unlike workers, bankers are not subject to relocation shocks. Moreover, as in Diamond and Dybvig (1983), all of workers’ savings are intermediated as financial intermediaries are able to provide their depositors with insurance against idiosyncratic liquidity risk. Each banker $j$ allocates its deposits into money and capital. By construction, there is a fixed number of firms in the banking sector. Moreover, in contrast to previous work such as Schreft and Smith, banks are Cournot competitors in capital markets. In addition, banks are subject to legal reserve or liquidity requirements. In particular, a bank is required to hold at least a fraction of its deposits ($\rho$) in cash reserves. All cash reserves are held at the monetary authority’s vaults and earn interest. In particular, the central bank pays a gross nominal interest rate $I_{i,t}$ for reserves held between $t$ and $t+1$, where $i = R, e$ designating whether reserves are required or excess, respectively. The interest rate on required reserves is higher than on excess reserves. That is, $I_{R,t} > I_{e,t}$.

The monetary authority imposes the nominal returns on cash reserves once and for all at the beginning of time. In this manner, interest rates on reserves are the primary policy instruments by the central bank. At the beginning of period $t+1$, the payments to the banking system for parking deposits at the central bank are paid from new money created and proportional income taxes on young agents. The tax rate, $\tau$, is assumed to be constant over time. In nominal terms, the central authority’s budget constraint in period $t+1$ is as follows:

$$M_{t+1} + P_{t+1} \tau w_{t+1} = I_{R,t} M_{R,t} + I_{e,t} M_{e,t}$$

where $M_{R,t} + M_{e,t} = M_t$, with $M_{R,t}$ and $M_{e,t}$ being the nominal stocks of
required and excess cash reserves held by the banking system in period \( t \). In real terms:

\[
\hat{m}_{t+1} + \tau_{t+1} w_{t+1} = \hat{m}_{R,t} R_{R,t} + \hat{m}_{e,t} R_{e,t}
\]

(2)

where \( \hat{m}_{t+1} = M_{t+1}/P_{t+1} \) is the real aggregate stock of money in \( t+1 \) and \( R_{i,t} = I_{i,t} P_{t}^{-1} \) is the real return on cash reserves. In sum, every period, the central authority adjusts its nominal money stock to satisfy its budget constraint.

4 Trade

4.1 Factor Markets

In period \( t \), a representative firm rents capital and hires workers in factor markets at rates \( r_t \) and \( w_t \), respectively. Both workers and firms view that their actions do not affect market prices. Thus, the inverse demand functions for labor and capital by a typical firm are expressed by:

\[
w_t = (1 - \alpha) AK_t^\alpha L_t^{-\alpha}
\]

(3)

and

\[
r_t = \alpha AK_t^{\alpha-1} L_t^{1-\alpha}
\]

(4)

where \( L_t = 1 \) in equilibrium.

4.2 A Typical Worker

At the beginning of period \( t \), each worker receives her labor income. A fraction of the income, \( \tau \), is paid in taxes to the government. Given that agents only value old-age consumption, all income is saved. Furthermore, as agents are subject to relocation shocks, all savings are intermediated. As the population size of workers is equal to mass one, the total amount of deposits in the banking system is: \( (1 - \tau) w_t \).

4.3 A Typical Bank’s Problem

At the beginning of period \( t \), each banker announces deposit rates taking the announced rates of return of other banks as given. A bank promises a gross real return on deposits, \( r_i^n \) if a young individual is relocated and a gross real return \( r_i^n \) if not. Given that banks offer similar financial services, each bank receives the same market share in the deposit market, attracting \( 1/N \) depositors or \( \frac{1}{N} (1 - \tau) w_t \) in deposits. Each bank allocates its deposits towards cash reserves and capital goods. Let \( m_t \) and \( k_{t+1} \), respectively denote the real amount of cash balances and capital goods held by each bank.\(^6\)

\(^6\)Given that banks solve the same problem, we omit the indexation for each bank.
Furthermore, unlike previous work such as Schreft and Smith (1997) and Ghossoub (2012), the rental market is characterized by Cournot (quantity) competition. That is, each bank recognizes that its own decisions about the amount of capital supplied will affect the market rental rate but that its choice does not affect that of other banks. In this manner, each intermediary faces the inverse demand for capital:

$$r_t = \alpha A \left( K_{t+1}^{N-1} + k_{t+1} \right)^{-1}$$

(5)

where $K_{t+1}^{N-1} = \sum_{j=1}^{N-1} k_{t+1,j}$ is the amount of capital provided by all other intermediaries in the banking system. Banks act as Cournot competitors in the market for capital where a representative bank treats $K_{t+1}^{N-1}$ as given. Moreover, $K_{t+1} = K_{t+1}^{N-1} + k_{t+1}$.

The role of market power by an intermediary enters in the choice of investment by an individual firm, $k_{t+1}$. Under a perfectly competitive capital market, intermediaries do not have any market power. Consequently, the marginal revenue from supplying resources to the capital market would simply be equal to $r_{t+1}$. For example, the rental rate represents marginal income earned from investment in Schreft and Smith (1997).

However, in an imperfectly competitive market, each intermediary is aware that they face a downward-sloping demand curve for capital. As a result, marginal revenue is given by $\frac{\partial r_t (k_{t+1}, K_{t+1}^{N-1})}{\partial k_{t+1}} k_{t+1} + r_t (k_{t+1}, K_{t+1}^{N-1})$. Therefore, due to the distortions from market power, the marginal income earned in the capital market is lower than under perfect competition.

In equilibrium, price competition among banks for depositors will force them to choose return schedules and portfolio allocations to maximize the expected utility of a representative depositor. A bank’s objective function is:

$$\max_{r_t, r_t^m, m_{t}, k_{t+1}} \pi \ln [r_t^m (1 - \tau) w_t] + (1 - \pi) \ln [r_t^m (1 - \tau) w_t]$$

(6)

subject to the following constraints.

First, a bank’s balance sheet at the beginning of period $t$ is expressed by:

$$\frac{1}{N} (1 - \tau) w_t = m_t + k_{t+1}$$

(7)

where $m_t = m_{R,t} + m_{e,t}$ and $m_{R,t} = \rho \frac{1}{N} (w_t - \tau)$. Furthermore, as relocated agents need cash to transact, total payments made to movers satisfy:

$$\frac{\pi}{N} r_t^m (1 - \tau) w_t = m_{R,t} R_{R,t} + m_{e,t} R_{e,t}$$

(8)

Defining $\gamma_t$ to be the fraction of deposits held by a bank in period $t$, the bank’s cash holdings must satisfy liquidity requirements imposed by the monetary authority:

$$\gamma_t \geq \rho$$

(9)
We begin by studying portfolio allocations in which money is strictly dominated in rate of return and bankers’ payments to non-movers are paid out of revenue from renting capital to firms in \( t + 1 \). As banks are required to hold a minimum amount of money cash reserves, money is strictly dominated in rate of return if \( I_t > I_{e,t} \). The constraint on payments to non-movers is therefore:

\[
\frac{1 - \tau}{N} r^n_t (1 - \tau) w_t = r \left( k_{t+1}, K_{t+1}^{N-1} \right) k_{t+1} \tag{10}
\]

Finally, given that the realization of the shock is private information, banks need to offer deposit contracts that are incentive compatible to prevent agents from misrepresenting their realizations of the relocation shock. That is:

\[
r_{t}^{m} \leq r_{t}^{n} \tag{11}
\]

In seeking to determine a bank’s portfolio allocation, suppose initially that the incentive compatibility constraint does not bind. The choice of capital investment by a single financial institution is such that:

\[
(1 - \pi) \left[ \frac{\partial r(k_{t+1}, K_{t+1}^{N-1})}{\partial k_{t+1}} k_{t+1} + r \left( k_{t+1}, K_{t+1}^{N-1} \right) \right] = \pi \frac{R_{e,t}}{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}} \tag{12}
\]

where the term on the left-hand-side of (12) is the additional gain in utility to non-relocated depositors from a marginal increase in capital investment. The term on the right-hand-side is the loss in utility to relocated depositors when the bank increases its capital investment by one unit. Upon using (5), the choice of capital can be expressed as:

\[
(1 - \pi) \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}} \right] = \frac{\pi R_{e,t}}{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}} \tag{13}
\]

where the term \( 1 - (1 - \alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}} \) represents the extent of market power by a representative intermediary in the capital market. Moreover, \( \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}} \) is the market share in the capital market of a typical bank. Solving for \( m_{e,t} \) from the above condition indicates that the amount of excess reserves held by a representative bank is:

\[
m_{e,t} = \frac{1}{N} (1 - \tau) w_t - \left( 1 + \frac{1 - \tau}{\pi} \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}} \right] \frac{R_{R,t}}{R_{e,t}} \right) m_{R,t} \tag{14}
\]

\(^7\)The complete solution to the bank’s problem is provided in the Appendix.
Or, equivalently, the fraction of deposits allocated towards excess reserves is:

\[
\gamma_{e,t} = 1 - \left( 1 + \frac{1-\pi}{\pi} \left[ 1 - (1 - \alpha) \frac{k_{t+1}^e}{k_{t+1} + K_{t+1}^e} \right] \frac{R_{R,t}}{R_{e,t}} \right) \gamma_{R,t} 
\]

(15)

where \( \gamma_{e,t} = m_{e,t}/((1 - \tau) w_t/N) \) and \( \gamma_{R,t} = m_{R,t}/((1 - \tau) w_t/N) = \rho \).

In this manner, \( \gamma_{e,t} > 0 \) if:

\[
\rho < \frac{1}{1 + \frac{1-\pi}{\pi} \left[ 1 - (1 - \alpha) \frac{k_{t+1}^e}{k_{t+1} + K_{t+1}^e} \right] \frac{R_{R,t}}{R_{e,t}}} 
\]

(16)

Thus, if reserve requirements are not particularly tight, then banks will tend to hold excess reserves.

Alternatively, the condition may be written in terms of the relative interest rate paid on required reserves. Let \( \mu = \frac{I_{R,t}}{I_{e,t}} \) denote the relative rate paid to required reserves. Then, (16) can be expressed as:

\[
\mu < \frac{1 - \rho}{\pi} \frac{1 - \alpha}{k_{t+1} + K_{t+1}^e} \frac{R_{R,t}}{R_{e,t}} = \mu_2 
\]

(17)

Notably, the reserve constraint binds (\( \gamma_t = \rho \)) if \( \mu \geq \mu_2 \) – if the relative rate paid to excess reserves is low enough, banks do not hold excess reserves. From (7) - (10) the relative return paid to depositors when banks do not hold excess reserves is:

\[
\frac{r_t^n}{r_t^m} = \frac{\pi}{1 - \frac{\pi}{\rho}} \frac{(1 - \rho) I_t}{I_{R,t}} 
\]

(18)

By comparison when \( \mu < \mu_2 \), the relative rate of return paid to excess (required) reserves is sufficiently high (low) so that banks hold excess reserves and the total reserves-deposit ratio comes from both required reserves and excess reserves:

\[
\gamma_t = \gamma_{e,t} + \gamma_{R,t} 
\]

(19)

Substituting from the expression for the fraction of excess reserves (\( \gamma_{e,t} \)), the reserves to deposit ratio is:

\[
\gamma_t = \frac{1 - \rho \frac{1-\pi}{\pi} \left( 1 - (1 - \alpha) \frac{k_{t+1}^e}{k_{t+1} + K_{t+1}^e} \right) \left( \frac{I_{R,t}}{I_{e,t}} - 1 \right)}{1 + \frac{1-\pi}{\pi} \left[ 1 - (1 - \alpha) \frac{k_{t+1}^e}{k_{t+1} + K_{t+1}^e} \right] \frac{R_{R,t}}{R_{e,t}}} 
\]

(20)

It is clear that \( \frac{\partial \gamma_t}{\partial \mu} < 0 \). Intuitively, a higher interest rate on excess reserves (lower \( \mu \)) raises the marginal cost of capital investment (lowers the cost of holding money) which encourages banks to hold a more liquid portfolio. In contrast,
a higher interest rate on required reserves lowers the marginal cost of capital investment as banks have more income to pay relocated agents. This, in turn, leads banks to allocate fewer resources towards money balances.

Furthermore, using (7) - (10) and (20), the relative return to depositors is such that:

\[
\frac{r_t^n}{r_t^m} = \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}} \right] \frac{I_t}{I_{e,t}}
\]

where \(I_t = r (k_{t+1}, K_{t+1}^{N-1}) \frac{P_{t+1}}{P_t} \) is the nominal return to capital between periods \(t\) and \(t+1\). In this manner, the incentive compatibility constraint is non-binding if: \(I_t > I_{e,t} \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{K_{t+1}^{N-1}} \right]^{-1} = I_1\).

On the other hand, the incentive compatibility constraint is binding to the point where depositors obtain full insurance against liquidity risk if \(I_t \in \left[ I_{e,t}, I_{e,t} \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{K_{t+1}^{N-1}} \right]^{-1} \right] \). Two cases arise here. Under case # 1, excess reserves are strictly dominated in rate of return where \(I_t \in \left[ I_{e,t}, I_{e,t} \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{K_{t+1}^{N-1}} \right]^{-1} \right] \). Then, using (7) - (10) and the fact that \(\frac{r_t^n}{r_t^m} = 1\), a bank’s liquidity holdings are such that:

\[
\gamma_t = \frac{\pi - I_t + (I_{e,t} - I_{R,t}) \rho}{(I_{e,t} + \frac{\pi}{\pi} I_t)}
\]

Under case #2 where \(I_t = I_{e,t} < I_{R,t}\), excess reserves and capital yield the same real return. Given the high return to money, banks hold part of their cash reserves for non-relocated agents in addition to movers. In particular, define \(\lambda_t\) to be the fraction of money balances paid to movers and \((1 - \lambda_t)\) the fraction given to non-movers. Therefore, payments to movers and non-movers are given respectively by:

\[
\frac{\pi}{N} r_t^m (1 - \tau) w_t = \lambda_t [m_{R,t} R_{R,t} + m_{e,t} R_{e,t}]
\]

and

\[
\frac{1 - \pi}{N} r_t^n (1 - \tau) w_t = r \left( k_{t+1}, K_{t+1}^{N-1} \right) k_{t+1} + \left( 1 - \lambda_t \right) [m_{R,t} R_{R,t} + m_{e,t} R_{e,t}]
\]

where from (2)

\[
\frac{P_{t+1}}{P_t} = \frac{\bar{m}_{R,t} R_t + \bar{m}_{e,t} R_e}{\bar{m}_{t+1} + \tau_{t+1} w_{t+1}}
\]

As previously mentioned, we only study equilibria where money is dominated in rate of return \((I_t \geq I_{e,t})\).
4.4 General Equilibrium

In a symmetric Nash equilibrium, each intermediary makes the same choice of investment. We define the symmetric Nash equilibrium level of investment of an individual intermediary as $k_{t+1}^* = k_{t+1}(w_t, P_{t+1}; N, \mu)$. Thus, $K_{t+1}^* = Nk_{t+1}^*(w_t, P_{t+1}; N, \mu)$. In equilibrium, all markets will clear. In particular, labor receives its marginal product, (3), and the labor market clears with $L_t = 1$. In this manuscript we focus on the behavior of the economy in the steady-state, where $m_{t+1} = m_t = \bar{m}$ and $K_{t+1} = K = \bar{K}$. Imposing steady-state on (26), the steady-state inflation rate is:

$$\frac{P_{t+1}}{P_t} = \frac{\rho I_R + (\gamma^* - \rho) I_e}{\gamma^* + \frac{1}{1+\tau}}$$

where

$$\gamma^* = \begin{cases} \frac{\rho}{1-\rho} & \text{if } \mu \\ \frac{1}{1+\frac{\tau}{1+\frac{\tau}{e}}[1-\frac{\tau}{1+\frac{\tau}{e}}]} & \text{if } \mu \\ \frac{\frac{\tau}{1+\frac{\tau}{e}} + (1 - \varphi^*) \rho}{\rho (\mu - 1)} & \text{if } \mu \leq \mu_0 \\ \frac{\frac{\tau}{1+\frac{\tau}{e}}}{\frac{\tau}{1+\frac{\tau}{e}} - 1} & \text{if } \mu > \mu_0 \end{cases}$$

In the first scenario where $\mu \geq \mu_2$, the relative return to excess reserves is so low that banks only hold required reserves. In the second situation in which $\mu \in (\mu_1, \mu_2)$, the relative interest rate on excess reserves is slightly higher and banks start to hold excess reserves. However, as the amount of money balances is relatively low, $r_1^e > r_1^m$. For relative returns such that $\mu \in (\mu_0, \mu_1)$, depositors obtain full insurance but money is still dominated in rate of return. In the final scenario, depositors obtain full insurance and $I_1^* = I_e$.

Finally, as we demonstrate in the appendix, using the pricing equation, (4), the definition of $I$, along with a bank’s balance sheet, (7) the equilibrium aggregate stock of capital is such that:

$$(K^*)^{1-\alpha} = \begin{cases} \left[\frac{1+\rho (\mu - 1)}{1+\frac{\tau}{1+\frac{\tau}{e}}[1-\frac{\tau}{1+\frac{\tau}{e}}]}\right]^{\frac{1}{1-\frac{\tau}{1+\frac{\tau}{e}}}} & \text{if } \mu \geq \mu_2 \\ \alpha A \left(1 - \frac{\pi}{1-\frac{\tau}{1+\frac{\tau}{e}}\rho (\mu - 1)}\right)^{-1} & \text{if } \mu \in (\mu_0, \mu_1) \\ \alpha A \left(\frac{1}{1+\frac{\tau}{1+\frac{\tau}{e}}[1-\frac{\tau}{1+\frac{\tau}{e}}]}\right) & \text{if } \mu \in (\mu_1, \mu_2) \\ \frac{1}{1+\frac{\tau}{1+\frac{\tau}{e}}[1-\frac{\tau}{1+\frac{\tau}{e}}]} & \text{if } \mu \in \left[1, \mu_0\right] \end{cases}$$
We proceed to establish existence and uniqueness of monetary steady-state equilibria in the following proposition. All the details and complete derivations are provided in the Appendix.

**Proposition 1.**

i. Suppose \( \mu \in [1, \mu_0] \), where \( \mu_0 \geq 1 \) if \( \frac{\xi}{\sigma} \geq \frac{\pi}{1-\pi} \left[ \frac{1}{1-\pi} \frac{\alpha}{\tau} - 1 \right] = \tilde{\tau} \). In addition, suppose that \( \tau < \frac{1-2\gamma}{1-\alpha} \) and \( \mu_0 < \tilde{\mu}_1 = 1 + \frac{\gamma}{\mu} \frac{1-\alpha}{\alpha} \). Under these conditions, a steady-state where \( \gamma^* > \rho \), \( \frac{\psi^*}{\psi} = 1 \), and \( I^* = I_e \) exists and is unique.

ii. Suppose \( \mu \in (\mu_0, \mu_1) \), with \( \mu_0 < \mu_1 \). Under this condition, a steady-state where \( \gamma^* > \rho \), \( \frac{\psi^*}{\psi} = 1 \), and \( I^* > I_e \) exists and is unique.

iii. Suppose \( \mu \in [\mu_1, \mu_2] \), where \( \mu_1 \leq \mu_2 \) if \( \rho \leq \frac{1}{1-\pi} \left( \frac{\alpha}{1-\alpha} \frac{\pi}{1-\pi} - \tau \right) = \hat{\rho} \). Under this condition, a steady-state where \( \gamma^* > \rho \), \( \frac{\psi^*}{\psi} > 1 \), and \( I^* > I_e \) exists and is unique.

iv. Suppose \( \mu \geq \mu_2 \). Under this condition, a steady-state where \( \gamma^* = \rho \), \( \frac{\psi^*}{\psi} > 1 \), and \( I^* > I_e \) exists and is unique.

In the first scenario, the interest rate paid to excess reserves is at its highest level, yet the rate paid on required reserves is at least as high as excess reserves. This scenario is possible if taxes on wage income are sufficiently high which provides the central authority with enough revenue to pay the amount of interest on reserves. On the other hand, the tax rate has an upper bound so that the large amount of interest payments on reserves does not lead to multiple steady-states. As the nominal interest rate on excess reserves is so high, then \( I^* = I_e \). Moreover, if \( \mu \) were to be less than 1, then money would no longer be dominated in rate of return.

In the second scenario, the relative return paid to required reserves is somewhat higher but banks still hold some excess reserves. In turn, depositors continue to receive full insurance against liquidity risk. The third scenario also involves banks holding excess reserves but depositors do not obtain full insurance because the reserve requirement is not particularly tight and the interest rate on excess reserves is lower than in the second scenario. Finally, in the last case, the interest rate on excess reserves is so low that banks only hold required reserves.

We proceed to examine how the degree of banking competition affects the economy in the following Proposition.

**Proposition 2.**

i. \( \gamma^* \), \( K^* \), and \( \frac{\psi^*}{\psi} \) do not depend on \( N \) if \( \mu \in [1, \mu_1] \).

ii. \( \gamma^* \) is decreasing in \( N \), but \( K^* \) and \( \frac{\psi^*}{\psi} \) are increasing in \( N \) if \( \mu \in (\mu_1, \mu_2) \).
iii. $\gamma^*, K^*, \text{ and } \frac{\gamma^*}{\mu}$ do not depend on $N$ if $\mu \geq \mu_2$.

Intuitively, when the relative return on excess reserves is high ($\mu \leq \mu_1$), banks hold a lot of excess reserves and provide full insurance against liquidity risk. Further, because the return to excess reserves is so high, there is little incentive for banks to engage in strategic behavior by withholding resources from the capital sector. Consequently, a change in the degree of banking competition will not produce any real effects.

Alternatively, if the return on excess reserves is over an intermediate range, $\mu \in (\mu_1, \mu_2)$, the return from holding excess money balances is low enough that banks exploit their market power in the capital sector. Thus, there are important direct effects from the degree of banking concentration on banks’ portfolios. First, when the degree of concentration in capital markets weakens, each intermediary recognizes that it has less market power and therefore allocates a larger fraction of its deposits towards capital investment. The change in the composition of banks’ portfolios towards less liquid assets results in less insurance to depositors against random relocation shocks. In turn, at the aggregate level, an increase in competition stimulates capital investment. The higher stock of capital puts downwards pressure on its return and upward pressure on wages and deposits. The increase in deposits further contributes towards capital accumulation.

Finally, if the return on excess reserves is too low, $\mu \geq \mu_2$, the liquidity constraint is binding and banks only hold required reserves. In this manner, banks do not distort capital markets and a change in the degree of banking competition has no real aggregate effects.

While the preceding analysis describes how the effects of banking concentration depend on monetary policy, the effects of changes in monetary policy have yet to be mentioned. In the following Proposition, we discuss the effects of monetary policy:

**Proposition 3.**

i. $\frac{d\gamma^*}{d\mu} > 0$, $\frac{dK^*}{d\mu} > 0$, $\frac{d\gamma^*}{d\mu} = 0$ if $\mu \in [1, \mu_0)$

ii. $\frac{d\gamma^*}{d\mu} = 0$, $\frac{dK^*}{d\mu} = 0$, $\frac{d\gamma^*}{d\mu} = 0$ if $\mu \in [\mu_0, \mu_1)$.

iii. $\frac{d\gamma^*}{d\mu} > 0$, $\frac{dK^*}{d\mu} > 0$, $\frac{d\gamma^*}{d\mu} > 0$ if $\mu \in [\mu_1, \mu_2)$.

iv. $\frac{d\gamma^*}{d\mu} = 0$, $\frac{dK^*}{d\mu} = 0$, $\frac{d\gamma^*}{d\mu} = 0$ if $\mu \geq \mu_2$.

Interestingly, the results in Proposition 3 indicate that the effects of monetary policy vary across $\mu$. In particular, when the return on excess reserves is sufficiently high, $\mu \in [1, \mu_0)$ the economy is at the Friedman rule and banks hold cash reserves on behalf of both non-movers and movers. In this case, lowering the return on excess reserves generates a substitution from money to physical capital – banks can continue to provide a complete risk sharing contract as they pay only a fraction of their cash reserves to movers given that the return on excess reserves is very high.
However, when the return on excess reserves is below some level, \( \mu \geq \mu_0 \), all payments to non-relocated agents are made from the return to physical capital. Therefore, if \( \mu \in [\mu_0, \mu_1) \), a higher return on required reserves implies a higher return to non-movers which promotes capital accumulation. In order to satisfy the complete risk sharing contract, banks must hold more money balances. As we show in the appendix, both effects cancel out each other and monetary policy does not influence capital formation.

Interestingly, when the return to money balances is over the intermediate range \([\mu_1, \mu_2)\), the incentive compatibility constraint is relaxed and banks do not offer full insurance to depositors. Instead, under a lower return to money, banks hold a less liquid portfolio, which results in higher capital formation. Finally, banks’ portfolios are constrained when the return to money is sufficiently low as they only hold required reserves. Thus, over the range \( \mu \geq \mu_2 \), monetary policy does not produce any real effects. That is, changes in the interest rate on excess reserves do not affect investment when banks do not have any incentive to hold excess reserves.

How do the effects of monetary policy depend on the degree of concentration in the financial sector? Does an increase in concentration render monetary transmission to be more or less effective in stimulating investment? A quick inspection of the effects of monetary policy over \( \mu \in [\mu_1, \mu_2) \) indicates that:

\[
\frac{dK^*}{d\mu} = \left[ \frac{\frac{1 - \pi}{\pi}}{\frac{1}{1 - \frac{\rho}{\omega}} + \frac{1 - \pi}{\pi} (1 - \tau) (1 - \alpha) A} \right] \frac{\frac{1}{\pi}}{1 - \alpha} \frac{\rho}{1 - \alpha} (1 + \rho (\mu - 1))^{\frac{1}{\pi} - 1} > 0
\]

Clearly, the efficacy of monetary policy is strictly increasing with the degree of banking competition. As the degree of banking concentration rises, banks internalize that they have a lot of market power. As a result, the impact of monetary policy on economic activity is weaker.

5 Welfare Analysis

We proceed to examine how the degree of competition and monetary policy affect economic welfare. In particular, we study the interaction between optimal monetary policy and banking competition. Following previous work such as Williamson (1986) and Ghossoub and Reed (2010), we use the expected utility of a typical generation of depositors as a proxy for welfare. As we demonstrate in the appendix, the expected utility of a typical depositor in the steady-state can be expressed as:

\[\text{Expected Utility} = \quad \text{Expected Utility} \]
Our analysis begins by studying the optimal interest rate policy on reserves, taking reserve requirements as given. The following Proposition sheds some light on this issue:

**Proposition 4.**

i. Suppose $\pi < \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho}$. Under these conditions, $\mu^* = 1 + \frac{1}{1-\rho}$, with $\mu^* \in [1, \mu_0]$.

ii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho} < \tilde{\rho}$. Under these conditions, we have a unique interior optimum, $\mu^* \in (\mu_1, \mu_2)$, where $\mu^*$ is decreasing in $N$.

iii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho \in (\hat{\rho}, \tilde{\rho})$. Under these conditions, $\mu^*$ takes any value $\in [\mu_2, \infty]$.

Interestingly, the optimal policy depends on the degree of liquidity risk in the economy. Figure 1 shows how welfare depends on the interest rate policy in the first case:

![Figure 1. Welfare and Monetary Policy, $\pi < \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho}$](image-url)
In particular, when the degree of liquidity risk is low ($\pi < \frac{1-\rho}{1-\rho_0}$) and reserve requirements are not tight ($\rho < \hat{\rho}$), banks allocate a lot of resources towards capital formation. As a result, the return to capital is relatively low. Thus, there is little gain from lowering the value of money to promote capital investment relative to the loss in risk sharing if the relative return paid to excess reserves is already low. Therefore, welfare is decreasing in $\mu$ over the range, $\mu \in (\mu_1, \mu_2)$. Instead, as capital formation is high, it is welfare-improving to raise the value of money by increasing the return on excess reserves so that $\mu$ approaches $\mu_1$ – in doing so, the return to capital would also rise.

Next, over the range $\mu \in [\mu_0, \mu_1]$, banks provide full insurance to depositors and the level of welfare is invariant to $\mu$. As discussed following Proposition 3, at rates on reserves where $\mu \in [1, \mu_0]$, the return on money is so high that banks hold cash balances on behalf of non-movers in addition to movers. Over this range, there is a trade-off between capital formation and payments on reserves. At lower values of $\mu$ below $\mu_0$, banks substitute away from investing in capital to holding more excess reserves. In fact, there is an optimal interest rate policy ($\mu^*$) which balances the different rates of return to both assets (money and capital).

Interestingly, the optimal interest rate rule is increasing in the tax rate on wages but decreasing in the reserve requirement. At higher tax rates, the after-tax income of depositors is lower. Consequently, in order to maximize welfare, it is optimal to pay a relatively lower rate on excess reserves so that capital formation would be higher – and therefore, wages would be higher – in order to help offset the loss of income from higher taxes. It is also decreasing in the fraction of required reserves – if required reserves are higher, then the amount of excess reserves would tend to be lower. An increase in $\mu^*$ would raise the returns on required reserves and promote welfare.

We next turn to case (ii) where there is relatively more liquidity risk than in case (i). When the degree of liquidity risk is above a certain level, capital investment is low. In this case, the effects of monetary policy on welfare depend on the required reserves ratio. Further, from our discussion of Proposition 3, monetary policy only has real effects when $\mu \in (\mu_1, \mu_2)$. Notably, in such a setting, a lower return on excess reserves (higher value of $\mu$) promotes capital formation and wages, which improves welfare. However, depositors also receive less insurance against relocation shocks which adversely affects their welfare. The optimal policy balances these effects. That is, there is an interior optimal policy on reserves when there is sufficient liquidity risk in the economy and reserve requirements are not particularly tight. As illustrated in Figure 2, $\mu^* \in (\mu_1, \mu_2)$.
In addition, the optimal policy is inversely related to the degree of banking competition. Intuitively, when the banking system gets more concentrated, banks hold more liquid portfolios and provide more insurance against liquidity shocks. This raises the need to lower the value of money to promote capital formation and welfare.

When \( \rho \) is high, there are significant gains from promoting capital formation as the capital stock in the economy is low. Therefore, welfare is strictly increasing with \( \mu \), when \( \mu \in (\mu_1, \mu_2) \). The optimal policy would be any value, \( \mu^* \geq \mu_2 \), as illustrated in Figure 3.
That is, if liquidity risk is high and reserve requirements are also relatively tight, the optimal policy attempts to remove incentives of banks to carry excess reserves. In turn, any interest rate policy where $\mu \geq \mu_2$ provides the same level of welfare since banks would only hold required reserves.

**Does more competition in the banking sector promote welfare? What is the welfare maximizing degree of banking competition, $N^*$?** The following Proposition sheds some light on these issues.

**Proposition 5.**

i. Suppose $\pi < \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho}$. Under this condition, $N^*$ takes any value $\in (1, \hat{N})$.

ii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho < \hat{\rho}$. Under these conditions, we have a unique interior optimum, $N^* \in (\hat{N}, \hat{N})$, with $\frac{dN^*}{d\rho} < 0$.

iii. Suppose $\pi > \frac{1-2\alpha}{1-\alpha}$ and $\rho \in (\hat{\rho}, \hat{\rho})$. Under these conditions, $N^*$ takes any value $\in [\bar{N}, \infty]$.

We begin by discussing case (i) where there is little liquidity risk. When the degree of liquidity risk is low, capital formation is high. Therefore, it is optimal to have a highly concentrated system as the gains in capital formation from more competition are small – this implies that the optimal size of the banking system is not any larger than $\hat{N}$. Further, as the degree of concentration does not affect real outcomes for $N \in (1, \hat{N}]$, then any level of concentration along this range maximizes welfare. In particular, perfect competition under these conditions is not optimal – instead, a relatively concentrated banking
system maximizes welfare so that banks withhold some resources from the capital market and aggregate income from the capital sector is higher than under perfect competition.

However, when the degree of liquidity risk is high, the optimal degree of competition depends on the required reserves ratio. In particular, when banks are not required to hold much liquidity, too much competition can harm total welfare as the loss in insurance outweighs the gains in capital formation. From our discussion of Proposition 2, the degree of banking competition only matters for the real economy when $\mu \in (\mu_1, \mu_2)$, which can be written as a condition on $N$, with $N \in (N, N^*).$ Over this range, a more competitive banking system promotes capital formation. However, banks in a less concentrated system provide less insurance against liquidity risk, which adversely affects welfare. The optimal degree of banking competition balances these trade-offs. In fact, the optimal degree of concentration under case (ii) is inversely related to the relative return on money. Intuitively, when the return on excess reserves is low (high $\mu$), banks provide little insurance to their risk averse depositors. Therefore, it is optimal to have a more concentrated banking system to promote risk sharing. As in case (i), again, perfect competition is not optimal. Yet, if banks are forced to hold a high amount of reserves, perfect competition can be optimal.

Given that the monetary authority has multiple tools at its discretion, we now turn to studying optimal reserve requirements in addition to the optimal interest rate policy. We examine this issue using numerical examples. The parameter values for the construction of the first two tables are: $\alpha = .34$, $A = 10$, $N = 20$, $\tau = 0.015$, and $I_e = 1.02$. In Table 1, $\pi = .4$, which corresponds to the case where $\pi < \frac{1-2\alpha}{2}$ . In this case, the Friedman rule is optimal. As shown in Table 1, there appears to be a continuum of optimal policies – the optimal combination favors paying relatively higher interest on excess reserves as reserve requirements are tighter. When the degree of liquidity risk is low, there will

<table>
<thead>
<tr>
<th>$\rho^*$</th>
<th>$\mu^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1.7614</td>
</tr>
<tr>
<td>0.04</td>
<td>1.3807</td>
</tr>
<tr>
<td>0.06</td>
<td>1.2538</td>
</tr>
<tr>
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<td>1.1904</td>
</tr>
<tr>
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<td>1.1523</td>
</tr>
<tr>
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<td>1.1269</td>
</tr>
<tr>
<td>0.14</td>
<td>1.1088</td>
</tr>
<tr>
<td>0.16</td>
<td>1.0952</td>
</tr>
</tbody>
</table>

Table 1.
Optimal Policy
Under $\pi = .4$
be relatively less individuals who will transact using money balances. As a
result, the optimal interest rate policy favors paying low rates on reserves. If
reserve requirements are not particularly tight, the optimal policy is to pay
a relatively low interest rate on excess reserves since banks will tend to hold
more cash on their balance sheets than required by the central bank. On the
other hand, as reserve requirements increase, banks will hold more required
reserves. As the optimal policy does not involve paying high rates on money
balances, the optimal relative interest rate on required reserves falls. The degree
of concentration does not play a role because banks provide full insurance to
their depositors.

In contrast to the examples in Table 1, Table 2 looks at policy when there
is more liquidity risk, \( \pi = 0.49 > \frac{1-\alpha}{1-\beta} \). In this case, incomplete risk sharing
is optimal but we continue to have a range of policies that maximizes social
welfare:

<table>
<thead>
<tr>
<th>( \rho^* )</th>
<th>( \mu^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
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</tr>
<tr>
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<td>1.9322</td>
</tr>
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<td>1.6215</td>
</tr>
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</tr>
<tr>
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</tr>
<tr>
<td>0.14</td>
<td>1.2663</td>
</tr>
<tr>
<td>0.16</td>
<td>1.2330</td>
</tr>
</tbody>
</table>

Table 2.
Optimal Policy
under \( \pi = 0.4 \)

In comparison to the previous case, there will be more individuals who are ex-
posed to liquidity shocks and derive income from the returns to money balances.
Thus, the optimal policy is to pay relatively higher rates on reserves than in
settings where there is little liquidity risk. As reserve requirements increase, the
degree of risk-sharing improves and it is optimal to pay lower rates on required
reserves.

Finally, Table 3 below looks at optimal policy when there is greater compe-
tition in the banking sector and \( N = 100 \):
In this scenario, the strategic incentives of banks to withhold resources from the capital sector are weaker and there will be less incentive to hoard cash reserves. Thus, their holdings of excess reserves are not as high as when the banking sector is more concentrated. In order to encourage banks to hold more excess reserves, the relative interest rate on required reserves is lower — in other words, the optimal interest rate on excess reserves is higher when the banking sector is more competitive.

### Table 3.
Optimal Policy under $\pi = .49$ and $N = 100$

<table>
<thead>
<tr>
<th>$\rho^*$</th>
<th>$\mu^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>2.1775</td>
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<tr>
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<td>1.5887</td>
</tr>
<tr>
<td>0.06</td>
<td>1.3925</td>
</tr>
<tr>
<td>0.08</td>
<td>1.2944</td>
</tr>
<tr>
<td>0.10</td>
<td>1.2355</td>
</tr>
<tr>
<td>0.12</td>
<td>1.1962</td>
</tr>
<tr>
<td>0.14</td>
<td>1.1682</td>
</tr>
<tr>
<td>0.16</td>
<td>1.1472</td>
</tr>
</tbody>
</table>

6 Conclusions

In recent years, the increased concentration of activity in the banking system has received much attention. However, such consolidation is part of a long-term pattern of consolidation activity. For example, Janicki and Prescott (2006) present evidence that the largest banks which made up less than 1% of active institutions in the United States held over 75% of assets in the banking system prior to the crisis. With this large of a role in economic activity, it is hard to believe that these institutions would not take advantage of their market power in the financial system. Moreover, institutions grew to be so large that they were deemed “too big to fail” during the crisis. Banking regulators and monetary policy authorities must confront the challenges associated with an increasing concentration of activity in the banking sector.

Our analysis demonstrates that banks in more concentrated systems have a tendency to hoard cash reserves in order to exploit their strategic advantages in the market for capital. However, Bencivenga and Smith (1991) argue that one of the key roles of intermediaries is to facilitate risk-sharing and promote investment activity. In stark contrast, concentrated banking systems deviate
from this function. Yet, that does not mean that perfect competition should be a regulatory goal. In some settings, depositors would benefit from consolidation. Nevertheless, the model also demonstrates that the optimal rate on excess reserves is generally lower as the banking sector consolidates. In this manner, we view that our work contributes to recent important research that examines the implications of the industrial organization of the banking system.\textsuperscript{9}

\textsuperscript{9}Tarullo (2011) calls for further research into the industrial organization of the financial system.
References


Gandel, S. 2013. By Every Measure, the Big Banks are Bigger. CNN Money. September 13.


Technical Appendix

1. A typical bank's problem. To begin, we solve the bank’s problem when the incentive compatibility constraint is relaxed. Upon using (7) – (10), along with (5) into the objective function and some algebra, we get:

\[
\max_{k_{t+1}} \pi \ln \left( R_{e,t} \left( \frac{1}{N} (1 - \tau) w_t - k_{t+1} \right) - (R_{e,t} - R_{R,t}) m_{R,t} \right) + (1 - \pi) \ln \left[ r \left( k_{t+1}, K_{R,t}^{N-1} \right) k_{t+1} \right] + \pi \ln \frac{N}{\pi} + (1 - \pi) \ln \frac{N}{1 - \pi}
\]

It is easily verified that the first order condition yields (12) in the text:

\[
- \frac{\pi R_{e,t}}{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}} + (1 - \pi) \left[ \frac{\partial \left( k_{t+1}, K_{R,t}^{N-1} \right) k_{t+1} + r \left( k_{t+1}, K_{R,t}^{N-1} \right) k_{t+1}}{r \left( k_{t+1}, K_{R,t}^{N-1} \right) k_{t+1}} \right] = 0
\]

where \( m_{R,t} R_{R,t} + m_{e,t} R_{e,t} = R_{e,t} \left( \frac{1}{N} (1 - \tau) w_t - k_{t+1} \right) - (R_{e,t} - R_{R,t}) m_{R,t} \)
given that \( m_t = m_{e,t} + m_{R,t} \) and \( \frac{1}{N} (1 - \tau) w_t - k_{t+1} = m_t \). Upon using (5), the first order condition can be written as:

\[
R_{e,t} k_{t+1} = \frac{1 - \pi}{\pi} \left[ 1 - (1 - \alpha) * \frac{k_{t+1}}{k_{t+1} + K_{R,t}^{N-1}} \right] [m_{R,t} R_{R,t} + m_{e,t} R_{e,t}]
\]

which is (13) in the text. Finally, using the bank’s balance sheet condition, (7), where \( k_{t+1} = \frac{1}{N} (1 - \tau) w_t - m_{R,t} - m_{e,t} \) into (13) with some algebra to get:

\[
m_{e,t} = \frac{1}{\pi} (1 - \tau) w_t - \left( 1 + \frac{1 - \pi}{\pi} \left[ 1 - (1 - \alpha) * \frac{k_{t+1}}{k_{t+1} + K_{R,t}^{N-1}} \right] \frac{R_{R,t}}{R_{e,t}} \right) m_{R,t}
\]

Given that \( m_{R,t} = \gamma_{R,t} \frac{1}{N} (1 - \tau) w_t \), with \( \gamma_{R,t} = \rho \), it directly follows that \( \gamma_{e,t} = \frac{m_{e,t}}{\pi (1 - \tau) w_t} \) is:

\[
\gamma_{e,t} = \frac{1 - \left( 1 + \frac{1 - \pi}{\pi} \left[ 1 - (1 - \alpha) * \frac{k_{t+1}}{k_{t+1} + K_{R,t}^{N-1}} \right] \frac{R_{R,t}}{R_{e,t}} \right) \gamma_{R,t}}{1 + \frac{1 - \pi}{\pi} \left[ 1 - (1 - \alpha) * \frac{k_{t+1}}{k_{t+1} + K_{R,t}^{N-1}} \right]}
\]

In this manner, \( \gamma_{e,t} \geq 0 \) if:

\[
\frac{R_{R,t}}{R_{e,t}} \leq \frac{\frac{1 - \rho}{\rho} \frac{\pi}{1 - \pi} \frac{k_{t+1}}{k_{t+1} + K_{R,t}^{N-1}}}{1 - (1 - \alpha) * \frac{k_{t+1}}{k_{t+1} + K_{R,t}^{N-1}}}
\]
where $\frac{R_{e,t}}{R_{t}} = \frac{I_{e,t}}{I_{t}} = \mu$. Therefore, banks hold excess reserves as long as:

$$\mu < \frac{1-\rho}{\rho} \frac{1}{1-\left(1-\alpha\right) \frac{k_{t+1}}{k_{t+1}+K_{t+1}+1}} = \mu_2$$

Next, the equilibrium amount of money is such that:

$$\gamma_t = \gamma_{e,t} + \gamma_{R,t}$$

where $\gamma_{R,t} = \rho$. Upon using the expression for $\gamma_{e,t}$ above, we get:

$$\gamma_t = \frac{1}{1-\frac{1-\alpha}{\pi} \left(1-\alpha\right) \frac{k_{t+1}}{k_{t+1}+K_{t+1}+1} \left[I_{e,t} - 1\right]}$$

which is (20) in the text. Next, the bank’s balance sheet condition, $k_{t+1} = \frac{1}{N} \left(1 - \tau\right) w_t - m_t$ can be equivalently stated as:

$$\frac{N}{1-\frac{1-\alpha}{\pi} \left(1-\alpha\right) \frac{k_{t+1}}{k_{t+1}+K_{t+1}+1} \left[I_{e,t} - 1\right]}$$

Next, using (8) and (10), the relative return to depositors is such that:

$$\frac{1}{N} \frac{r_{e,t}^{n} (1-\tau) w_t}{r_{t}^{n} (1-\tau) w_t} = \frac{r_{e,t}}{m_{R,t} R_{R,t} + m_{e,t} R_{e,t}} \left(k_{t+1}, K_{t+1}^{N-1}\right)$$

or equivalently:

$$\frac{1}{N} \frac{r_{e,t}^{n} (1-\tau) w_t}{r_{t}^{n} (1-\tau) w_t} = \frac{1}{1-\frac{1-\alpha}{\pi} \left(1-\alpha\right) \frac{k_{t+1}}{k_{t+1}+K_{t+1}+1} \left[I_{e,t} - 1\right]}$$

Using the definition of $\gamma_t$, along with the bank’s balance sheet, (7) to get:

$$\frac{r_{e,t}^{n}}{r_{t}^{n}} = \frac{1}{1-\frac{1-\alpha}{\pi} \left(1-\alpha\right) \frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}} \left[I_{e,t} - 1\right]}$$

or equivalently:

$$\frac{r_{e,t}^{n}}{r_{t}^{n}} = \frac{1}{1-\frac{1-\alpha}{\pi} \left(1-\alpha\right) \frac{k_{t+1}}{k_{t+1}+K_{t+1}^{N-1}} \left[I_{e,t} - 1\right]}$$

where $I_t = r_{t} \left(k_{t+1}, K_{t+1}^{N-1}\right) P_{t+1}$. Next, using the fact that $\gamma_t = \gamma_{e,t} + \gamma_{R,t}$, the relative return to depositors can be expressed as:
Finally, we substitute for \( \gamma_t \) from (20) along with some simplification to get:

\[
\frac{r^n_t}{r^m_t} = \frac{\pi}{1 - \pi} \frac{1 - \gamma_t}{\gamma_t + \rho \left( \frac{I_t}{I_e,t} - 1 \right)} \frac{I_t}{I_{e,t}} \tag{37}
\]

which is equation (21) in the text. Clearly, \( \frac{r^n_t}{r^m_t} > 1 \) if \( I_t > \frac{I_{e,t}}{1 - (1 - \alpha) \frac{\rho}{k_t+1 + K_{t+1}^{N-1}}} \).

Next, suppose \( \rho \geq \rho_2 \). As discussed above, under this condition, \( \gamma_{e,t} = 0 \) and \( \gamma_t = \rho \). Therefore, from the bank’s balance sheet, (7) and the definition of \( \gamma_t \), the fraction of deposits allocated towards capital is such that:

\[
\frac{k_{t+1}}{\pi (1 - \tau) w_t} = 1 - \rho \tag{38}
\]

Moreover, from (36) and the fact that \( \gamma_t = \rho \), the relative return to depositors is such that:

\[
\frac{r^n_t}{r^m_t} = \frac{\pi}{1 - \pi} \frac{(1 - \rho)}{\rho} \frac{I_t}{I_{e,t}} \tag{39}
\]

Subsequently, suppose \( I_t \in \left( I_{e,t}, I_{e,t} \left[ 1 - (1 - \alpha) \frac{k_{t+1}}{k_{t+1} + K_{t+1}^{N-1}} \right]^{-1} \right) \). In this case, the bank’s incentive compatibility constraint is binding, with \( \frac{r^n_t}{r^m_t} = 1 \) and excess reserves are dominated in rate of return. From (36) and the fact that \( \frac{r^n_t}{r^m_t} = 1 \), it is easy to verify that:

\[
\gamma_t = \frac{\frac{\pi}{1 - \pi} I_t - \rho (I_{e,t} - I_{e,t})}{I_{e,t} \left( \frac{\pi}{1 - \pi} I_t \right)}
\]

which is (22) in the text. In addition:

\[
\gamma_{e,t} = \gamma_t - \rho = \frac{\frac{\pi}{1 - \pi} (1 - \rho) I_t - \rho I_{e,t}}{I_{e,t} + \frac{\pi}{1 - \pi} I_t} \tag{40}
\]

Moreover, \( I_t = r \left( k_{t+1}, K_{t+1}^{N-1} \right) \frac{P_{t+1}}{f_t} \), where

\[
\frac{P_{t+1}}{P_t} = \frac{\rho I_R + \gamma_{e,t} I_e}{\gamma_t + \frac{\tau}{1 - \tau}}
\]

Using the expressions for \( \gamma_{e,t} \) and \( \gamma_t \):
\[
\frac{P_{t+1}}{P_t} = \frac{\left( \rho I_R + \frac{\tau}{1-\tau} (1-\rho) I_t - \rho R_{t, t} I_e \right)}{\frac{\tau}{1-\tau} I_t + (1-\rho) R_{t, t} + \frac{\tau}{1-\tau} I_e} + \frac{\tau}{1-\tau}
\]

which can be written as:
\[
\frac{P_{t+1}}{P_t} = \frac{\rho I_R + (1-\rho) I_e}{\frac{1-\rho}{1-\tau} I_t + \frac{1}{\tau} \left[ \left( \rho + \frac{\tau}{1-\tau} \right) I_{e,t} - \rho R_{t, t} \right]} I_t
\]

Plugging this information into the expression for the nominal return to capital, \( I_t = r (k_{t+1}, K_{t+1}^{N-1}) \frac{P_{t+1}}{P_t} \):
\[
I_t = (1-\tau) \left[ \rho I_R + (1-\rho) I_e \right] r (k_{t+1}, K_{t+1}^{N-1}) - (1-\tau) \frac{1-\pi}{\pi} \left[ \left( \rho + \frac{\tau}{1-\tau} \right) I_{e,t} - \rho R_{t, t} \right]
\]

(41)

2. Proof of Proposition 1. We begin by providing the conditions for case (iii). Notably, suppose that \( \frac{\gamma^*}{\gamma} > 1 \) and \( \gamma^* > \rho \). First, from the derivation of the bank’s problem, we showed that \( \gamma^* > \rho \) if \( \mu < \mu_2 \). Next, we established that \( \frac{\gamma^*}{\gamma} > 1 \) if \( I_t > \frac{\gamma^*}{\gamma} \frac{I_{e,t}}{I_t} \). In a symmetric steady-state Nash equilibrium, this condition can be written as:
\[
I > I_e \frac{1-\alpha}{1-\pi}
\]
where
\[
I = r \frac{P_{t+1}}{P_t}
\]

and \( \frac{P_{t+1}}{P_t} \) is given by (27) and \( r = \alpha AK^{\alpha-1} \) under the functional form for the production technology. Using the expressions for \( \gamma_e \) and \( \gamma^* \), from (15) and (20), along with some algebra:
\[
\frac{P_{t+1}}{P_t} = \frac{\rho I_R + (1-\rho) I_e}{1-\rho \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] \left( \frac{I_{e,t}}{I_e} - 1 \right) + \frac{\tau}{1-\tau} \left( 1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] \right)}
\]

(42)

In addition, imposing symmetry and steady-state on (32):
\[
\frac{1}{(1-\tau)} \frac{K}{w} = 1 - \frac{\rho \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] (\mu - 1)}{1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right]}
\]

(43)

where \( \frac{K}{w} = \frac{K^{1-\alpha}}{(1-\alpha) \bar{A}} = \frac{\alpha}{(1-\alpha) \bar{A}} \). Substituting this information into (43) :

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\[ r = \frac{1}{(1 - \tau) (1 - \alpha)} \frac{\alpha}{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)} \quad (44) \]

Equivalently:

\[ K^{1-\alpha} = (1 - \tau) (1 - \alpha) A \left( 1 - \frac{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)}{1 + \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]} \right) \quad (45) \]

which can be written as:

\[ K^{1-\alpha} = (1 - \tau) (1 - \alpha) A \frac{\frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] \left[ 1 + \rho (\mu - 1) \right]}{1 + \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]} \]

which is the expression found in the text.

Using (42) and (44) and the definition of \( I \):

\[ I = \frac{\alpha}{\frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]} \frac{1}{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)} + \frac{\tau}{1 - \tau} I_e \quad (46) \]

In this manner, \( I > \frac{I_e}{1 - \frac{1 - \omega}{N}} \) if:

\[ \frac{\frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]}{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)} \frac{1}{1 + \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]} + \frac{\tau}{1 - \tau} I_e > \frac{I_e}{1 - \frac{1 - \omega}{N}} \quad (47) \]

With some simplification, (47) can be written as:

\[ \mu > 1 + \frac{1 - \left( \frac{\alpha}{\frac{1-x}{x} (1 - \tau) (1 - \alpha) - \frac{\tau}{1 - \tau}} \right) \left[ 1 + \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] \right]}{\rho \frac{\frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]}{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)}} = \mu_1 \]

where \( \mu_2 > \mu_1 \) if:

\[ \frac{\frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]}{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)} > 1 + \frac{1 - \left( \frac{\alpha}{\frac{1-x}{x} (1 - \tau) (1 - \alpha) - \frac{\tau}{1 - \tau}} \right) \left[ 1 + \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] \right]}{\rho \frac{\frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right]}{1 - \rho \frac{1-x}{x} \left[ 1 - \frac{1 - \omega}{N} \right] (\mu - 1)}} \]

which can be reduced to:

\[ \rho < \frac{1}{1 - \tau} \left( \frac{\alpha}{\frac{1-x}{x} (1 - \alpha) - \frac{\tau}{1 - \tau}} \right) = \hat{\rho} \]

This provides the proof of case (iii) in the Proposition.

Next, we move to case (ii). Suppose \( \mu \leq \mu_1 \), where \( r^n = r^m \). We proceed to find conditions under which \( I > I_e \). From (41) above, in the steady-state:
\[ I = (1 - \tau) \left[ \rho I_R + (1 - \rho) I_e \right] - (1 - \tau) \frac{1 - \pi}{\pi} \left[ \left( \rho + \frac{\tau}{1 - \tau} \right) I_{e,t} - \rho I_{R,t} \right] \]  \hspace{1cm} (48)

Using (7) and (22), with some simplification, we have:

\[ \frac{K}{(1 - \tau) w} = \frac{(1 - \rho) I_{e,t} + \rho I_{R,t}}{1 + \frac{\pi}{1 - \tau} I} \]  \hspace{1cm} (49)

From our work above, \( \frac{K}{w} = \frac{1}{(1 - \alpha) AK} = \frac{\alpha}{(1 - \alpha) \pi} \). Plugging this information into (49):

\[ I = \frac{1 - \pi}{\pi} \frac{(1 - \rho) I_{e,t} + \rho I_{R,t}}{(1 - \tau)(1 - \alpha)} - \frac{1 - \pi}{\pi} I_e \]  \hspace{1cm} (50)

Setting (41) = (50) along with some algebra:

\[ r = \left( \frac{1 - \alpha}{\alpha} - \frac{\pi}{1 - \pi} \right)^{-1} \]  \hspace{1cm} (51)

or equivalently:

\[ K^{1 - \alpha} = \alpha A \left( \frac{1 - \alpha}{\alpha} - \frac{\pi}{1 - \pi} \right) \]  \hspace{1cm} (52)

In regards to the nominal return to capital, using (51) into (50) we get:

\[ I = \frac{1 - \pi}{\pi} \frac{(1 - \tau)(1 - \alpha)(1 - \rho) I_{e,t} + \rho I_{R,t}}{(1 - \alpha)} - \frac{1 - \pi}{\pi} I_e \]  \hspace{1cm} (53)

From (53), it is trivial to show that \( I > I_e \) if:

\[ \mu > \frac{\alpha \left( \frac{1 - \alpha}{\alpha} - \frac{\pi}{1 - \pi} \right)}{(1 - \pi)(1 - \tau)(1 - \alpha) \rho} \cdot \frac{(1 - \rho)}{\rho} = \mu_0 \]

Next, \( \mu_0 \leq \mu_1 \) if

\[ \frac{\alpha \left( \frac{1 - \alpha}{\alpha} - \frac{\pi}{1 - \pi} \right)}{(1 - \pi)(1 - \tau)(1 - \alpha) \rho} \cdot \frac{(1 - \rho)}{\rho} \leq 1 + \frac{1 - \left( \frac{\alpha}{1 - \tau(1 - \alpha)} - \frac{\pi}{1 - \tau} \right)}{\rho \frac{\pi}{1 - \tau} \left[ 1 - \frac{\alpha}{N} \right]} \]

With a few lines of algebra, this condition can be written as:

\[ 1 - \frac{1 - \alpha}{N} \leq 1 \]

which always holds as \( N > 1 \). This completes the proof for case (ii).

We continue by moving on to case (i). Clearly, at \( \mu = \mu_0 \), \( I = I_e \). In addition, \( \mu_0 \geq 1 \) if:
We proceed to get an expression for the real return to capital. From (25), in the steady-state, the Friedman rule implies that:

\[ r \frac{P_{t+1}}{P_t} = I_e \]  

Using the expression for inflation from (27):

\[ \frac{P_{t+1}}{P_t} = \frac{(\rho I_R + \gamma e I_e)}{\gamma + \frac{\tau}{1-\tau}} \]

Upon substituting into (54):

\[ \gamma = \frac{\rho (\mu - 1) - \frac{1}{r} \frac{\tau}{1-\tau}}{(\frac{1}{r} - 1)} \]  

Moreover, from the bank’s balance sheet, (7):

\[ \gamma = 1 - \frac{\alpha}{(1-\alpha)(1-\tau) r} \]  

The equilibrium value(s) of \( r \) at the Friedman rule solve the system of equations given by (55) and (56).

We begin by characterizing the (56) locus. The locus (56) satisfies the following: \( \gamma = 0 \) when \( r = r_0 = \frac{\alpha}{(1-\alpha)(1-\tau)} \) and \( \frac{d\gamma}{dr} > 0 \). Moreover, \( \lim_{r \to \infty} \gamma \to 1 \).

Next, we characterize the (55) locus. The sign of \( \gamma \) depends on the relationship between \( r \) and \( \hat{r} = \frac{\tau}{\rho (\mu - 1)} \). On one hand, \( \hat{r} \) may be larger than 1. Under this case, if \( r \in (1, \hat{r}) \), then \( \gamma > 0 \). Alternatively, \( \hat{r} \) may be less than 1. In this case, \( \gamma > 0 \) if \( r \in (\hat{r}, 1) \).

Note that for \( r < 1 < \hat{r}, \gamma < 0 \). If \( \hat{r} > 1 \), it must be that in an equilibrium \( r \in (1, \hat{r}) \). Moreover, \( \lim_{r \to 1} \gamma \to \infty \) and \( \frac{d\gamma}{dr} < 0 \). In addition, \( \gamma (\hat{r}) = 0 \). By comparison, if \( \hat{r} < 1 \), it must be that in equilibrium, \( r < 1 \). For \( r \in (\hat{r}, 1) \), \( \gamma > 0, \frac{d\gamma}{dr} > 0, \lim_{r \to 1} \gamma \to \infty \) and \( \gamma (\hat{r}) = 0 \).

Given the characterization of each locus, a unique intersection between both loci occurs when \( r_0 < \hat{r} \) (we can get up to two intersections if \( \hat{r} < r_0 \)). Clearly, \( r_0 \leq \hat{r} \) if:

\[ \frac{\alpha}{(1-\alpha)(1-\tau)} \leq \frac{\tau}{\rho (\mu - 1)} \]

which can be written as:

\[ \mu \leq 1 + \frac{\tau}{\rho} \frac{1-\alpha}{\alpha} = \hat{\mu}_1 \]  

Moreover, \( \hat{r} > 1 \) if:

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\[ \mu < 1 + \frac{\tau}{\rho(1 - \tau)} = \mu_0 \]  

Next, combining, (55) and (56) with some algebra, the following polynomial yields the values of \( r \):

\[ r^2 + \frac{1}{(1 - \tau)(1 + \rho(\mu - 1))} \frac{\alpha}{1 - \alpha} - \frac{1}{(1 - \tau)(1 + \rho(\mu - 1))} \frac{1}{1 - \alpha} \rho \tau = 0 \]

which has two roots:

\[

t_1 = \frac{1}{(1 - \tau)(1 + \rho(\mu - 1))} \frac{1}{1 - \alpha} \left( 1 + \frac{\sqrt{1 - 4(1 - \tau)(1 + \rho(\mu - 1))}}{(1 - \tau)(1 - \alpha)} \right) \\

\]

\[

t_2 = \frac{1}{(1 - \tau)(1 + \rho(\mu - 1))} \frac{1}{1 - \alpha} \left( 1 - \frac{\sqrt{1 - 4(1 - \tau)(1 + \rho(\mu - 1))}}{(1 - \tau)(1 - \alpha)} \right) \\

\]

The discriminant under the square root is positive if:

\[ \mu < \left( \frac{1}{(1 - \alpha) \alpha 4(1 - \tau) - 1} \right) \frac{1}{\rho} + 1 = \mu_2 \]

where \( \mu_2 > \mu_0 \) if:

\[ \left( \frac{1}{(1 - \alpha) \alpha 4(1 - \tau) - 1} \right) \frac{1}{\rho} + 1 > \frac{\rho + \frac{\tau + \tau}{(1 - \tau)} - \frac{\tau + (1 - \rho)}{\frac{1 - \alpha}{1 - \alpha} - \frac{1 - \alpha}{1 - \tau}}}{\rho (1 + \frac{\frac{1 - \alpha}{1 - \alpha}}{1 - \tau})} \]

with some algebra this condition can be written as:

\[ \alpha^2 - \alpha (1 - \pi) + \frac{(1 - \pi)^2}{4} > 0 \]

which holds for all \( \alpha \in [0, 1] \). In this manner for all \( \mu < \mu_0 \), the discriminant in the expressions for \( r \) is positive.

Next, \( \mu_0 < \mu_1 \) if:

\[ 1 + \frac{\tau}{1 - \tau \rho} < 1 + \frac{\tau}{\rho} \frac{1 - \alpha}{\alpha} \]

which can be written as:

\[ \tau < \frac{1 - 2\alpha}{1 - \alpha} \]

As we focus on the case of a unique equilibrium at the Friedman rule, our analysis above requires that: \( r_0 \leq \hat{r} \) or equivalently: \( \mu \leq 1 + \frac{\tau}{\rho} \frac{1 - \alpha}{\alpha} = \mu_1 \). A
sufficient condition is that \( \tilde{\mu}_1 > \mu_0 \). Upon using the expressions for \( \tilde{\mu}_1 \) and \( \mu_0 \), this condition is written as:

\[
\tau \left( \frac{1 - \alpha}{\alpha} \right)^2 \left[ \frac{(1 - 2\alpha)}{(1 - \alpha)} - \tau \right] + \frac{\pi}{1 - \pi} \left( \frac{1}{1 - \pi} - \frac{(1 - \alpha)}{\alpha} \right) > 0
\]

which always holds when \( \tau \geq \frac{1 - 2\alpha}{1 - \alpha} \). We assume that this condition also holds when \( \tau < \frac{1 - 2\alpha}{1 - \alpha} \). Given this analysis, a unique Friedman rule equilibrium exists if \( \mu \in [1, \mu_0) \). Notably, if \( \mu_0 < \tilde{\mu}_0 \), the Friedman rule equilibrium is such that \( r > 1 \). By comparison, if \( \mu_0 > \tilde{\mu}_0 \), the Friedman rule equilibrium is such that \( r > 1 \) when \( \mu \in [1, \tilde{\mu}_0] \) and \( r < 1 \) if \( \mu \in [\tilde{\mu}_0, \mu_0] \). Upon using the definitions of \( \mu_0 \) and \( \tilde{\mu}_0 \), if:

\[
\frac{\alpha \left( \frac{1 - \alpha}{\alpha} - \pi \right) \rho}{(1 - \pi)(1 - \tau)(1 - \alpha) \rho} < 1 + \frac{\tau}{1 - \tau} \rho
\]

which can be written as: \( \tau > \frac{1 - 2\alpha}{1 - \alpha} \). Finally, it is easily verified that \( \mu_0 \geq 1 \) if \( \tau > 1 - \pi \left[ \frac{1 - \alpha}{1 - \pi} - 1 \right] = \tilde{\tau} \), with \( \tilde{\tau} > 0 \) if \( \pi \geq \frac{1 - 2\alpha}{1 - \alpha} = \tilde{\pi} \). In this manner, when the conditions in Proposition 1, case i hold, a unique Friedman rule equilibrium exists. The equilibrium is represented by \( r_1 \) above. Upon substituting for the production technology, the expression for \( K \) in (29) is easily obtained. This completes the proof of Proposition 1.

3. Derivation and characterization of the welfare function. To begin, consider case iii, where \( \mu \in [\mu_1, \mu_2] \) under which \( \frac{r^n}{n!} > 1 \) and \( \gamma > \rho \). From (6), the expected utility of depositors is such that:

\[
U_t = \pi \ln \left[ r^m_t (1 - \tau) w_t \right] + (1 - \pi) \ln \left[ r^n_t (1 - \tau) w_t \right]
\]

Substituting the binding constraints, (8) and (10):

\[
U_t = \pi \ln \left[ N m_t R_t + m_t R_t \right] + (1 - \pi) \ln \left[ N r^k_t \right]
\]

Imposing steady-state and symmetry:

\[
U = \pi \ln \left[ N m_R R + N m_R \right] + (1 - \pi) \ln \left[ \frac{rK}{(1 - \pi)} \right]
\]

(61)

Subsequently, using the definition of \( \gamma \), the fact that \( I_i = R_t \frac{m_t}{P_{t+1}} \), with \( i = e, R \), and (27):

\[
U = \pi \ln \left[ \frac{\gamma + \frac{r}{\tau}}{\pi} \right] (1 - \tau) w + (1 - \pi) \ln \left[ \frac{rK}{(1 - \pi)} \right]
\]

(62)

With some simplification, we have:
\[ U = \alpha \ln K + \pi \ln \left[ \gamma + \frac{\tau}{1 - \tau} \right] + \pi \ln \left( \frac{1 - \tau}{1 - \alpha} \right) A \frac{A}{\pi} + (1 - \pi) \ln \frac{\alpha A}{1 - \pi} \] (63)

where \( K^{1-\alpha} = \left[ \frac{1 + \rho (\mu - 1)}{1 + \frac{1 - \alpha}{\pi} [1 - \frac{1 - \alpha}{\pi}]} \right] \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] (1 - \tau) (1 - \alpha) A \) from (29) and:

\[
\gamma = \frac{1 - \rho \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] (\mu - 1)}{1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}
\]

Upon differentiating (63):

\[
\frac{dU}{d\mu} = \frac{1}{K} \frac{dK}{d\mu} + \frac{\pi}{\gamma + \frac{\pi}{\pi}} \frac{d\gamma}{d\mu}
\] (64)

where from the expression for \( K \),

\[
(1 - \alpha) K^{-\alpha} \frac{dK}{d\mu} = \frac{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] (1 - \tau) (1 - \alpha) A \rho}{1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}
\]

Upon using the expression for \( K \) and some simplification:

\[
\frac{1}{K} \frac{dK}{d\mu} = \rho \frac{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}{(1 - \alpha) [1 + \rho (\mu - 1)]}
\] (65)

Next, from the expression for \( \gamma \), (28):

\[
\gamma = \frac{1 - \rho \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] (\mu - 1)}{1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}
\]

Differentiating with respect to \( \mu \):

\[
\frac{d\gamma}{d\mu} = -\rho \frac{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}{1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}
\] (66)

Plugging (65) and (66) into (64) with some algebra:

\[
\frac{dU}{d\mu} = \frac{\alpha \rho}{(1 - \alpha) [1 + \rho (\mu - 1)]} \left( 1 - \rho \frac{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] (\mu - 1) + \frac{\pi}{1 - \pi} \left( 1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] \right)} \right)
\]

\[
\frac{dU}{d\mu} \geq 0 \text{ if:}
\]

\[
\frac{\alpha \rho}{(1 - \alpha) [1 + \rho (\mu - 1)]} \left( 1 - \rho \frac{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]}{1 - \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] (\mu - 1) + \frac{\pi}{1 - \pi} \left( 1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right] \right)} \right) \geq 0
\] (68)

which can be re-written as:
\[
\mu \leq \frac{1}{1 - \frac{\alpha}{\pi}} + \frac{\pi}{1 - \frac{\alpha}{\pi}} \left( 1 - \frac{\alpha}{\pi} \right) + \frac{\pi}{1 - \frac{\alpha}{\pi}} \left( 1 - \frac{\alpha}{\pi} \right) + \left[ \frac{\pi}{1 - \frac{\alpha}{\pi}} + 1 \right] \frac{1 - \frac{\alpha}{\pi} \rho}{\rho} = \hat{\mu}
\]

(69)

Recall that \( \mu_2 = \frac{1 - \rho}{\pi} \) and \( \mu_1 = 1 + \frac{1}{\rho} \left[ \frac{1}{\pi} - 1 \right] \). The behavior of the welfare function with respect to \( \mu \) over the \( \mu \in (\mu_1, \mu_2) \) interval clearly hinges on the position of \( \hat{\mu} \) relative to \( \mu_1 \) and \( \mu_2 \). Three possibilities may arise, one where \( \hat{\mu} < \mu_1 \), another where \( \hat{\mu} \in (\mu_1, \mu_2) \), and a third case where \( \hat{\mu} > \mu_2 \). First, \( \hat{\mu} \leq \mu_1 \) if:

\[
1 + \frac{\pi}{1 - \frac{\alpha}{\pi}} \left( 1 - \frac{\alpha}{\pi} \right) - \frac{\pi}{1 - \frac{\alpha}{\pi}} \left( 1 - \frac{\alpha}{N} \right) + 1 \leq 1 + \frac{1}{\rho} \left[ \frac{1}{\pi} - 1 \right] \left[ 1 - \frac{\alpha}{N} \right]
\]

Upon simplifying, it can be verified that this condition becomes:

\[
\pi \leq \frac{1 - 2\alpha}{1 - \alpha}
\]

From our work above, \( \mu_1 \leq \mu_2 \) if:

\[
\rho \leq \left( \frac{\alpha}{1 - \alpha} \frac{1}{1 - \frac{\alpha}{\pi}} \frac{1}{1 - \frac{\alpha}{\pi}} \right) = \hat{\rho}
\]

In this manner, whenever \( \pi \leq \frac{1 - 2\alpha}{1 - \alpha} \) and \( \rho < \hat{\rho}, \hat{\mu} \leq \mu_2 \) and \( \frac{d\mu}{d\rho} \leq 0 \) for \( \mu \in [\mu_1, \mu_2] \).

Now, suppose \( \pi > \frac{1 - 2\alpha}{1 - \alpha} \). Under this condition, \( \hat{\mu} > \mu_1 \). Next, we need to compare \( \hat{\mu} \) to \( \mu_2 \). In particular, \( \hat{\mu} \leq \mu_2 \) if:

\[
1 + \frac{\pi}{1 - \frac{\alpha}{\pi}} \left( 1 - \frac{\alpha}{\pi} \right) - \frac{\pi}{1 - \frac{\alpha}{\pi}} \left( 1 - \frac{\alpha}{N} \right) + 1 \leq \frac{1 - \rho}{1 - \frac{\alpha}{\pi}} \frac{\pi}{1 - \frac{\alpha}{\pi}} \frac{1 - \frac{\alpha}{\pi}}{1 - \frac{\alpha}{N}}
\]

with some algebra, this condition becomes:

\[
\rho \leq \frac{\pi}{1 - \frac{\alpha}{\pi}} = \hat{\rho}
\]

Using the definitions of \( \hat{\rho} \) and \( \hat{\mu} \), with a few lines of algebra, \( \hat{\rho} > \hat{\rho} \) if \( \pi > \frac{1 - 2\alpha}{1 - \alpha} \).

Two cases emerge here. First, consider that \( \rho < \hat{\rho} < \hat{\rho} \). Under this condition, \( \hat{\mu} \in (\mu_1, \mu_2) \) and we have a unique interior local optimum, \( \mu^* \in (\mu_1, \mu_2) \). Second, if \( \rho \in (\hat{\rho}, \hat{\rho}) \), \( \hat{\mu} > \mu_2 \) and \( \frac{d\mu}{d\rho} > 0 \) for \( \mu \in (\mu_1, \mu_2) \).
Next, consider case iv, where $\mu \geq \mu_2$, with $\frac{r^n}{r_m} > 1$ and $\gamma = \rho$. Over this parameter space, we established that $K^{1-\alpha} = (1-\rho)(1-\tau)(1-\alpha)A$ and $\frac{P_{t+1}}{P_t} = \frac{\rho P_n}{r^{1+1}}$. Plugging this information into the welfare function, (62):

$$U = \pi \ln \left( \frac{\rho + \frac{r^n}{r_m}}{\pi} \right) (1-\tau)w(K) + (1-\pi) \ln \frac{r(K)K}{1-\pi}$$

Since $\frac{dK}{dt} = 0$, it directly follows that, $\frac{dU}{dt} = 0$.

Under case ii, where $\mu \in (\mu_0, \mu_1]$, with $\frac{r^n}{r_m} = 1$ and $I > I_e$, we established that $K^{1-\alpha} = A \left( \frac{1-\alpha}{\alpha} - \frac{1-\pi}{\pi} \right)$. Given that depositors receive complete risk sharing, the welfare function is such that:

$$U = \ln \left[ r^n(1-\tau)w \right]$$

By (10) in a symmetric steady-state equilibrium, $r^n(1-\tau)w = \frac{r(K)K}{1-\pi}$. Therefore:

$$U = \ln \frac{r(K)K}{1-\pi}$$

Clearly, $\frac{dU}{dt} = 0$ as $\frac{dK}{dt} = 0$.

Finally, at $\mu = \mu_0$, where $\frac{r^n}{r_m} = 1$ and $I = I_e$, from the work above, the expected utility is:

$$U_t = \ln \left[ r^m(1-\tau)w_t \right] = \ln c_{t+1}$$

We proceed to derive an expression for $c_{t+1}$. To begin, using (23) and (24) under complete risk sharing:

$$\frac{\lambda N \left[ m_{R,t}R_{R,t} + m_{e,t}R_{e,t} \right]}{\pi} = \frac{r(k_{t+1}, K_t^{N-1}) k_{t+1} + (1-\lambda_{t}) \left[ m_{R,t}R_{R,t} + m_{e,t}R_{e,t} \right]}{1-\pi}$$

With a few lines of algebra, (70) becomes:

$$\lambda_t = \pi \left( \frac{r(k_{t+1}, K_t^{N-1}) k_{t+1} + 1}{m_{R,t}R_{R,t} + m_{e,t}R_{e,t}} \right)$$

At the Friedman rule, $r(k_{t+1}, K_t^{N-1}) = R_{e,t}$. Therefore, (71) is:

$$\lambda_t = \pi \left( \frac{k_{t+1}}{m_{R,t}R_{e,t} + m_{e,t}} + 1 \right)$$

From (23) in the steady-state:

$$c = r^m(1-\tau)w = \frac{\lambda N \left[ m_{R, R} + m_{e}R_e \right]}{\pi}$$

Upon substituting for $\lambda_t$ in the steady-state and imposing symmetry:
\[ c = (K + N [m_R \mu + m_e]) R_e \]  
which can be written as:

\[ c = (1 - \gamma + \rho \mu + \gamma_e) r (K) (1 - \tau) w \]  

Substituting for the functional form from the production function and utilizing \( \gamma_e = \gamma - \rho \):

\[ c = \frac{(1 - \tau) (1 + \rho \mu - \rho) \alpha A (1 - \alpha) A}{K^{1 - 2\alpha}} \]

Equivalently:

\[ c = \frac{(1 - \tau) \alpha A (1 - \alpha) A}{(\alpha A)^{\frac{1 - 2\alpha}{1 - \alpha}}} \left( 1 + \rho \mu - \rho \right) r^{\frac{1 - 2\alpha}{1 - \alpha}} \]  

where \( r = r_1 \) derived above. In this manner:

\[ U = \ln \left( \frac{(1 - \tau) \alpha A (1 - \alpha) A}{(\alpha A)^{\frac{1 - 2\alpha}{1 - \alpha}}} \left( 1 + \rho \mu - \rho \right) r^{\frac{1 - 2\alpha}{1 - \alpha}} \right) \]

which is the expression in the text. Differentiating \( U \) with respect to \( \mu \) to get:

\[ \frac{dU}{d\mu} = \frac{1}{c} \frac{dc}{d\mu} \]  

where

\[ \frac{dc}{d\mu} = \frac{(1 - \tau) \alpha A (1 - \alpha) A}{(\alpha A)^{\frac{1 - 2\alpha}{1 - \alpha}}} \left( \rho r^{\frac{1 - 2\alpha}{1 - \alpha}} + \frac{1 - 2\alpha}{1 - \alpha} \left( 1 + \rho \mu - \rho \right) \frac{dr}{d\mu} r^{\frac{1 - 2\alpha}{1 - \alpha} - 1} \right) \]

Upon substituting into (76) and some simplification to get:

\[ \frac{dU}{d\mu} = \frac{\rho + \frac{1 - 2\alpha}{1 - \alpha} \left( 1 + \rho \mu - \rho \right) \frac{1}{r} \frac{dr}{d\mu}}{\left( 1 + \rho \mu - \rho \right)} \]  

Given that the term in the denominator of (77) is positive, \( \frac{dU}{d\mu} \geq 0 \) if:

\[ 1 \geq - \frac{1 - 2\alpha}{1 - \alpha} \frac{1 + \rho (\mu - 1)}{\rho} \frac{1}{r} \frac{dr}{d\mu} \]  

Recall the expression for \( r \):

\[ r_1 = \frac{1}{2 (1 - \alpha) (1 - \tau)} \frac{1 + [1 - 4 (1 - \tau) [1 + \rho (\mu - 1)] (1 - \alpha) \alpha]^{\frac{1}{2}}}{[1 + \rho (\mu - 1)]} \]

Upon differentiating with respect to \( \mu \) and simplifying, we get:
this manner, the characterization of the welfare function. From our work above, the degree of banking competition only matters when \( \mu \in [\mu_0, \mu_1] \) and \( \frac{dU}{d\mu} = 0 \) if \( \mu \geq \mu_2 \). Therefore, the optimal monetary policy hinges on the behavior of \( U \) at the Friedman rule and over the interval \( \mu \in (\mu_1, \mu_2) \). From our work above, \( \frac{dU}{d\mu} < 0 \) when \( \mu \in (\mu_1, \mu_2) \) if \( \pi \leq \frac{1-2\alpha}{1-\alpha} \) since \( \hat{\mu} < \mu_2 \). However, \( \frac{dU}{d\mu} \geq (\prec) 0 \) if \( \mu \leq (>) \hat{\mu}_0 \) over \( \mu \in [1, \mu_0) \). Therefore, \( \hat{\mu}_0 \) is the global optimum. In comparison, if \( \pi > \frac{1-2\alpha}{1-\alpha} \), \( \frac{dU}{d\mu} > 0 \) if \( \mu \in [1, \mu_0) \). By comparison, if \( \pi < \frac{1-2\alpha}{1-\alpha} \), \( \hat{\mu}_0 \) is a local optimum over the range \( \mu \in [1, \mu_0) \). This completes the characterization of the welfare function.

4. **Proof of Proposition 4.** From our characterization of the welfare function, \( \frac{dU}{d\mu} = 0 \) if \( \mu \in [\mu_0, \mu_1] \) and \( \frac{dU}{d\mu} = 0 \) if \( \mu \geq \mu_2 \). Therefore, the optimal monetary policy hinges on the behavior of \( U \) at the Friedman rule and over the interval \( \mu \in (\mu_1, \mu_2) \). From our work above, \( \frac{dU}{d\mu} < 0 \) when \( \mu \in (\mu_1, \mu_2) \) if \( \pi \leq \frac{1-2\alpha}{1-\alpha} \) since \( \hat{\mu} < \mu_2 \). However, \( \frac{dU}{d\mu} \geq (\prec) 0 \) if \( \mu \leq (>) \hat{\mu}_0 \) over \( \mu \in [1, \mu_0) \). Therefore, \( \hat{\mu}_0 \) is the global optimum. In comparison, if \( \pi > \frac{1-2\alpha}{1-\alpha} \) and \( \rho < \hat{\rho} < \hat{\rho} \), \( \frac{dU}{d\mu} \geq (\prec) 0 \) if \( \mu \leq (>) \hat{\mu} \), where \( \hat{\mu} \in (\mu_1, \mu_2) \) under the conditions stated. In this manner, \( \hat{\mu} \) is a global optimum. This necessarily happens as \( \frac{dU}{d\mu} > 0 \) if \( \mu \in (1, \mu_0) \) when \( \pi > \frac{1-2\alpha}{1-\alpha} \). Finally, suppose \( \pi > \frac{1-2\alpha}{1-\alpha} \) and \( \rho \in (\hat{\rho}, \hat{\rho}) \). Under these conditions, \( \frac{dU}{d\mu} > 0 \), when \( \mu \in (\mu_1, \mu_2) \). Consequently, it is optimal to set \( \mu^* \geq \mu_2 \). This completes the proof of Proposition 4.

5. **Proof of Proposition 5.** From our work above, the degree of banking competition only matters when \( \mu \in (\mu_1, \mu_2) \). Recall that the incentive compatibility constraint is non-binding if:

\[
\mu > \mu_1 = 1 + \frac{1 - \left( \frac{\alpha}{1-\tau(1-\alpha)} - \frac{\tau}{1-\tau} \right)}{\rho \left( \frac{1}{1-\tau} - \frac{\rho(1-\tau)}{\rho-\alpha(1-\tau)} \left( \frac{1}{1-\tau} - \frac{\rho(1-\tau)}{\rho-\alpha(1-\tau)} \right) \right)} [1 - \frac{\rho}{\rho(1-\tau)} (1-\alpha) \frac{1}{N}]
\]

which can be expressed as a condition on \( N \):

\[
N > \frac{1}{\left( \frac{(1-\alpha)}{\left( \frac{1}{1-\tau} - \frac{\rho(1-\tau)}{\rho-\alpha(1-\tau)} \left( \frac{1}{1-\tau} - \frac{\rho(1-\tau)}{\rho-\alpha(1-\tau)} \right) \right)} \right)} = \frac{(1-\alpha)}{\left( \frac{1}{1-\tau} - \frac{\rho(1-\tau)}{\rho-\alpha(1-\tau)} \left( \frac{1}{1-\tau} - \frac{\rho(1-\tau)}{\rho-\alpha(1-\tau)} \right) \right)}
\]

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Analogously, the reserves constraint is non-binding if: \( \mu < \mu_2 = \frac{1 - \rho}{\mu + \frac{1 - \alpha}{N}} \)
which can be written as a condition on \( N \):

\[
N < \frac{1 - \alpha}{1 - \frac{1 - \alpha}{\mu}} = \tilde{N}
\]

In this manner, \( \mu \in (\mu_1, \mu_2) \Longleftrightarrow N \in (N, \tilde{N}) \). That is, the degree of concentration only matters for real activity if \( N \) lies in this range.

Differentiating the welfare function, (63) with respect to \( N \) to obtain:

\[
\frac{dU}{dN} = \alpha \frac{1}{K} \frac{dK}{dN} + \frac{\pi}{\gamma + \frac{\tau}{1-\tau}} \frac{d\gamma}{dN}
\]

where \( K^{1-\alpha} = \left[ \frac{1+\rho(\mu-1)}{1+\frac{1-\alpha}{\mu}} \right] \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] (1-\tau) (1-\alpha) A \) from the work above and

\[
\gamma = \frac{1 - \rho \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] (\mu - 1)}{1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right]}
\]

Using the expressions for \( K \) and \( \gamma \) with some algebra:

\[
\alpha \frac{1}{K} \frac{dK}{dN} = \frac{1}{N^2 \left[ 1 - \frac{1-\alpha}{N} \right]} \frac{\alpha}{\left[ 1 - \frac{1-\alpha}{N} \right] \left( 1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] \right)}
\]

and

\[
\frac{d\gamma}{dN} = -\frac{(\mu-1) \rho + 1}{(1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right])^2} \frac{1 - \pi}{\pi} \frac{1 - \alpha}{N^2}
\]

Substituting this information into \( \frac{dU}{dN} \) to get:

\[
\frac{dU}{dN} = \frac{1}{(1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right]) \left[ 1 - \frac{1-\alpha}{N} \right]} \frac{\alpha}{\left[ 1 - \frac{1-\alpha}{N} \right] \left( 1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] \right)} \frac{1 - \pi}{\pi} (1 - \alpha)
\]

In this manner, \( \frac{dU}{dN} \geq 0 \) if:

\[
\frac{\alpha}{\left[ 1 - \frac{1-\alpha}{N} \right]} \geq \frac{\pi}{\gamma + \frac{\tau}{1-\tau}} \frac{[(\mu-1) \rho + 1]}{\left( 1 + \frac{1-\pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right] \right)} \frac{1 - \pi}{\pi} (1 - \alpha)
\]

Using the expression for \( \gamma \) and some simplification, \( \frac{dU}{dN} \geq 0 \) if:

\[
\frac{1}{1-\tau} > \left[ (\mu-1) \rho + 1 \right] \frac{\left( \frac{1 - \alpha}{\alpha} \right)}{(1-\tau) \frac{1 - \alpha}{\alpha} (1 - \alpha) \frac{1 - \pi}{\pi} \left[ 1 - \frac{1-\alpha}{N} \right]} \equiv \Phi(N)
\]

It is clear that \( \Phi'(N) > 0 \). Moreover,
\[ \Phi(N) = \left( (\mu - 1) \rho \left( \frac{\pi (1 - \alpha)}{\alpha} + 1 \right) + \frac{\pi (1 - \alpha)}{\alpha} - \frac{\alpha}{1 - \tau} \right) \left[ 1 - \left( \frac{\alpha}{1 - \tau} \frac{1 - \pi}{(1 - \tau) (1 - \alpha)} - \frac{\tau}{1 - \tau} \right) \right] \]

(80)

and

\[ \Phi(\tilde{N}) = \left( (\mu - 1) \rho + 1 \right) \pi \left( \frac{1 - \alpha}{\alpha} \right) + \rho (\mu - 1) - \frac{\tau}{1 - \tau} \frac{11 - \rho}{\mu} \right] \]

(81)

Three cases are possible, here. First, if \( \Phi(N) > \frac{1}{1 - \tau} \), then \( \frac{dU}{dN} < 0 \) for \( N \in (N, \tilde{N}) \). Second, if \( \Phi(N) < \frac{1}{1 - \tau} \) but \( \Phi(\tilde{N}) > \frac{1}{1 - \tau} \), then \( \frac{dU}{dN} \geq (>) 0 \) if \( N \leq (>) \tilde{N} \) with \( \tilde{N} \in (N, \tilde{N}) \). Finally, if \( \Phi(\tilde{N}) < \frac{1}{1 - \tau} \), \( \frac{dU}{dN} > 0 \) for \( N \in (N, \tilde{N}) \).

To begin, from (80), \( \Phi(N) > \frac{1}{1 - \tau} \) if:

\[ (\mu - 1) \rho \pi \left\{ \left( \frac{1 - \alpha}{\alpha} \right) - \left( \frac{\pi}{(1 - \tau)} + \frac{\alpha}{(1 - \tau) (1 - \alpha)} \right) \right\} > \frac{\alpha}{1 - \tau} \left( \frac{1 - \alpha}{\alpha} - \frac{\tau}{1 - \tau} \right) \frac{1 - \rho}{1 - \tau} \]

where \( (\frac{1 - \alpha}{\alpha}) - \left( \frac{\pi}{(1 - \tau)} + \frac{\alpha}{(1 - \tau) (1 - \alpha)} \right) > 0 \) if \( \pi < \frac{1 - 2 \alpha}{(1 - \alpha)} \). By comparison, the term on the right-hand-side is negative if \( \pi < \frac{(1 - 2 \alpha)}{(1 - \alpha)} \). In this manner, \( \Phi(N) > \frac{1}{1 - \tau} \) if \( \pi < \frac{(1 - 2 \alpha)}{(1 - \alpha)} \). The opposite holds. Therefore when \( \pi < \frac{(1 - 2 \alpha)}{(1 - \alpha)} \), \( \frac{dU}{dN} < 0 \) for \( N \in (N, \tilde{N}) \), and the optimal degree of banking competition, \( N^* \) is such that: \( N^* \in [1, \tilde{N}] \). This completes the proof of case (i).

Next, \( \Phi(N) < \frac{1}{1 - \tau} \) if \( \pi > \frac{(1 - 2 \alpha)}{(1 - \alpha)} \). Using the definition of \( \tilde{N} \), \( \Phi(\tilde{N}) < \frac{1}{1 - \tau} \) if:

\[ \left( \frac{\pi}{(1 - \tau)} + \frac{\alpha}{(1 - \tau) (1 - \alpha)} \right) \left[ \frac{\pi (1 - \alpha)}{\alpha} + 1 - \frac{\alpha}{1 - \tau} \frac{1}{1 - \rho} \right] \mu < 1 - \frac{1}{\rho} \left[ \frac{\pi (1 - \alpha)}{\alpha} - \frac{\tau}{1 - \tau} \right] + \frac{\pi (1 - \alpha)}{\alpha} \]

where \( \left[ \frac{\pi (1 - \alpha)}{\alpha} + 1 - \frac{\alpha}{1 - \tau} \frac{1}{1 - \rho} \right] > 0 \) if \( \rho < \frac{\pi (1 - \alpha)}{(1 - \alpha) [1 - \pi + (1 - \alpha) \frac{\alpha}{\alpha}]} = \tilde{\rho} \), whereas the term on the right hand side is positive if \( \rho > \frac{\pi (1 - \alpha)}{(1 - \alpha) [1 - \pi + (1 - \alpha) \frac{\alpha}{\alpha}]} \). In this manner, \( \Phi(\tilde{N}) < \frac{1}{1 - \tau} \) if \( \rho > (\tilde{\rho}) \). Therefore, if \( \pi > \frac{(1 - 2 \alpha)}{(1 - \alpha)} \) and \( \rho < \tilde{\rho} \), \( \frac{dU}{dN} \geq (>) 0 \) if \( N \leq (>) \tilde{N} \) with \( \tilde{N} \in (N, \tilde{N}) \). Under these condition, \( \tilde{N} = N^* \) is optimal. Finally, if \( \rho \in (\tilde{\rho}, \hat{\rho}) \), \( \Phi(\tilde{N}) < \frac{1}{1 - \tau} \) and \( \frac{dU}{dN} > 0 \) for \( N \in (N, \tilde{N}) \). Therefore, it is optimal to set \( N^* : N^* \in [\tilde{N}, \infty] \). This completes the proof of Proposition 5.