Chapter 1

MULTICRITERIA DECISION AID/ANALYSIS IN FINANCE

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Abstract Over the past decades the complexity of financial decisions has increased rapidly, thus highlighting the importance of developing and implementing sophisticated and efficient quantitative analysis techniques for supporting and aiding financial decision making. Multicriteria decision aid (MCDA), an advanced branch of operations research, provides financial decision makers and analysts with a wide range of methodologies well-suited for the complexity of modern financial decision making. The aim of this chapter is to provide an in-depth presentation of the contributions of MCDA in finance focusing on the methods used, applications, computation, and directions for future research.

Keywords: Multicriteria decision aid, finance, portfolio theory, multiple criteria optimization, outranking relations, preference disaggregation analysis

1. Introduction

Over the past decades, the globalization of financial markets, the intensification of competition among organizations, and the rapid social and technological changes that have taken place have only led to increasing uncertainty and instability in the business and financial environment. Within this more recent context, both the importance of financial decision making and the complexity of the process by which financial decision making is carried out have increased. This is clearly evident by the variety and volume of new financial products and services that have appeared on the scene.

In this new era of financial reality, researchers and practitioners acknowledge the requirement to address financial decision-making problems through integrated and realistic approaches utilizing sophisticated analytical techniques. In this way, the connections between financial theory, the tools of operations research, and mathematical modelling have become more entwined. Techniques from the fields of optimization, forecasting, decision support systems, MCDA, fuzzy logic, stochastic processes, simulation, etc. are now commonly considered valuable tools for financial decision making.

The use of mathematics and operations research in finance got its start in the 1950s with the introduction of Markowitz's portfolio theory [111] [113]. Since then, in addition to portfolio selection and management, operations research has contributed to financial decision making problems in other areas including venture capital investments, bankruptcy prediction, financial planning, corporate mergers and acquisitions, country risk assessment, etc. These contributions are not limited to academic research; they are now often found in daily practice.

Within the field of operations research, MCDA has evolved over the last three decades into one of its pillar disciplines. The development of MCDA is based upon the common finding that a sole objective, goal, criterion, or point of view is rarely used to make real-world decisions. In response, MCDA is devoted to the development of appropriate methodologies to support and aid decision makers across ranges of situations in which multiple conflicting decision factors (objectives, goals, criteria, etc.) are to be considered simultaneously.

The methodological framework of MCDA is well-suited to the growing complexities encountered in financial decision making. While there have been in finance MCDA stirrings going back twenty to thirty years, the topic of MCDA, as can be seen from the bulk of the references, really hasn't come into its own until recently. As for early stirrings, we have, for example, Bhaskar [**?**] in which microeconomic theory was criticized for largely pursuing a single criterion approache arguing that things like profit maximization are too naive to meet the evolving decision-making demands in many financial areas. Also, in another paper [17], the unavoidable presence of multiple objectives in capital budgeting was noted and the necessity for developing ways to deal with the unique challenges posed by multiple criteria was stressed. It is upon what has taken place since these early roots, and on what are today promising directions in MCDA in finance, that this contribution is focused.

Such observations and findings have motivated researchers to explore the potentials of MCDA in addressing financial decision-making problems. The objective of this chapter is to provide a state-of-the-art comprehensive review of the research made up to date on this issue. Section 2 presents discussions to justify the presence of MCDA in financial decision making. Section 3, focuses on MCDA in resource allocation problems (continuous problems) as in the field of portfolio management. Section 4, presents the contribution of MCDA methodologies in supporting financial decisions that require the evaluation of a discrete set of alternatives (firms, countries, stocks, investment projects, etc.). Finally, Section 5 concludes the chapter and discusses possible future research directions on the implementation of multicriteria analysis in financial institutions and firms.

2. Financial Decision Making

Financial-economic decision problems come in great variety. Individuals are involved in decisions concerning their future pensions, the financing of their homes, and investments in mutual funds. Firms, financial institutions, and advisors are involved in cross-country mergers, complicated swap contracts, and mortgage-backed securities, to name just a few.

Despite the variety, such decisions have much in common. Maybe "money" comes first to mind, but there are typically other factors that suggest that financial-economic problems should most appropriately be treated as multiple criteria decision problems in general: multiple actors, multiple policy constraints, and multiple sources of risk (see e.g., Spronk & Hallerbach [**?**], and Hallerbach & Spronk [66; 68], Martel & Zopounidis [117], Zopounidis [185], and Steuer & Na [164]).

Two other common elements in financial decisions are that their outcomes are distributed over time and uncertainty, and thus involve risk. A further factor is that most decisions are made consciously, with a clear and constant drive to make "good", "better" or even "optimal" decisions. In this drive to improve on financial decisions, we stumble across an area of tension between decision making in practice on the one hand and the potential contributions of finance theory and decision tools on the other. Although the bulk of financial theory is of a descriptive nature, thus focusing on the "average" or "representative" decision maker, we observe a large willingness to apply financial theory in actual decision-making. At the same time, knowledge about decision tools that can be applied in a specific decision situation, is limited. Clearly, there is need of a framework that can provide guidance in applying financial theory, decision tools, and common sense to solving financial problems.

2.1. Issues, Concepts, and Principles

> Finance is a sub field of economics distinguished by both its focus and its methodology. The primary focus of finance is the workings of the capital markets and the supply and the pricing of capital assets. The methodology of finance is the use of close substitutes to price financial contracts and instruments. This methodology is applied to value instruments whose characteristics extend across time and whose payoffs depend upon the resolution of uncertainty. (Ross [137], p. 1)

The field of finance is concerned with decisions with respect to the efficient allocation of scarce capital resources over competing alternatives. The allocation is efficient when the alternative with the highest value is chosen. Current value is viewed as the (present) value of claims on future cash flows. Hence we can say that financial decisions involve the valuation of future, and hence uncertain or "risky," cash flow streams. Cash flow stream X is valued by comparing it with cash flow streams {A,...,Z} that are traded on financial markets. When a traded cash flow stream Y has been identified that is a substitute for X, then their values must be the same. After all, when introducing X to the market, it cannot be distinguished value-wise from Y. Accepting the efficient market hypothesis (stipulating that all available information is fully and immediately incorporated in market prices), the market price of Y equals the value of Y, and hence the value of X. This explains the crucial role of financial markets.

The valuation of future cash flow streams is a key issue in finance. The process of valuation must be preceded by evaluation: without analyzing the characteristics of a cash flow stream, no potential substitute can be identified. Since it is uncertain what the future will bring, the analysis of the risk characteristics will be predominant. Moreover, as time passes, the current value must be protected against influences that may erode its value. This in turn implies the need for risk management. There are basically three areas of financial decisions:

1 **Capital budgeting**: to what portfolio of real investment projects should a firm commit its capital? The central issues here are how

to evaluate investment opportunities, how to distinguish profitable from non-profitable projects and how to choose between competing projects.

2 **Corporate financing**: this encompasses the capital structure policy and dividend policy and addresses questions as: how should the firm finance its activities? What securities should the firm issue or what financial contracts should the firm engage in? What part of the firm's earnings should be paid as cash dividends and what part reinvested in the firm? How should the firm's solvency and liquidity be maintained?

3 **Financial investment**: this is the mirror image of the previous decision area and involves choosing a portfolio of financial securities with the objective to change the consumption pattern over time.

In each of these decision areas the financial key issues of valuation, risk analysis and risk management, and performance evaluation can be recognized, and from the above several financial concepts emerge: financial markets, efficient allocation and market value. In approaching the financial decision areas, some financial principles or maxims are formulated. The first is self-interested behavior: economic subjects are driven by *non-satiation* ("greed"). This ensures the goal of value maximization. Prices are based on financial markets, and under the efficient market hypothesis, prices of securities coincide with their value. Value has time and risk dimensions. With regard to the former, *time preference* is assumed (a dollar today is preferred to a dollar tomorrow). With respect to the latter, *risk aversion* is assumed (a safe dollar is preferred to a risky dollar). Overall risk may be reduced by *diversification*: combining risky assets or cash flow streams may be beneficial. In one way or another, the trade-off between expected return and risk that is imposed by market participants on the evaluation of risky ventures will translate into a *risk-return trade-off* that is offered by investment opportunities in the market.

Since value has time and risk aspects, the question arises about what mechanisms can be invoked to incorporate these dimensions in the valuation process. There are basically two mechanisms. The first is the arbitrage mechanism. Value is derived from the presumption that there do not exist arbitrage opportunities. This no-arbitrage condition excludes sure profits at no cost and implies that perfect substitutes have the same value. This is the *law of one price*, one of the very few laws in financial economics. It is a strong mechanism, requiring very few assumptions on market subjects, only non-satiation. Examples of valuation models built on no-arbitrage are the Arbitrage Pricing Theory for primary financial assets and the Option Pricing Theory for derivative securities. The second is the equilibrium mechanism. In this case value is derived from the market clearing condition that demand equals supply. The latter mechanism is much weaker than the former: the exclusion of arbitrage opportunities is a necessary but by no means a sufficient condition for market equilibrium. In addition to non-satiation also assumptions must be made regarding the risk attitudes of all market participants. Examples of equilibrium-based models are the Capital Asset Pricing Model and its variants. Below we discuss the differences between the two valuation approaches in more detail. It suffices to remark that it is still a big step from the principles to solving actual decision problems.

2.2. Focus of Financial Research

An alternative, albeit almost circular, definition of finance is provided by Jarrow [85], p.1.

> Finance theory (...) includes those models most often associated with financial economics. (...) [*A*] practical definition of financial economics is found in those topics that appear with some regularity in such publications as *Journal of Finance, Journal of Financial and Quantitative Analysis, Journal of Financial Economics*, and *Journal of Banking and Finance.*

Browsing through back volumes of these journals and comparing them to the more recent ones reveals a blatant development in nature and focus. In early days of finance, the papers were descriptive in a narrative way and in the main focused on financial instruments and institutions. Finance as a decision science emerged in the early 1950s, when Markowitz [110; 111] studied the portfolio selection decision and launched what now is known as “modern portfolio theory.” In the 1960s and the early 1970s, many financial economic decision problems were approached by operational research techniques; see for example Ashford, Berry & Dyson [4] and McInnes & Carleton [118] for an overview. However, since then, this type of research has became more and more absorbed by the operations research community and in their journals.

But what direction did finance take? Over the last 25 years mathematical models have replaced the verbal models and finance has founded itself firmly in a neo-classical micro-economic tradition. Over this period we observe a shift to research that is descriptive in a sophisticated econometrical way and that focuses on the statistical characteristics of (mainly well-developed) financial markets where a host of financial instruments is traded. Bollerslev [20], p. 41, aptly describes this shift as follows.

> A cursory look at the traditional econometrics journals (...) severely underestimates the scope of the field [of financial econometrics], as many of the important econometric advances are now also published in the premier finance journals - the *Journal of Finance*, the *Journal of Financial Economics*, and the *Review of Financial Studies* – as well as a host of other empirically oriented finance journals.

The host of reported research addresses the behavior of financial market prices. The study of the pricing of primary securities is interesting for its own right, but it is also relevant for the pricing of derivative securities. Indeed, the description of the pricing of primary assets and the development of tools for pricing derivative assets mark the success story of modern finance.

The body of descriptive finance theory has grown enormously. According to modern definitions of the field of finance, the descriptive nature is even predominant.

> The core of finance theory is the study of the behavior of economic agents in allocating and deploying their resources, both spatially and across time, in an uncertain environment. (Merton [120], p. 7)

Compared to Ross' [137] definition cited earlier, the focus is purely positive. The question arises to what extent the insights gained from descriptive finance – how sophisticated they may be from a mathematical, statistical or econometric point of view – can serve as guidelines for financial decisions in practice. Almost thirty years ago, in the preface of their book *The Theory of Finance*, Eugene Fama and Merton Miller defended their omission of detailed examples, purporting to show how to apply the theory to real-world decision problems, as

> (...) a reflection of our belief that the potential contribution of the theory of finance to the decision-making process, although substantial, is still essentially indirect. The theory can often help expose the inconsistencies in existing procedures; it can help keep the really critical questions from getting lost in the inevitable maze of technical detail; and it can help prevent the too easy, unthinking acceptance of either the old clichs or new fads. But the theory of finance has not yet been brought, and perhaps never will be, to the cookbook stage. (Fama & Miller [54], p. viii)

Careful inspection of current finance texts reveals that in this respect not much has changed. However, pure finance theory and foolproof financial recipes are two extremes of a continuum. The latter cookbook stage will never be achieved, of course, and in all realism and wisdom this alchemic goal should not be sought for. But what we dearly miss is an extensive body of research that bridges the apparent gap between the extremes: research that shows how to solve real-world financial decision problems without violating insights offered by pure finance theory on the one hand and without neglecting the peculiarities of the specific decision problem on the other.

On another matter, the role of assumptions in modelling is to simplify the real world in order to make it tractable. In this respect the art of modelling is to make assumptions where they most contribute to the model's tractability and at the same time detract from the realism of the model as little as possible. The considerations in this trade-off are fundamentally different for positive (descriptive) models on the one hand and conditional-normative models on the other. In the next section we elaborate further on the distinctions between the two types of modelling as concerns the role of assumptions.

2.3. Descriptive vs. Conditional-Normative Modelling

In a positive or descriptive model simplified assumptions are made in order to obtain a testable implication of the model. The validity of the model is evaluated according to the inability to reject the model's implications at some level of significance. So validity is of an empirical nature, solely judged by the implications of the model. Consider the example of an equilibrium asset-pricing model. As a starting point, assumptions are made with respect to the preferences of an imaginary investor and the risk-return characteristics of the investment opportunities. These assumptions are sufficiently strong to allow solving the portfolio optimization problem. Next a homogeneity condition is imposed: all investors in the market possess the same information and share the same expectations. This allows focusing on "a representative investor". Finally the equilibrium market clearing condition is imposed: all available assets (supply) must be incorporated in the portfolio of the representative investor (demand). The first order conditions of portfolio optimality then stipulate the trade-off between risk and expected return that is required by the investor. Because of the market clearing, the assets offer the same trade-off. Hence a market-wide relationship between risk and return is established and this relationship is the object of empirical testing. As long as the pricing relationship is not falsified the model is accepted, irrespective of whether the necessary assumptions are realistic or not. When the model is falsified, deduction may help to amend the assumptions where after the same procedure is followed. This hypothetic-deductive cycle ends when the model is no longer falsified by the empirical data at hand.

In a conditional-normative model, simplifying assumptions are also made in order to obtain a tractable model. These assumptions relate to the preferences of the decision maker and to the representation of the set of choice alternatives. The object of the conditional-normative modelling is not to infer a testable implication but to obtain a decision rule. This derived decision rule is valid and can normatively be applied conditional on the fact that the decision maker satisfies the underlying assumptions; cf. Keynes [92].

In order to support decisions in finance, obviously both the preferences of the decision maker and the characteristics of the choice alternatives should be understood and related to each other. Unfortunately, the host of financial-economic modelling is of a positive nature and focuses on the "average" decision maker instead of addressing the particular (typically non-average) decision maker. The assumptions underlying financial theory at best describe "average individuals" and "average decision situations" and hence are not suited to describe specific individual decision problems. The assumptions made to simplify the decision situation often completely redefine the particular problem at hand. The real world is replaced by an over-simplified model-world. As a consequence, not the initial problem is solved but a synthesized and redefined problem that is not even recognized by the decision maker himself. The over-simplified model becomes a Procrustes bed for the financial decision maker who seeks advice.

For example, it is assumed that a decision maker has complete information and that this information can be molded into easily manipulated probability distributions. Even worse, positive knowledge and descriptive theories that by definition reflect the outcomes of decisions made by some representative decision maker are used to prescribe what actions to take in a specific decision situation. For example, equilibrium asset pricing theories predict the effects of decisions and actions of many individuals on the formation of prices in financial markets. Under the homogeneity condition the collection of investors is reduced to the representative investor. When the pricing implications of the model are simply used to guide actual investment behavior, then the decision maker is forced into the straitjacket of this representative investor.

Unfortunately we observe that conditional-normative financial modelling is only regarded as a starting point for descriptive modelling and is not pursued for its own sake. After almost twenty years, Hastie's [71] lament has not lost its poignancy.

> In American business today, particularly in the field of finance, what is needed are approximate answers to the precise problem rather than precise answers to the approximate problem.

Apart from the positive modelling of financial markets as described above, there is one other field in finance in which the achievements of applied modelling are apparent: option pricing theory, the set of models that enable the pricing of derivative securities and all kinds of contingent claims. Indeed, the option pricing formulas developed by Black & Scholes [18] and Merton [119] mark a huge success in the history of financial modelling. Contingent claims analysis made a flying start, and

> .... when judged by its ability to explain the empirical data, option pricing theory is the most successful theory not only in finance, but in all of economics. (Ross [137], p. 24)
>
> Given a theory that works so well, the best empirical work will be to use it as a tool rather than to test it. (Ross [137], p. 23)

Indeed, modern-day derivatives trading would be unthinkable without the decision support of an impressive coherent toolbox for analyzing the risk characteristics of derivatives and for pricing them in a consistent way. Compared to this framework, the models and theories developed and tested for primary assets look pale. What is the reason for the success of derivatives research?

For an explanation we turn to the principal tool used in option pricing theory: no-arbitrage valuation. By definition derivative securities derive their value from primary underlying assets. Under some mild assumptions, a dynamic trading strategy can be designed in which the derivative security is exactly replicated with a portfolio of the primary security and risk-free bonds. Under the no-arbitrage condition, the current value of the derivative security and the replicating portfolio should be identical. Looking from another perspective, a suitably chosen hedge combination of the derivative and the underlying security produces a risk-free position. On this position the risk-free rate must be earned, otherwise there exist arbitrage opportunities. Since the position is risk free, risk attitudes and risk aversion do not enter the story. Therefore a derivative security will have the same value in a market environment with risk neutral investors as in a market with risk averse investors. This in turn implies that a derivative can be priced under the assumption that investors are risk neutral. As a consequence, no assumptions are required on preferences (other than non-satiation), utility functions, the degree of risk aversion, and risk premia. Thus, option pricing theory can escape from the burden of modelling of preference structures. Instead, research attention shifts to analyzing price dynamics on financial markets. An additional reason for the success in derivatives research is that the analytical and mathematical techniques are similar to those used in the physical sciences (see for example Derman [35]).

Of course, even in derivatives modelling some assumptions are required. This introduces model risk. When the functional relationships stipulated in the model are wrong, or when relevant input parameters of the model are incorrectly estimated, the model produces the wrong value and the wrong risk profile of the derivative. To an increasing degree, financial institutions are aware that great losses can be incurred because of model risk. Especially in risk management and derivatives trading model risk is a hot item (see Derman [34]). This spurred Merton to ventilate this warning.

> At times, the mathematics of the models become too interesting and we lose sight of the models' ultimate purpose. The mathematics of the models is precise, but the models are not, being only approximations to the complex, real world. Their accuracy as a useful approximation to that world varies considerably across time and place. The practitioner should therefore apply the models only tentatively, assessing their limitations carefully in each application. (Merton [120], p. 14)

Ironically this quote was taken just after the very successful launch of Long Term Capital Management (LTCM), the hedge fund of which Merton and Myron Scholes were the founding partners. In 1998, LTCM collapsed and model risk played a very important role in this debacle.

Summarizing we draw the conclusion that successful applied financial modelling does exist, and blossoms in the field of derivatives. Here also the validity of the assumptions is crucial, this in contrast to positive modelling. However, in the field of derivatives with replicating strategies and arbitrage-based valuation, the concept of "absence of risk" is well defined and no preference assumptions are needed in the modelling process. For modelling decisions regarding the underlying primary assets, in contrast, assumptions on the decision maker's preferences and on the "risk" attached to the outcomes of the choice alternatives are indispensable. For these types of financial problems, the host of simplifying assumptions that are made in the descriptive modelling framework invalidate the use of the model in a specific decision situation. Thus we face the following challenge: how can we retain the conceptual foundation of the financial-economic framework and still provide sound advice that can be applied in multifarious practice? As a first step we will sketch the relationship between decision sciences and financial decision-making.

2.4. Decision Support for Financial Decisions

Over the last fifty years or so, the financial discipline has shown continuously rapid and profound changes, both in theory and in practice.

Many disciplines have been affected by globalization, deregulation, privatization, computerization, and communication technologies. Hardly any field has been influenced as much as finance. After the mainly institutional and even somewhat *ad hoc* approaches before the fifties, Markowitz [110; 111] has opened new avenues by formalizing and quantifying the concept of "risk". In the decades that followed, a lot of attention was paid to the functioning of financial markets and the pricing of financial assets including options. The year 1973 gave birth to the first official market in options (CBOE) and to crucial option pricing formulas that have become famous quite fast (Black-Scholes and Cox-Ross-Rubinstein, see Hull [75]), both in theory and practice. At that time, financial decision problems were structured by (a) listing a number of mutually exclusive decision alternatives, (b) describing them by their (estimated) future cash flows, including an estimation of their stochastic variation and later on including the effect of optional decisions, and (c) valuing them by using the market models describing financial markets.

In the seventies, eighties and nineties, the financial world saw enormous growth in derivative products, both in terms of variety and in terms of market volumes. Financial institutions have learned to work with complex financial products. Academia has contributed by developing many pricing models, notably for derivatives. Also, one can say that financial theory has been rewritten in the light of contingent claims ("optional decisions") and will soon be further reshaped by giving more attention to game elements in financial decisions. The rapid development of the use of complex financial products has certainly not been without accidents. This has led regulators to demand more precise evaluations and the reporting of financial positions (cf. e.g., the emergence of the Value-at-Risk concept, see Jorion [90]).

In addition to the analysis of financial risk, the structured management of financial risk has come to the forefront. In their textbook, Bodie & Merton [19] describe the threefold tasks of the financial discipline as Valuation, Risk Management, and Optimization. We would like to amend the threefold tasks of financial management to Valuation, Risk Management, and Decision Making. The reason is that financial decision problems often have to be solved in dynamic environments where information is not always complete, different stakeholders with possibly conflicting goals and constraints play a role and clear-cut optimization problems cannot always be obtained (and solved).

At the same time, many efforts from the decision-making disciplines are misdirected. For instance, some approaches fail to give room for the inherent complexity of the decision procedure given the decision maker's specific context. Other approaches concentrate on the beauties of a particular decision method without doing full justice to the peculiarities of the decision context. Aside from being partial in this respect, useful principles and insights offered by financial-economic theory are often not integrated in the decision modelling. It is therefore no surprise that one can observe in practice unstructured *ad hoc* approaches as well as complex approaches that severely restrict the decision process.

2.5. Relevance of MCDA for Financial Decisions

The central issue in financial economics is the efficient allocation of scarce capital and resources over alternative uses. The allocation (and redistribution) of capital takes place on financial markets and is termed "efficient" when market value is maximized. Just as water will flow to the lowest point, capital will flow to uses that offer the highest return. Therefore it seems that the criterion for guiding financial decisions is one-dimensional: maximize market value or maximize future return.

From a financial-economic perspective, the goal of the firm, for example, is very much single objective. Management should maximize the firm's contribution to the financial wealth of its shareholders. Also the shareholders are considered to be myopic. Their only objective is to maximize their single-dimensional financial wealth. The link between the shareholders and the firm is footed in law. Shareholders are the owners of the firm. They possess the property rights of the firm and are thus entitled to decide what the firm should aim for, which according to homogeneity is supposed to be the same for all shareholders, i.e., maximize the firm's contribution to the financial wealth of the shareholders. The firm can accomplish this by engaging in investment projects with positive net present value. This is the neo-classical view on the role of the firm and on the relationship between the firm and its shareholders in a capitalist society. Figure 1.1 depicts a simplified graphical representation of this line of thought.

It is important to note that this position is embedded in a much larger framework of stylized thinking in among others economics (general equilibrium framework) and law (property rights theory and limited liability of shareholders). Until today, this view is seen as an ideal by many; see for example Jensen [86]. Presently, however, the societal impact of the firm and its governance structure is a growing topic of debate. Here we will show that also in finance there are many roads leading to Rome, or rather to the designation MCDA. Whether one belongs to the camp of Jensen or to the camp of those advocating socially responsible entrepreneurship, one has to deal with multiple criteria.

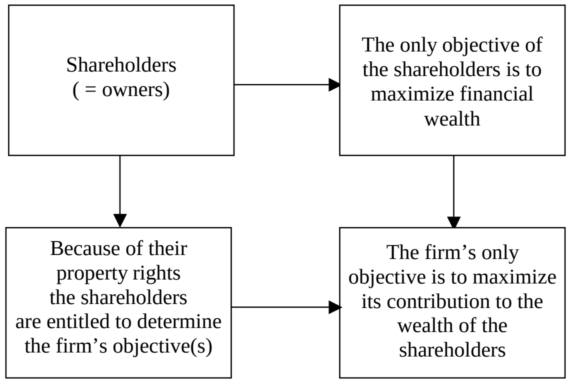

Figure 1.1. The neo-classical view on the objective of the firm

There is a series of situations in which the firm chooses (or has to take account of) a multiplicity of objectives and (policy) constraints. An overview of these situations is depicted in Figure 2. One issue is who decides on the objective(s) of the firm. If there is a multiplicity of parties who may decide what the firm is aiming for, one generally encounters a multitude of goals, constraints and considerations that - more often than not - will be at least partially conflictive. A clear example is the conflicting objectives arising from agency problems (Jensen & Meckling, [87]). This means that many decision problems include multiple criteria and multiple actors (viz. group decision making, negotiation theory, see Box 3 in Figure 1.2). Sometimes, all those who decide on what the firm should aim for agree upon exactly the same objective(s). In fact, this is what neo-classical financial theory assumes when adopting shareholder value maximization (Box 1 in Figure 1.2). In practice, there are many firms that explicitly strive for a multiplicity of goals, which naturally leads to decision problems with multiple criteria (Box 2 in Figure 1.2).

However, although these firms do explicitly state to take account of multiple objectives, there are still very few of these firms that make use of tools provided by the MCDA literature. In most cases firms maximize one objective subject to (policy) constraints on the other objectives. As such there is nothing wrong with such a procedure as long as the location of these policy constraints is chosen correctly. In practice, however, one often observes that there is no discussion at all about the location of the policy constraints. Moreover, there is often no idea about the trade-offs between the location of the various constraints and the objective function that is maximized. In our opinion, multiple criteria decision methodologies may help decision makers to gain better insights in the

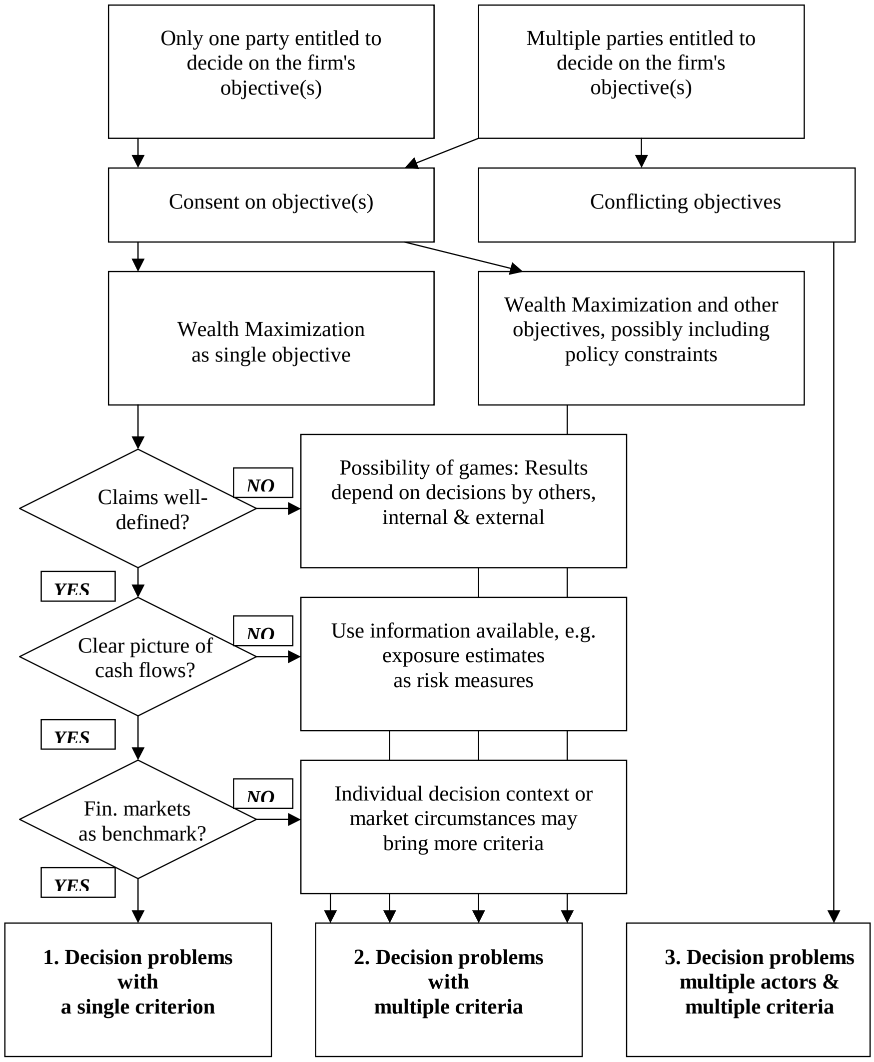

Figure 1.2. Situations leading to MCDA in the firm

trade-offs they are confronted with.

Now let us get back to the case in which the owner(s) / shareholders do have only one objective in mind: wealth maximization. Although this is by definition the most prominent candidate for single criteria decision-making, we will argue that even in this case there are many circumstances in which the formulation as a multiple criteria decision problem is opportune.

In order to contribute maximally to the wealth of its shareholders, an individual firm should maximize the value of its shares. The value of these shares is determined on the financial markets by the forces of demand and supply. Shares represent claims on the future residual cash flows of the firm (and also on a usually very limited right on corporate control). In the view of the financial markets, the value of such a claim is determined relative to the claims of other firms that are traded on these markets. The financial markets' perception of the quality of these cash flow claims is crucial for the valuation of the shares. Translated to the management of the individual firm, the aim is not only to maximize the quality of the future residual cash flows of the firm but also to properly communicate all news about these cash flows to the financial markets. Only by the disclosure of such information can informational asymmetries be resolved and the fair market value of a cash flow claim be determined. In evaluating the possible consequences of its decision alternatives, management should estimate the effects on the uncertain (future) cash flows followed by an estimation of the financial markets' valuation of these effects. Then (and only then) the decision rule of management is very simple: choose the decision alternative that generates the highest estimated market value.

The first problem that might arise while following the above prescription is that residual claims cannot always be defined because of "gaming effects" (see Figure 1.2, Box 2). In other words, the future cash flows of the firm do not only depend on the present and future decisions of the firm's management, but also on the present and future decisions of other parties. An obvious example is the situation of oligopolistic markets in which the decisions of the competitors may strongly influence each other. Similar situations may arise with other external stakeholders such as powerful clients, powerful suppliers, and powerful financiers. Games may also arise within the firm, for instance between management and certain key production factors. The problem with game situations is that their effect on a firm's future cash flows caused by other parties involved cannot be treated in the form of simple constraints or as cost factors in cash flow calculations. MCDA may help to solve this problem by formulating multi-dimensional profiles of the consequences of the firm's decision alternatives. In these profiles, the effects on parties other than the firm are also included. These multi-dimensional profiles are the keys to open the complete MCDA toolbox.

A second problem in dealing with the single-objective wealth maximization problems is that the quality of information concerning the firm's future cash flows under different decision alternatives is far from complete. In addition, the available information may be biased or flawed. One way to approach the incomplete information problem is suggested by Spronk & Hallerbach [**?**]. In their multi-factorial approach, different sources of uncertainty should be identified after which the exposures of the cash flows to these risk sources are estimated. The estimated exposures can next be included in a multi-criteria decision method. In the case that the available information is not conclusive, different "views" on the future cash flows may develop. Next each of these views can be adopted as representing a different dimension of the decision problem. The resulting multi-dimensional decision problem can then be handled by using MCDA (see Figure 1.2, Box 2).

A third potential problem in wealth maximization is that the financial markets do not always provide relevant pricing signals to evaluate the wealth effects of the firm's decisions, for example, because of market inefficiencies. This means that the firm may want to include attributes in addition to the market's signals in order to measure the riskiness and wealth effects of its decisions.

2.6. A Multicriteria Framework for Financial Decision

In our view it, is the role of financial modelling to support financial decision making, as described in Hallerbach and Spronk [69], to build pointed models that take into account the peculiarities of the precise problem. The goal here is to bridge the gaps between decision-making disciplines, the discipline of financial economics, and the need for adequate decision support.

2.6.1 Principles. This framework is built on the principle that assumptions should be made where they help the modelling process the most and hurt the particular decision problem the least.[1] We call this the *Principle of Low Fat Modelling.* When addressing a decision situation, make use of all available information, but do not make unrealistic assumptions with respect to the availability of information. Do not make unrealistic assumptions that disqualify the decision context at hand. There should be ample room to incorporate idiosyncrasies of the decision context within the problem formulation, thus recognizing that the actual (non-average) decision maker is often very different from the "representative" decision maker. The preferences of the decision maker may not be explicitly available and may not even be known in detail by the decision maker himself. The uncertainty a decision maker faces with respect to the potential outcomes of his decisions may not be readily represented by means of a tractable statistical distribution. In many real-life cases, uncertainty can only be described in imprecise terms and available information is far from complete. And when the preferences of the decision maker are confronted with the characteristics of the decision alternatives, the conditional-normative nature of derived decision rules and advice should be accepted.

A second principle underlying our framework is the *Principle of Eclecticism.* One should borrow all the concepts and insights from modern financial theory that help to make better financial decisions. Financial theory can provide rich descriptions of uncertainty and risk. Examples are the multi-factor representation of risk in which the risk attached to the choice alternatives is conditioned on underlying factors such as the contingent claims approach in which the decision outcomes are conditioned on the opportunity to adjust or revise decisions in the future or game theory in which the outcomes are also conditioned on potential (conflicting) decisions made by other parties. But it is not the availability of theoretical insights that determines their application; it depends on the specific decision context at hand.

By restricting one thinking to a prechoosen set of problem characteristics, there is obviously more "to be seen" but at the same time it is possible to make observation errors, and maybe more worrisome, the problem and its context may be changing over time. This calls for the *Principle of Permanent Learning*, which stresses the process nature of decision making in which both the representation of the problem and the problem itself can change over time. Therefore, there is a permanent need to critically evaluate the problem formulation, the decisions made and their performance. Obviously, decision making and performance evaluation are two key elements in the decision-making process. As argued in Spronk and Vermeulen [159], performance evaluation of decisions should be structured such that the original idiosyncrasies of the problem (i.e., at the time the decision is made) are fully taken into account at the moment of evaluation, (i.e., *ex post*). By doing so, one increases the chance of learning from errors and misspecifications in the past.

2.6.2 Allocation decisions. Financial decisions are allocation decisions, in which both time and uncertainty (and thus risk) play a crucial role. In order to support decisions in finance, both the preferences of the decision maker and the characteristics of the choice alternatives should be adequately understood and related to each other. A distinction can be made between "pure" financial decisions in which cash flows and market values steer the decision and "mixed" financial decisions in which other criteria are also considered. In financial theory, financial decisions are considered to be pure. In practice, most decisions are mixed. Hallerbach & Spronk [68] show that many financial decisions are mixed and thus should be treated as multiple criteria decision problems.

The solution of pure financial decisions requires the analysis, valuation, and management of risky cash flow streams and risky assets. The solution of mixed financial problems involves, in addition, the analysis of other effects. This implies that, in order to describe the effects of mixed decisions, multi-dimensional impact profiles should be used (cf. Spronk & Hallerbach [**?**]). The use of multi-dimensional impact profiles naturally opens the door to MCDA. Another distinction that can be made is between the financial decisions of individuals on one hand the financial decisions of companies and institutions on the other. The reason for the distinction results from the different ways in which decision makers steer the solutions. Individual decisions are guided by individual preferences (e.g., as described by utility functions), whereas the decisions of corporations and institutions are often guided by some aggregate objective (e.g., maximization of market value).

2.6.3 Uncertainty and risk[2]. In each of the types of financial decisions just described, the effects are distributed over future time periods and are uncertain. In order to evaluate these possible effects, available information should be used to develop a "picture" of these effects and their likelihood. In some settings there is complete information but more often information is incomplete. In our framework, we use multi-dimensional risk profiles for modelling uncertainty and risk. This is another reason why multicriteria decision analysis is opportune when solving financial decision problems. *Two questions* play a crucial role:

1. Where does the uncertainty stem from or, in other words, what are the sources of risk?

2. When and how can this uncertainty be changed?

The answer to the first question leads to the decomposition of uncertainty. This involves attributing the inherent risk (potential variability in the outcomes) to the variability in several underlying state variables or factors. We can thus view the outcomes as being *generated* by the factors. Conversely, the stochastic outcomes are *conditioned* on these factors. The degree in which fluctuations in the factors propagate into fluctuations in the outcomes can be measured by response coefficients. These sensitivity coefficients can then be interpreted as exposures to the underlying risk factors and together they constitute the multi-dimensional risk profile of a decision alternative.

The answer to the second question leads to three prototypes of decision problems:

(1) The decision maker makes and implements a final decision and waits for its outcome. This outcome will depend on the evolution of external factors, beyond the decision maker's control.

(2) The decision maker makes and implements a decision and observes the evolution of external factors (which are still beyond the decision maker's control). However, depending on the value of these factors, the decision maker may make and implement additional decisions. For example, a decision maker may decide to produce some amount of a new and spectacular software package and then, depending on market reaction, he may decide to stop, decrease, or increase production.

(3) As in (2), but the decision maker is not the sole player and thus has to take account of the potential impact of decisions made by others sometime in the future (where the other(s) are of course confronted with a similar type of decision problem). The interaction between the various players in the field gives rise to dynamic game situations.

2.6.4 A bird's-eye view of the framework. In Figure 1.3, a bird's-eye view of the framework is presented. The framework integrates several elements in a process-oriented approach towards financial decisions. The left side of Figure 1.3 represents the elements that lead to decisions, represented by the Resolution/Conclusion box at the lower left hand side. As mentioned above, performance evaluation (shown at the lower right hand side of the figure) is an integral part of the decision-making process. However, in this article we do not pay further attention to performance evaluation or to the feedback leading from performance evaluation to other elements of the decision-making process.

Financial decision problems will often be put as allocation problems. At this stage, it is important to determine whether the problem is a mixed or pure financial problem. Also, one should know who decides and which objectives are to be served by the decisions.

In the next step, the problem is defined more precisely. Many factors play a role here. For instance, the degree of upfront structure in the problem definition, the similarity with other problems, time and commitment from the decision makers, availability of time, similarity to

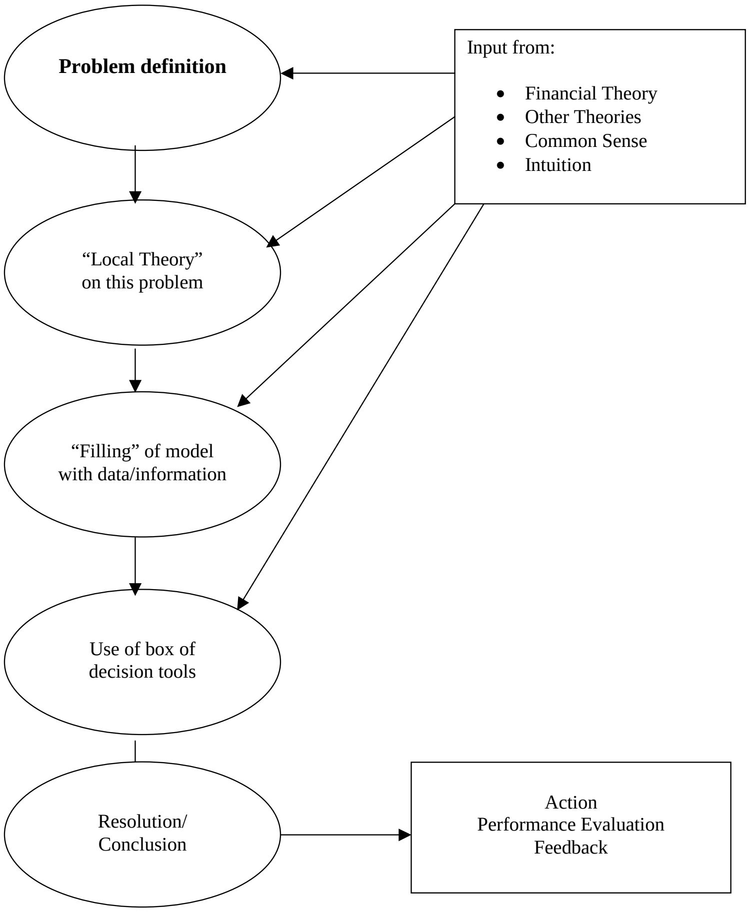

Figure 1.3. A bird’s-eye view of the framework.

problems known in theory and so on. In this stage, the insights from financial theory often have to be supplemented (or even amended) by insights from other disciplines and by the discipline of common sense. The problem formulation can thus be seen as a theoretical description (we use the label “local theory”) of the problem.

After the problem formulation, data have to be collected, evaluated and sometimes transformed into estimates. These data are then used as inputs for the formalization of the problem description. The structure of the problem, together with the quality and availability of the data determines what tools can be used and in which way. As explained above, the use of multi-dimensional impact profiles almost naturally leads to the use of multicriteria decision analysis.

2.6.5 The framework and modern financial theory. In our framework we try to borrow all concepts and insights from modern financial theory that help to make better financial decisions. Financial theory provides rich and powerful tools for describing uncertainty and risk. Examples are the multi-factor representation of risk, which leads to multi-dimensional impact profiles that can be integrated within multicriteria decision analysis. A very important contribution of financial theory is the contingent claims approach in which the decision outcomes are conditioned on the opportunity to adjust or revise decisions in the future. This comes together with financial markets where contingent claims are being traded in volume. This brings us to the role of financial markets as instruments to trade risks, to redistribute risks, and even to decrease or eliminate risk. We believe and hope that contingent claims thinking will also be used in other domains than finance. In the first place because of what it adds when describing decision problems. Secondly, new markets may emerge in which also non-financial risks can be handled in a better way.

In addition to helping to better describe decision problems, financial theory provides a number of crucial insights. The most obvious (which is clearly not limited to financial economics) is probably the concept of "best alternative opportunity" thinking. Whenever making an evaluation of decision alternatives, one should take into account that the decision maker may have alternative opportunities (often but not exclusively provided by markets), the best of which sets a benchmark for the evaluation of the decision alternatives considered.

Other concepts are the efficient market hypothesis and the no-arbitrage condition. These point both to the fact that in competitive environments, it is not obvious that one can outsmart all the others. So if you find ways to make easy money, you should at least try to answer the question why you have been so lucky and how the environment will react.

3. MCDA in Portfolio Decision-Making Theory

We now turn our attention to the area of finance known as portfolio theory. In portfolio theory, we study the attributes of collections of securities called portfolios and how investors process these attributes in order to determine the securities that are ultimately selected to form a portfolio.

At the core of portfolio theory is the portfolio selection problem. Formulated as an optimization problem, this is a problem that has been studied extensively. However, the problem that has been the subject of so much study for over fifty years is only two-dimensional, able to address only the two criteria of risk (as measured by standard deviation) and return. To more realistically model the problem and be better prepared for the future which will only be more complicated, we now discuss the issues involved in generalizing portfolio selection to include criteria beyond standard deviation and return. In this way, MCDA in the form of multiple criteria optimization enters the picture. The word "multiple" of course means two or more, but in this paper we will generally use it to mean more than two. We now explore the possibilities of multiple objectives in portfolio selection and discuss the effects of recognizing multiple criteria on the traditional assumption and practice of portfolio selection in finance.

In this portion of the paper, we are organized as follows. In Subsection 3.1 we describe the risk-return portfolio selection problem in finance. In Subsection 3.2 we show how the problem, although with only two objectives, can be re-cast in a multiple criteria optimization framework. In Subsection 3.3 we discuss two popular variants of the portfolio selection model, the short-sales permitted and short-sales prohibited models, and in Subsection 3.4 we discuss the bullet-shaped feasible regions that so often accompany portfolio optimization problems. In Subsection 3.5 we review some of key assumptions of portfolio analysis and discuss the sensitivity of the nondominated set to changes in various factors. With the sensitivity of the nondominated set indicating the presence of additional criteria beyond risk and return, an expanded multiple criteria portfolio optimization formulation is proposed in Subsection 3.6. With the nondominated frontier transformed into a nondominated surface, Subsection 3.7 reports on the idea that the "modern portfolio analysis" of today is probably best seen in the large as the projection onto the risk-return plane of the real multiple criteria portfolio selection problem in higher dimensional space. In Subsection 3.8 we comment on some future research directions in MCDA in portfolio analysis.

3.1. Portfolio Selection Problem

In finance, due to Markowitz [111; 112; 113], we have the canonical portfolio selection problem as follows. Assume

(a) n securities
(b) an initial sum of money to be invested
(c) the beginning of a holding period

(d) end of a holding period.

Let $x_1, \ldots, x_n$ denote *investment proportion* weights. These are the proportions of the initial sum to be invested at the beginning of the holding period in the n securities to form a portfolio. Also, for each $i \in \{1, \ldots, n\}$, let r_i be the random variable for the percent return realized on security i between the beginning of the holding period and the end of the holding period. Then r_p, the random variable for the percent return realized on the portfolio between the beginning of the holding period and the end of the holding period, is given by

$$r_p = \sum_{i=1}^{n} r_i x_i$$

Unfortunately, r_p is not deterministic because it is based on upon the r_i. Thus it is not possible to know at the beginning of the holding period the value to be achieved by r_p at the end of the holding period. However, it is assumed that at the beginning of the holding period we have in our possession all expected values $E\{r_i\}$, predicted variances σ_{ii}, and predicted covariances σ_{ij} for the n securities.

Since r_p is not deterministic and an investor would presumably wish to protect against low values of r_p from in fact turning out to be the case, the approach considered prudent in portfolio selection is to seek a solution (that is, of investment proportion weights) that produces both a high expected value of r_p and a low predicted standard deviation value of r_p. Using the $E\{r_i\}$, σ_{ii} and σ_{ij}, the expected value of r_p is given by

$$E\{r_p\} = \sum_{i=1}^{n} E\{r_i\} x_i \tag{1.1}$$

and the predicted standard deviation of r_p is given by

$$\sigma\{r_p\} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j} \tag{1.2}$$

As for constraints, there is always the "sum-to-one" constraint

$$\sum_{i=1}^{n} x_i = 1 \tag{1.3}$$

Depending on the model being built, there may be constraints in addition to the above and restrictions on the variables such as

$$\ell_i \geq x_i \geq \mu_i \text{ for all } i \tag{1.4}$$

The way (1.1)-(1.4) is solved is as follows. First compute the set of all of the model's "nondominated" combinations of expected return and standard deviation. Then, after examining the set, which is portrayed graphically in the form of a curved line, the investor selects the nondominated combination that he or she feels strikes the best balance between expected return $E\{r_p\}$ and predicted standard deviation $\sigma\{r_p\}$.

With $E\{r_p\}$ to be maximized and $\sigma\{r_p\}$ to be minimized, (1.1)-(1.4) is a multiple objective program. Although the power of multiple criteria optimization is not necessary with two-objective programs (because they can be addressed with single criterion techniques), the theory of multiple criteria optimization is necessary when wishing to generalize portfolio selection, as we do, to take into account additional criteria.

3.2. Multiple Criteria Optimization

In operations research, there is the multiple criteria optimization problem. In its formulation, apart from having more than one objective, it looks like any other mathematical programming problem, but its solution is more involved. To handle both maximization and minimization objectives, we have

$$\begin{aligned} \max \textit{ or } \min \{f_1(\mathbf{x}) &= z_1\} \\ &\vdots \\ \max \textit{ or } \min \{f_k(\mathbf{x}) &= z_k\} \\ \text{s.t.} \qquad \mathbf{x} &\in S \end{aligned} \tag{MC}$$

where $\mathbf{x} \in R^n$, k is the number of objectives, and the z_i are *criterion values*. In multiple criteria optimization there are two feasible regions. One is $S \subset R^n$ in *decision space* and the other is $Z \subset R^k$ in *criterion space*. Let $\mathbf{z} \in R^k$. Then *criterion vector* $\mathbf{z} \in Z$ if and only if there exists an $\mathbf{x} \in S$ such that $\mathbf{z} = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x}))$. In this way, Z is the set of all *images* of the $\mathbf{x} \in S$

Criterion vectors in Z are either *nondominated* or *dominated*, and points in S are either *efficeint* or *inefficeint*. Letting $J^+ = \{\, i \mid f_i(\mathbf{x})$ is to be maximized$\}$ and $J^- = \{j \mid f_j(\mathbf{x})$ is to be minimized$\}$, we have

Definition 1 *Assume formulation* (MC). *Then* $\bar{\mathbf{z}} \in Z$ *is a* nondominated *criterion vector if and only if there does not exist another* $\mathbf{z} \in Z$ *such that* **(i)** $z_i \geq \bar{z}_i$ *for all* $i \in J^+$, *and* $z_j \leq \bar{z}_j$ *for all* $j \in J^-$, *and* **(ii)** $z_i > \bar{z}_i$ *or* $z_j < \bar{z}_j$ *for at least one* $i \in J^+$ *or* $j \in J^-$. *Otherwise,* $\bar{\mathbf{z}} \in Z$ *is* dominated.

The set of all nondominated criterion vectors is designated N and is called the *nondominated set*.

Definition 2 *Let $\bar{\mathbf{x}} \in S$. Then $\bar{\mathbf{x}}$ is* efficient *in* (MC) *if and only if its* image *criterion vector $\bar{\mathbf{z}} = (f_1(\bar{\mathbf{x}}), \ldots, f_k(\bar{\mathbf{x}}))$ is nondominated, that is, if and only if $\bar{\mathbf{z}} \in N$. Otherwise, $\bar{\mathbf{x}}$ is* inefficient.

The set of all efficient points is designated E and is called the *efficient set*. Note the distinction that is be made with regard to terminology. While nondominance is, of course, a criterion space concept, in multiple criteria optimization, efficiency is only a decision space concept.

To define optimality in a multiple criteria optimization problem, let $U\colon R^k \to R$ be the decision maker's utility function. Then, any $\mathbf{z}^{\text{o}} \in Z$ that maximizes U over Z is an *optimal criterion vector*, and any $\mathbf{x}^{\text{o}} \in S$ such that $(f_1(\mathbf{x}^{\text{o}}), \ldots, f_k(\mathbf{x}^{\text{o}})) = \mathbf{z}^{\text{o}}$ is an *optimal solution.* We are interested in the efficient and nondominated sets because if U is such that *more-is-always-better-than-less* for each z_i, $i \in J^+$, and *less-is-always-better-than-more* for each z_j, $j \in J^-$, then any $\mathbf{z}^{\text{o}}$ optimal criterion vector is such that $\mathbf{z}^{\text{o}} \in N$, and any feasible *inverse image* $\mathbf{x}^{\text{o}}$ is such that $\mathbf{x}^{\text{o}} \in E$. The significance of this is that to find an optimal criterion vector $\mathbf{z}^{\text{o}}$, it is only necessary to find a best point in N. Since N is normally a portion of the surface of Z, this is much better than having to search all of Z. After a $\mathbf{z}^{\text{o}}$ has been found, it is only necessary to obtain an $\mathbf{x}^{\text{o}} \in S$ inverse image to know what to implement to achieve the k simultaneous performances indicated by the values in $\mathbf{z}^{\text{o}}$.

Unfortunately, although N is a portion of the surface of feasible Z in criterion space, locating the best solution in N, when $k > 2$, is often a non-trivial task because of the size of N. As a result, a large part of the field of multiple criteria optimization is concerned with procedures, mostly interactive, for searching N for an optimal or *near-optimal* solution, where a near-optimal solution is close enough to being optimal to terminate the decision process.

Thus, in this framework, the portfolio selection problem of (1.1)-(1.4) now appears as the two-objective multiple criteria optimization problem

$$
\begin{aligned}
\min \{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \sigma_{ij} x_j = z_1\} \qquad\qquad \text{(MC-Orig)} \\
\max \{\sum_{i=1}^{n} E\{r_i\} x_i = z_2\} \\
\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1 \\
\ell_i \geq x_i \geq \mu_i \text{ for all } i
\end{aligned}
$$

In (MC-Orig), z_1 is predicted variance and z_2 is expected return. But why variance instead of standard deviation? Whereas standard deviation is more intuitive, mathematical programming formulations as in (MC-Orig) most often have the risk objective expressed in terms of variance since quadratic routines are typically employed in a workhorse software capacity when analyzing the problem. Square roots can always be taken manually later.

3.3. Two Model Variants

Two model variants of (1.1)-(1.4) have evolved as classics. One is the *unrestricted* model

$$\min \{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j = z_1\} \qquad \text{(MC-Unrestr)}$$
$$\max \{\sum_{i=1}^{n} E\{r_i\} x_i = z_2\}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$
$$\text{all } x_i \text{ unrestricted}$$

meaning that no constraints beyond the sum-to-one constraint are allowed in the model. The other is the *variable-restricted* model

$$\min \{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j = z_1\} \qquad \text{(MC-Bounds)}$$
$$\max \{\sum_{i=1}^{n} E\{r_i\} x_i = z_2\}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} x_i = 1$$
$$\ell_i \geq x_i \geq \mu_i \text{ for all } i$$

in which lower and upper bounds exist on the weights. The significant aspect of the unrestricted model is that there are no limits on the negativities of the weights, meaning that *unlimited* short selling is permitted. To illustrate the short selling of a security, let $x_3 = -.2$. This would say the following to an investor. Borrow a position in security 3 to the extent of 20% of the initial sum to be invested. Then immediately sell it to generate extra cash. Now with the 120% of the initial sum, invest it as dictated by the other x_i weights to complete the portfolio.

The unrestricted model is the clear favorite in teaching and academic research. In research, the unrestricted model has long provided fertile ground for academics because of its beautiful mathematical properties. For example, as long as the variance/covariance matrix

$$\mathbf{V} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & & \\ \vdots & & & \vdots \\ \sigma_{n1} & & \cdots & \sigma_{nn} \end{bmatrix}$$

is nonsingular, every imaginable piece of information about the model appears to be derivable in closed form (see for example Roll [136], pp. 158–165). In teaching, this is an advantage because via this model portfolio selection can be taught without having to have mathematical programming included in the curriculum (which it hardly ever is any more in finance, even in Ph.D. programs).

One the other hand, the variable-restricted model is the clear favorite in practice. For instance, in the US, short selling is prohibited by law in the $6 trillion mutual fund business. It is also prohibited in the management of pension assets. And even in hedge funds where short selling is part of their strategy, it is all but impossible to imagine any situation in which there wouldn't be limits. The question is, when trying to locate an optimal solution, how much difference might there be in the locations of the nondominated frontiers of the two models, and might any differences be cause for concern?

3.4. Bullet-Shaped Feasible Regions

When looking through the portfolio chapters of almost any university investments text, it would be hard to miss seeing graphs of bullet-shaped regions, often with dots in them, as in Figure 1.4(*top*). When unbounded (which in almost all books they are), and with standard deviation on the horizontal and expected return $E\{r_p\}$ on the vertical, these are all graphs of the feasible region Z of (MC-Unrestr) in criterion space. The dots within Z typically signify the criterion vectors $(\sigma\{r_i\}, E\{r_i\})$ of individual securities.

In contrast, the feasible region Z of an (MC-Bounds) formulation is as in Figure 1.4 (*bottom*). As a subset of the Z of its (MC-Unrestr) counterpart, the Z of an (MC-Bounds) formulation is easily recognizable by its "scalloped" rightmost boundary.

To see why a feasible region Z of (MC-Unrestr) is continuous, bullet-shaped, and unbounded, let us first consider the two securities A and B in Figure 1.5. The unbounded line sweeping through A and B, which is

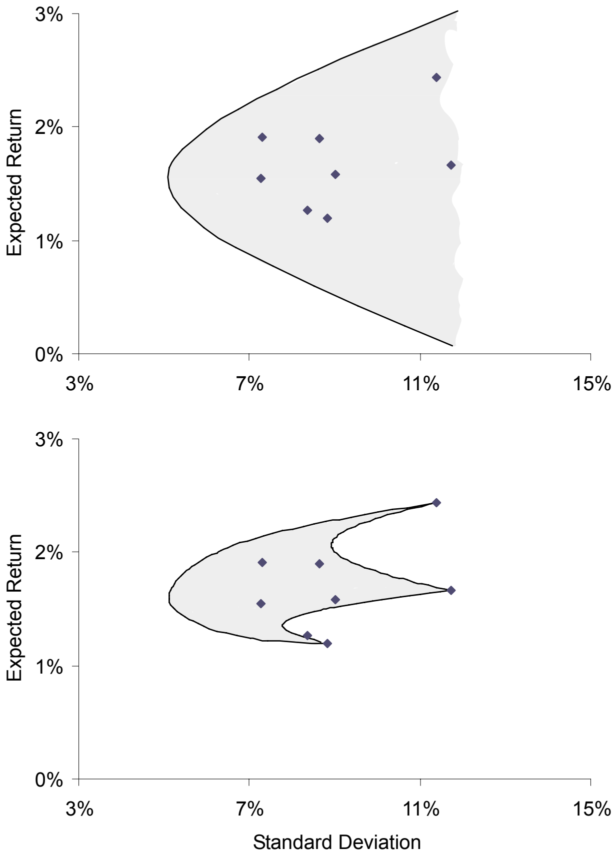

Figure 1.4. Feasible regions Z of (MC-Unrestr) and (MC-Bounds) for the same eight securities

actually a hyperbola, is the set of criterion vectors of all two-stock portfolios resulting from all linear combinations of A and B whose weights sum to one. In detail, all points on the hyperbola strictly between A and B correspond to weights $x_a > 0$ and $x_b > 0$; all points on the hyperbola above and to the right of A correspond to weights $x_a > 1$ and $x_b < 0$; and all points on the hyperbola below and to the right of B correspond to weights $x_a < 0$ and $x_b > 1$. The degree of "bow" toward the vertical axis of the hyperbola is a function of the correlation coefficient ρ_{be} between A and B. This is seen by looking at the components of the $(\sigma\{r_{ab}\}, E\{r_{ab}\})$ criterion vector of any two-stock portfolio which are given by

$$\sigma\{r_{ab}\} = \sqrt{\sigma_{aa}x_a^2 + 2\rho_{ab}\sigma_a\sigma_b x_a x_b + \sigma_{bb}x_b^2} \qquad (1.5)$$

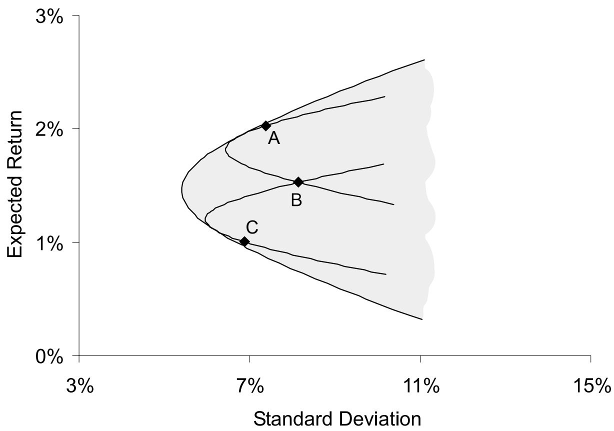

Figure 1.5. Continuous, bullet-shaped, and unbounded feasible region Z created by securities W, B and E

and

$$E\{r_{ab}\} = E\{r_a\}x_a + E\{r_b\}x_b \tag{1.6}$$

in which $\sigma_a = \sqrt{\sigma_{aa}}$ and $\sigma_b = \sqrt{\sigma_{bb}}$. Whereas $E\{r_{ab}\}$ is linear, the positive value of $\sigma\{r_{ab}\}$ decreases nonlinearly as a function of ρ_{be} as its value goes from 1 to -1, and hence the bowing effect.

Through B and C there is another hyperbola. And since through any point on the hyperbola through A and B and any point on the hyperbola through B and C there is another hyperbola (not shown), and so forth, the feasible region fills in and takes on its bullet shape whose boundary, in the case of (MC-Unrestr), is a (single) hyperbola as well.

With regard to the feasible region Z of (MC-Bounds), the hyperbolic lines through the criterion vectors of any two financial products are not unbounded. In each case, they end in each direction at some point because of the bounds on the variables. While still filling in to create a bullet-shaped Z, the leftmost boundary, instead of being formed by a single hyperbola, is in general piecewise hyperbolic. The rightmost boundary, instead of being unbounded takes on it trademark “scalloped” effect.

Because predicted standard deviation is to be minimized and expected return is to be maximized, we look to the “northwest” of Z for the nondominated set. This causes the nondominated set to be the upper portion of the leftmost boundary (the portion that is nonnegatively sloped). In finance, they call this the "efficient frontier." However, this causes a terminological conflict with the distinction indicated earlier about efficiency/inefficiency being reserved for points in decision space and nondominated/dominated being reserved for vectors in criterion space. Rather than the efficient frontier, we will refer to it as the "nondominated frontier," not only because this is consistent with the terminology of multiple criteria optimization discussed earlier, but because nondominated is the more intuitive term in criterion space.

3.5. Assumptions and Nondominated Sensitivities

The assumptions surrounding the use of (MC-Unrestr) and (MC-Bounds), and theories based upon them, in finance are largely as follows.

- **(a)** There are many investors, each small, none of which can affect prices.
- **(b)** There are no taxes.
- **(c)** There are no transactions costs.
- **(d)** Each investor's asset universe is all publicly traded securities.
- **(e)** All investors are rational mean-variance optimizers.
- **(f)** All investors have utility functions whose indifference curves are convex-to-the-origin.
- **(g)** All investors share the same expected returns, predicted variances, and predicted covariances about the future. This is called *homogeneous expectations.*
- **(h)** All investors have the same single holding period.
- **(i)** Each security is infinitely divisible.

We now discuss the sensitivity of the nondominated frontier to factors that have implications about the appropriateness of this set of the assumptions. Sensitivity is measured by noting what happens to the nondominated frontier as the parameter associated with a given factor changes. We start by looking at the sensitivity of the nondominated frontier to changes in an upper bound common to all investment proportion weights. Then we discuss the likely sensitivities of the nondominated frontier to changes in other things such a portfolio dividend requirement, a social responsibility attribute to be possessed by a portfolio, and other matters of concern. The computer work required for testing such sensitivities is outlined in the following 7-step algorithmic procedure.

1. Start the construction of what we recognize in multiple criteria optimization as an e-constraint program by converting the ex-

pected return objective in (MC-Unrestr) and (MC-Bounds) to a $\geq$ constraint with right-hand side ρ.

2. Install in the e-constraint program whatever constraints are required to accommodate the factor parameter to be varied.
3. Set the factor parameter to its starting value.
4. Set ρ to its starting value.
5. Solve the e-constraint program and take the square root of the outputted variance to form the nondominated point $(\sigma\{r_p\}, \rho)$.
6. If ρ has reached its ending value, go to Step 7. Otherwise, increment ρ and go to Step 5.
7. Connect on a graph all of the nondominated points obtained from the current value of the factor parameter to achieve a display of the nondominated frontier of this factor parameter value. If the factor parameter has reached its ending value, stop. Otherwise increment the factor parameter and go to Step 4.

To illustrate with regard to the testing of the sensitivity of the nondominated frontier to changes in the common upper bound on the x_i, we form the e-constraint program

$$\min \{ \sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j = \sigma^2\{r_p\} \} \tag{Exp-1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} E\{r_i\} x_i \geq \rho$$

$$\sum_{i=1}^{n} x_i = 1$$

$$\ell \leq x_i \leq \mu \quad i \in \{1, \ldots, n\}$$

in which $n = 20$; $\ell = -.05$ in all runs to permit mild short selling; and μ is set in turn to $1.00, .15, .10$ to generate three frontiers. Running for 25 different ρ values (experimenter's choice) for each μ-value, the three nondominated frontiers of Figure 1.6 result. The topmost frontier is for $\mu = 1.00$, the middle frontier is for $\mu = .15$, and the bottommost frontier is for $\mu = .10$.

As seen in Figure 1.6, the nondominated frontier undergoes major changes as we step through the three values of μ. Hence there is considerable sensitivity to the value of μ. Since, in the spirit of diversification, investors would presumably prefer smaller values of μ to larger values as long as portfolio performance is not seriously deteriorated in other aspects, we can see that an examination of the tradeoffs among risk, return, and μ are involved before a final decision can be made. Since an

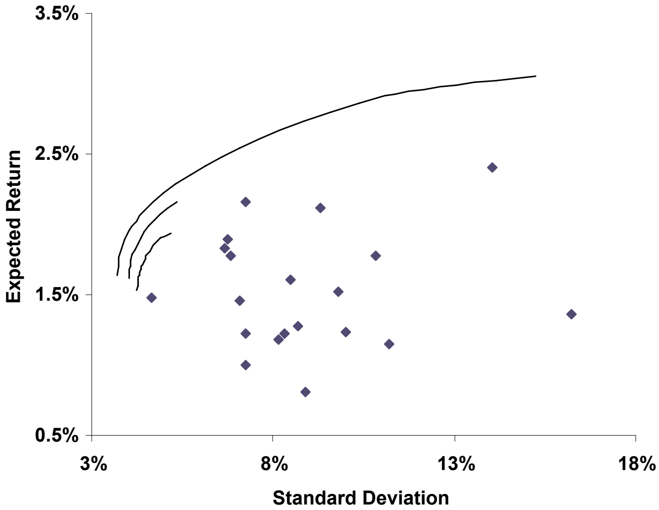

Figure 1.6. Nondominated frontiers as a function of changes in the value of upper bound parameter μ

investor would probably have no way of knowing in advance his or her optimal value of μ without reference to its effects on risk and return, we have demonstrated that μ should probably be considered a criterion to be minimized.

Using the same 7-step algorithmic procedure, other experiments (results not shown) could be conducted. For example, if we wished to test the sensitivity of the nondominated frontier to changes in a expected portfolio dividend yield requirement, we would form the following e-constraint program

$$\min\{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i\sigma_{ij}x_j = \sigma^2\{r_p\}\} \tag{Exp-2}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{i=1}^{100} E\{r_i\}x_i \geq \rho \\ & \sum_{i=1}^{n} E\{d_i\}x_i \geq \delta \\ & \sum_{i=1}^{n} x_i = 1 \\ & \ell \leq x_i \leq \mu \quad i \in \{1,\ldots,n\} \end{aligned}$$

in which d_i is the random variable for the dividend yield realized on security i between the beginning and end of the holding period and δ is the expected portfolio dividend yield requirement. A similar type of formulation could be set up for social responsibility.

For both dividends and social responsibility we can probably expect to see nondominated frontier sensitivities along the lines of that for μ. If this is indeed the case, this would signal that dividends and social responsibility should probably be added to the list of criteria as well. With μ, we now see how it is easy to have more criteria than two in investing. Whereas the assumptions at the beginning of the subsection assume a two-criterion world, we are led to see new things by virtue of these experiments. One is that the assumption about risk and return being the only criteria is certainly under seige. Another is that, in the company of μ, dividends, and social responsibility, the last of which can be highly subjective, *individualism* should be given more play. By individualism, no investor's criteria, opinions, or assessments need conform to those of another. In direct conflict with the assumption about homogeneous expectations – which nobody believes in anyway – at the security level, individualism allows an investor to have differing opinions about any security's expected return, variance, covariance with any other security, liquidity, dividend outlook, social responsibility score, and so forth. At the portfolio level, for example, individualism allows investors to possess different lists of criteria, have differing objective functions for even the same criteria, work from different asset universes, and enforce different attitudes about the nature of allowable short selling or the number of securities to be tolerated in a portfolio. Therefore, in contrast to current theory, with different lists of criteria, different objective functions, and different sets of constraints, all investors would not face the same feasible region with the same nondominated set. Each would have his or her own portfolio problem with its own optimal solution. The benefit of this enlarged outlook would be that portfolio theory would then not only have to focus on explaining equilibrium solutions, but on customized solutions as well.

3.6. Expanded Formulations and New Assumptions

Generally, in multiple criteria, we recognize a constraint from an objective as follows. If when modelling as a constraint we realize that we can not easily fix a right-hand side value without knowing how other output measures turn out, then we are probably looking at an objective.

With individualism, an investor could easily be looking at the expanded multiple criteria optimization problem as follows

$$\begin{aligned}
\min\ &\{f_1(\mathbf{x}) = \text{risk}\} && \text{(MC-Expand)}\\
\max\ &\{f_2(\mathbf{x}) = \text{return}\}\\
\max\ &\{f_3(\mathbf{x}) = \text{dividends}\}\\
\min\ &\{f_4(\mathbf{x}) = \text{maximum investment proportion weight}\}\\
\max\ &\{f_5(\mathbf{x}) = \text{social responsibility}\}\\
\min\ &\{f_6(\mathbf{x}) = \text{number of securities in portfolio}\}\\
\min\ &\{f_7(\mathbf{x}) = \text{short selling}\}\\
\text{s.t.}\ &\quad \mathbf{x} \in S
\end{aligned}$$

With regard to the number of securities in a portfolio, this can easily be a criterion. For individuals or mutual funds, every extra security is a paperwork headache and a distraction. Not only is resource time required to monitor the security, but there are also the monthly statements to absorb and file, annual reports to decide whether to read or not, proxy matters (shareholder proposals, mergers, name changes, spin offs, etc.) to be evaluated and voted upon, tax consequences to be dealt with, and the like. For most investors, they would just as soon wish to minimize much of the hassle. Also, as reflected by the last objective in (MC-Expand), investors open to the idea of short selling might nevertheless wish to minimize it if possible.

Updating to take a new look at portfolio selection, the following is proposed as a more appropriate set of assumptions with which to approach the study of portfolio theory when multiple criteria and individualism are to be taken into account.

- **(a)** There are many investors, each small, none of which can affect prices.
- **(b)** There are no taxes.
- **(c)** There are no transactions costs.
- **(d)** An investor's asset universe can be any subset of all publicly traded securities, even for large investors usually not more than a few hundred.
- **(e)** Investors may possess any mix of up to about six or eight objectives.
- **(f)** All investors have utility functions whose indifference curves are convex-to-the-origin.
- **(g)** Heterogeneity of expectations is the rule. That is, investors can be expected to have widely different forecasts about any secu-

rity attribute including expected returns, predicted variances, and predicted covariances, expected dividends, and so forth.

(h) Short selling is allowed but to only some limited extent.

The first three assumptions remain the same as they are nice to retain in that they establish benchmarks against which some of the world's imperfections can be measured. The assumption about convex-to-the-origin utility function contours is also retained as we see no compelling difficulty with it at the present time, but all the rest have either been modified or deleted.

3.7. Nondominated Surfaces

If multiple criteria exist in portfolio selection, then the nondominated set of current-day finance that exists as a frontier in R^2 is a *surface* in R^n. What evidence might there be to support this? In current-day finance there is the "market portfolio". By theory, the market portfolio contains every security in proportion to its market capitalization (number of shares times price), is somewhere in the midst of the nondominated frontier, and is supposed to be everyone's optimal portfolio when not including the risk-free asset. Since the market portfolio is impractical, indices like the S&P 500 are used as surrogates. But empirically, the surrogates, which should be essentially as desirable as the market portfolio, have always been found to be deep below the nondominated frontier, in fact so deep below that this cannot be explained by chance variation. Whereas this is an anomaly in conventional risk-return finance, this is exactly what we would expect in multiple criteria finance.

To take a glimpse at the logic as to why this is what we would expect, consider the following. In a risk-return portfolio problem, let us assume that the feasible region Z is the ellipse in Figure 1.7. Here, the nondominated frontier is the portion of the boundary of the ellipse in the second quadrant positioned at the center of the ellipse. Similarly, in a q-criterion portfolio problem (with $q - 2$ objectives beyond risk and return), let us assume that the feasible region is an *ellipsoid* in q-space. Here, the nondominated surface is the portion of the surface of the ellipsoid in an orthant positioned at the center of the ellipsoid. Now assume that the market portfolio, which by theory is nondominated, is in the middle of the nondominated set. If this is the case, then the market portfolio would be at $\mathbf{z}^2$ on the ellipse. However, if (i) there is a third objective, (ii) the feasible region is ellipsoidal in three-space, and (iii) the market portfolio is in the middle of the nondominated surface in R^3, then the market portfolio would *project* onto risk-return space at $\mathbf{z}^3$. If (i) there is a fourth objective, (ii) the feasible region is ellipsoidal

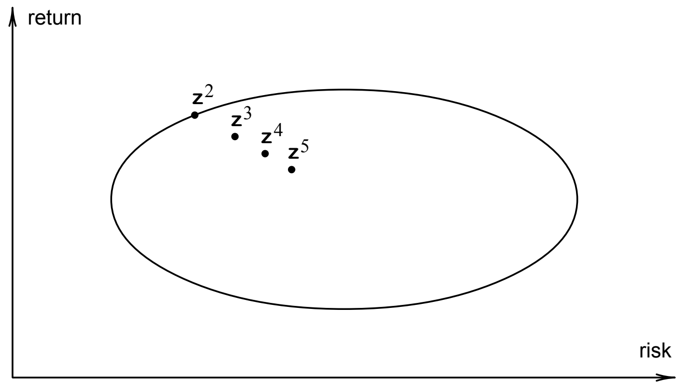

Figure 1.7. An ellipsoidal feasible region projected onto two-dimensional risk-return space

in four-space, and (iii) the market portfolio is in the middle of the non-dominated surface in R^4, then the market portfolio would project onto risk-return space at $\mathbf{z}^4$. With five objectives under the same conditions, then the market portfolio would project onto risk-return space at $\mathbf{z}^5$, and so forth, becoming deeper and deeper.

Consequently, it may not be unreasonable to conjecture that what is, to use a term from Elton and Gruber [51], the "modern portfolio theory" of today is only a first-order approximation, a projection onto the risk-return plane, of the real multiple criteria problem from higher dimensional criterion space.

3.8. Further Research in MCDA in Portfolio Analysis

In addition to further study into multiple criteria and individualism in investing, we also find intriguing for future research the area of special variable treatments in portfolio optimization. By special variable treatments, we mean conditions on the variables such as the following.

- **a.** No fewer than a given number of securities, and no more than a given number of securities, can be in a portfolio (either long or short).
- **b.** No more than a given number of securities can be sold short.
- **c.** If a stock is in a portfolio, then its weight must be in market cap proportion to the weights of all other stocks in the portfolio.
- **d.** No more than a given proportion of a portfolio can be involved in stocks sold short.

- **e.** Some or all of the x_i are semi-continuous. That is, an x_i is either zero or in a given interval $[a, b], a > 0$.
- **f.** No more than a given number of stocks may have a given upper bound. For instance, at most one stock (but which one is not known beforehand) may constitute as much as 25% of a portfolio, with all other stocks having an upper bound of 5%.

While some of these can be modelled with auxiliary 0-1 variables, others may only be amenable to local search algorithms as in Gandibleux, Caballero and Molina [57]. Having at one's disposal well-researched methods for dealing with such special variable treatments would extend the power at our new look at portfolio analysis when focusing on customized portfolio solutions. Another area of interest is the use of mean absolute deviation (MAD), which can be modelled linearly, as the risk measure in place of variance (standard deviation). Finally, it may be that multiple criteria and behavioral finance (see for example Shefrin [147]) reinforce one another as both area see much more going on in investing than the traditional.

References to some of the older classical papers on portfolio selection and to papers by authors who have most recently been taking a new look at portfolio theory along the lines discussed here are in the bibliography.

4. MCDA in Discrete Financial Decision-Making Problems

Several decision-making problems, including financial decision-making problems, require the evaluation of a finite set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$, which may include firms, investment projects, stocks, credit applications, etc. These types of problems are referred to as "discrete" problems. The outcome of the evaluation process may have different forms, which are referred to as "problematics" [141]: (1) problematic α: Choosing one alternative, (2) problematic β: Sorting the alternatives in well defined groups defined in a preference order, (3) problematic γ: Ranking the alternatives from the best to the worst, and (4) problematic δ: Describing the alternatives in terms of their performance on the criteria. The selection of an investment project is a typical example of a financial decision-making problem where problematic α (choice) is applicable. The prediction of business failure is an example of problematic β (classification of firms as healthy or failed), the comparative evaluation and ranking of stocks according to their financial and stock market performance is an example of problematic γ, whereas the description of the financial characteristics of a set of firms is a good example of problematic δ.

The selection of one of these problematics depends solely on the objective of the analysis and the decision-making context. In each case, the evaluation process involves the aggregation of all the pertinent decision factors $F = (g_1, g_2, \ldots, g_n)$, which are referred to as "evaluation criteria" or simply "criteria". Formally, a criterion g_j is a non-decreasing real-valued function that describes an aspect of the global performance of the alternatives and defines how the alternatives are compared to each other, as follows:

$$\begin{aligned} g_{ij} > g_{kj} \Leftrightarrow a_i \succ a_k \quad & (a_i \text{ is preferred to } a_k) \\ g_{ij} = g_{kj} \Leftrightarrow a_i \sim a_k \quad & (a_i \text{ is indifferent to } a_k) \end{aligned}$$

where g_{ij} denotes the performance of alternative a_i on criterion g_j.

The aggregation of all criteria into an overall evaluation index can be performed in many different ways depending on the form of the criteria aggregation model. Within the MCDA field one can distinguish three main forms of aggregation models: (1) outranking relations (relational form), (2) utility functions (functional form), (3) decision rules (symbolic form). In all cases, the aggregation model is developed so as to respect the decision maker's judgment policy. To ensure that this purpose is achieved some information on the preferential system of the decision maker must be specified, such as the criteria weights. The required preferential information can be specified either through direct procedures in which a decision analyst elicits it directly from the decision maker, or through indirect procedures in which the decision maker provides examples of the decisions that he takes and the decision analyst analyzes them to determine the required preferential parameters which are most consistent with the decision maker's global evaluations. The latter approach is known in the MCDA field as "preference disaggregation analysis" [82].

The subsequent subsections in this portion of the paper present several MCDA discrete evaluation approaches which are suitable for addressing financial decision-making problems. The presentation is organized in terms of the criteria aggregation model employed by each approach (outranking relations, utility functions, decision rules).

4.1. Outranking Relations

The foundations of the outranking relations theory have been set by Bernard Roy during the late 1960s through the development of the ELECTRE family of methods (**EL**imination **E**t **C**hoix **T**raduisant la **RE**alité; [139]). Since then, they have been widely used by MCDA researchers, but mostly in Europe and Canada.

An outranking relation is a binary relation that enables the decision maker to assess the strength of the outranking character of an alternative a_i over an alternative a_k. This strength increases if there are enough arguments (coalition of the criteria) to confirm that a_i is at least as good as a_k, while there is no strong evidence to refuse this statement.

Outranking relations techniques operate into two stages. The first stage involves the development of an outranking relation among the considered alternatives, while the second stage involves the exploitation of the developed outranking relation to choose the best alternatives (problematic α), to sort them into homogenous groups (problematic β), or to rank them from the most to the least preferred ones (problematic γ).

Some of the most widely known outranking relations methods include the family of the ELECTRE methods [140] and the family of the PROMETHEE methods [23]. These methods are briefly discussed below. A detailed presentation of all outranking methods can be found in the books of Roy and Bouyssou [142] and Vincke [173].

ELECTRE methods. The family of ELECTRE methods was initially introduced by Roy [139], through the development of the ELECTRE I method, the first method to employ the outranking relation concept. Since then several extensions have been proposed, including ELECTRE II, III, IV, IS and TRI [140]. These methods address different types of problems, including choice (ELECTRE I, IS), ranking (ELECTRE II, III, IV) and sorting/classification (ELECTRE TRI).

Given a set of alternatives $A = \{a_1, a_2, \dots a_m\}$ any of the above ELECTRE methods can be employed depending on the objective of the analysis (choice, ranking, sorting/classification). Despite their differences, all the ELECTRE methods are based on the identification of the strength of affirmations of the form $Q =$"alternative a_i is at least as good as alternative a_k". The specification of this strength requires the consideration of the arguments that support the affirmation Q as well as the consideration of the arguments that are against it. The strength of the arguments that support Q is analyzed through the "concordance test". The measure used to assess this strength is the global concordance index $C(a_i, a_k) \in [0, 1]$. The closer is C to unity, the higher is the strength of the arguments that support the affirmation Q. The concordance index is estimated as the weighted average of partial concordance indices defined for each criterion:

$$C(a_i, a_k) = \sum_{j=1}^{n} w_j c_j (g_{ij} - g_{kj})$$

where w_j is the weight of criterion g_j ($\sum w_j = 1, w_j \geq 0$) and $c_j(g_{ij} - g_{kj})$ is the partial concordance index defined as a function of the difference $g_{ij} - g_{kj}$ between the performance of a_i and a_k on criterion g_j. The partial concordance index measures the strength of the affirmation Q'="a_i is at least as good as a_k on the basis of criterion g_j". The partial index is normalized in the interval $[0, 1]$, with values close to 1 indicating that Q' is true and values close to 0 indicating that Q' is false.

Except for assessing the strength of the arguments that support the affirmation Q, the strength of the arguments against Q is also assessed. This is performed through the "discordance test", which leads to the calculation of the discordance index $D_j(g_{ij} - g_{kj})$ for each criterion g_j. Conceptually, the discordance index $D_j(g_{ij} - g_{kj})$ measures the strength of the indications against the affirmation Q'. The higher is the discordance index the more significant is the opposition of the criterion on the validity of the affirmation Q. If the strength of this opposition for criterion g_j is above a critical level (veto threshold), then the criterion vetoes the validity of the affirmation Q irrespective of the performance of the considered pair of alternatives (a_i, a_k) on the other criteria.

Once the concordance and discordance tests are performed, their results (concordance index C, discordance indices D_j) are combined to construct the final outranking relation. The way that this combination is performed, as well as the way that the results are employed to choose, rank, or sort the alternatives depends on the specific ELECTRE method that is used. Details on these issues can be found in the works of Roy [140; 141] as well as in the book of Roy and Bouyssou [142].

PROMETHEE methods. The development of the PROMETHEE family of methods (**P**reference **R**anking **O**rganization **METH**hod of **E**nrichment **E**valuations) began in the mid 1980s with the work of Brans and Vincke [23] on the PROMETHEE I and II methods.

The PROMETHEE method leads to the development of an outranking relation that can be used either to choose the best alternatives (PROMETHEE I) or to rank the alternatives from the most preferred to the least preferred (PROMETHEE II). For a given set of alternatives A, the evaluation process in PROMETHEE involves the pairwise comparisons (a_i, a_k) to determine the preference index $\pi(a_i, a_k)$ measuring the degree of preference for a_i over a_k, as follows:

$$\pi(a_i, a_k) = \sum_{j=1}^{n} w_j P_j(g_{ij} - g_{kj}) \in [0, 1]$$

The preference index is similar to the global concordance index of the ELECTRE methods. The higher is the preference index (closer to unity) the higher is the strength of the preference for a_i over a_k. The calculation of the preference index depends on the specification of the criteria weights w_j ($\sum w_j = 1$, $w_j \geq 0$) and the preference functions P_j for each criterion g_j. The preference functions are increasing functions of the difference $g_{ij} - g_{kj}$ between the performances of a_i and a_k on criterion g_j. The preference functions are normalized between 0 and 1. The case $P_j(a_i, a_k) \approx 1$ indicates a strong preference for a_i over a_k in terms of the criterion g_j, whereas the case $P_j(a_i, a_k) \approx 0$ indicates weak preference. Generally, the preference functions may have different forms, depending on the judgment policy of the decision maker. Brans and Vincke [23] proposed six specific forms (generalized criteria) which seem sufficient in practice.

The results of the comparisons made for all pairs of alternatives (a_i, a_k) are organized in a graph (value outranking graph). The nodes of the graph represent the alternatives under consideration, whereas the arcs between nodes a_i and a_k represent the preference of alternative a_i over a_k (if the direction of the arc is $a_i \rightarrow a_k$) or the opposite (if the direction of the arc is $a_k \rightarrow a_i$). Each arc is associated with a flow representing the preference index $\pi(a_i, a_k)$. The sum of all flows leaving a node a_i is called the leaving flow $\phi^+(a_i)$. The leaving flow provides a measure of the outranking character of alternative a_i over all the other alternatives in A. In a similar way, the sum of all flows entering a node a_i is called the entering flow $\phi^-(a_i)$. The entering flow measures the outranked character of alternative a_i compared to all the other alternatives in A.

On the basis of these flows the heuristic procedures of PROMETHEE I and II are employed to choose the best alternatives (PROMETHEE I) or to rank the alternatives from the most preferred to the least preferred (PROMETHEE II). The choice of the best alternatives in the PROMETHEE I method involves the definition of the preference (P), indifference (I) and incomparability (R) relations of the basis of the leaving and entering flows of the outranking graph [23]. The ranking of the alternatives in the PROMETHEE II method is based on the difference between the leaving and the entering flow $\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$, which provides the net flow for a node (alternative) a_i. The net flow constitutes the overall evaluation index of the performance of the alternatives. The most preferred alternatives are the ones with the higher net flows, whereas the alternatives with the lower net flows are considered as the least preferred ones.

4.2. Utility Functions-Based Approaches

The multiattribute utility theory (MAUT; [91]) extends the traditional utility theory to the multi-dimensional case. Even from the early stages of the MCDA field, the strong theoretical foundations of the MAUT framework have been among the cornerstones of the development of MCDA and its practical implementation. The objective of MAUT is to model and represent the decision maker's preferential system into a utility/value function $U(a_i)$. The utility function is defined on the criteria space, such that:

$$U(a_i) > U(a_k) \Leftrightarrow a_i \succ a_k \quad (a_i \text{ is preferred to } a_k) \tag{1.7}$$

$$U(a_i) = U(a_k) \Leftrightarrow a_i \sim a_k \quad (a_i \text{ is indifferent to } a_k) \tag{1.8}$$

The most commonly used form of utility function is the additive one:

$$U(a_i) = p_1 u_1(g_{i1}) + p_2 u_2(g_{i2}) + \ldots + p_n u_n(g_{in}) \tag{1.9}$$

where, $u_1, u_2, \ldots, u_n$ are the marginal utility functions corresponding the evaluation criteria. Each marginal utility function $u_j(g_j)$ defines the utility/value of the alternatives for each individual criterion g_j. The constants $p_1, p_2, \ldots, p_n$ represent the criteria trade-offs that the decision maker is willing to take. These constants are often considered to represent the weights of the criteria and they are defined such that they sum-up to unity.

A detailed description of the methodological framework underlying MAUT and its applications is presented in the book of Keeney and Raiffa [91].

Generally, the process for developing an additive utility function is based on the cooperation between the decision analyst and the decision maker. This process involves the specification of the criteria trade-offs and the form of the marginal utility functions. The specification of these parameters is performed through interactive procedures, such as the midpoint value technique [91]. The realization of such interactive procedures is often facilitated by the use of multicriteria decision support systems, such as the MACBETH system [13].

However, the implementation of such interactive procedures in practice can be cumbersome, mainly because it is rather time consuming and it depends on the willingness of the decision maker to provide the required information and the ability of the decision analyst to elicit it efficiently. The preference disaggregation approach of MCDA (PDA; [82]) provides the methodological framework to cope with this problem. PDA refers to the analysis (disaggregation) of the global preferences (judgement policy) of the decision maker in order to identify the criteria aggregation model that underlies the preference result (ranking or classification/sorting). Similarly to MAUT, preference disaggregation analysis uses common utility decomposition forms to model the decision maker's preferences. Nevertheless, instead of employing a direct procedure for estimating the global utility model (MAUT), preference disaggregation analysis uses regression-based techniques (indirect estimation procedure). More specifically, in PDA the parameters of the utility decomposition model are estimated through the analysis of the decision maker's overall preference on some reference alternatives A', which may include either examples of past decisions or a small subset of the alternatives under consideration. The decision maker is asked to provide a ranking or a classification of the reference alternatives according to his decision policy (global preferences). Then, using regression-based techniques the global preference model is estimated so that the decision maker's global evaluation is reproduced as consistently as possible by the model. A comprehensive bibliography on preference disaggregation methods can be found in Jacquet-Lagrèze and Siskos [82; 83].

PDA methods are particularly useful in addressing financial decision-making problems [186]. The repetitive character of financial decisions and the requirement for real-time decision support are two features of financial decisions which are consistent with the PDA framework. Thus, several PDA methods have been extensively used in addressing financial decision problems, mainly in cases where a ranking or sorting/classification of the alternatives is required. The following subsections provide a brief description of some representative PDA methods which have been used in financial problems.

UTA method. The UTA method (**UT**ilités **A**dditives; [81]) is an ordinal regression method developed to address ranking problems. The objective of the method is to develop an additive utility function which is as consistent as possible with the decision maker's judgment policy. The input to the method involves a set of reference alternatives A'. For each reference alternative the decision maker is asked to provide his global evaluation so as to form a total pre-order of the alternatives in A': $a_1 \succ a_2 \succ \ldots \succ a_m$ (the indifference relation "$\sim$" as defined in (1.8) can also be used in the pre-order). The developed utility model is assumed to be consistent with the decision maker's judgment policy if it is able to reproduce the given pre-order of the reference alternatives as consistently as possible. In that regard, the utility model should be developed so that: $U(a_1) > U(a_2) > \ldots > U(a_m)$.

In developing the utility model to meet this requirement, there are two types of possible errors which may occur [151]): (1) the under-estimation error when the developed model assigns a reference alternative to a lower (better) rank than the one specified in the given pre-order (the alternative is under-estimated by the decision maker), and (2) the over-estimation error when the developed model assigns a reference alternative to a higher (worse) rank than the one specified in the given pre-order (the alternative is over-estimated by the decision maker). The objective of the model development process is to minimize the sum of these errors. This is performed through linear programming techniques [81].

UTADIS method. The UTADIS method (**UT**ilités **A**dditives **DIS**criminantes; [47; 80]) is a variant of the UTA method, developed for sorting/classification problems. Similarly to the UTA method, the decision maker is asked to provide his global evaluation on a set of reference alternatives A'. In this case, however, the decision maker is not asked to rank the alternatives is A'. Instead, he classifies the reference alternatives into homogenous groups $C_1, C_2, \ldots, C_q$ defined in an ordinal way, such that $C_1 \succ C_2 \succ \cdots \succ C_q$ (i.e., group C_1 includes the most preferred alternatives, whereas group C_q includes the least preferred ones). Within this context, the developed additive utility model will be consistent with the decision maker's global judgment if the following conditions are satisfied:

$$\left.\begin{array}{ll} U(a_i) \geq t_1, & \forall\ a_i \in C_1 \\ t_1 > U(a_i) \geq t_2, & \forall\ a_i \in C_2 \\ \dots\dots\dots\dots & \dots\dots \\ U(a_i) < t_{q-1}, & \forall\ a_i \in C_q \end{array}\right\} \tag{1.10}$$

where $t_1 > t_2 > \ldots > t_{q-1}$ are thresholds defined in the global utility scale $[0, 1]$ to discriminate the groups (each t_k is the lower bound of group C_k). Similarly, to the UTA method, the under-estimation and over-estimation errors are also used in the UTADIS method to measure the differences between the model's results and the predefined classification of the reference alternatives. In this case, the two types of errors are defined as follows: (1) the under-estimation error $\sigma_i^+ = \max\{0, t_k - U(a_i)\}, \forall\ a_i \in C_k,\ k = 1, 2, \ldots, q-1$, (2) the over-estimation error $\sigma_i^- = \max\{0, U(a_i) - t_{k-1}\}, \forall\ a_i \in C_k,\ k = 2, 3, \ldots, q$. The additive utility model is developed to minimize these errors using a linear programming formulation [47].

Recently, several new variants of the original UTADIS method have been proposed (UTADIS I, II, III) to consider different optimality criteria during the development of the additive utility classification model [47; 189].

MHDIS method. The MHDIS method (**M**ulti-group **H**ie-rarchical **DIS**crimination [193]) extends the PDA framework of the UTADIS method in complex sorting/classification problems involving multiple groups (of course the method is also applicable in the simple two-group case). As the name of the method implies, MHDIS addresses sorting problems through a hierarchical procedure, during which the groups are distinguished progressively, starting by discriminating group C_1 (most preferred alternatives) from all the other groups $\{C_2, C_3, \ldots, C_q\}$, and then proceeding to the discrimination between the alternatives belonging to the other groups. At each stage of this sequential/hierarchical process two additive utility functions are developed for the classification of the alternatives. Assuming that the classification of the alternatives should be made into q ordered classes $C_1 \succ C_2 \succ \cdots \succ C_q$, $2(q-1)$ additive utility functions are developed. These utility functions have the following additive form:

$$U_k(a_i) = \sum_{j=1}^{n} u_{kj}(g_{ij}), \quad U_{\sim k}(a_i) = \sum_{j=1}^{n} u_{\sim kj}(g_{ij}) \tag{1.11}$$

Both functions are defined between 0 and 1. The function U_k measures the utility for the decision maker of a decision to assign an alternative into group C_k, whereas the second function $U_{\sim k}$ corresponds to the classification into the set of groups $C_{\sim k} = \{C_{k+1}, C_{k+2}, \ldots, C_q\}$. The rules used to perform the classification of the alternatives are the following:

$$\left.\begin{array}{l} \text{If } U_1(a_i) > U_{\sim 1}(a_i) \text{ then } a_i \in C_1 \\ \quad \text{Else if } U_2(a_i) > U_{\sim 2}(a_i) \text{ then } a_i \in C_2 \\ \quad\quad \ldots\ldots\ldots\ldots\ldots\ldots \\ \quad\quad\quad \text{Else if } U_{q-1}(a_i) > U_{\sim (q-1)}(a_i) \text{ then } a_i \in C_{q-1} \\ \quad\quad\quad\quad \text{Else } a_i \in C_q \end{array}\right\} \tag{1.12}$$

Except for the hierarchical classification framework, the MHDIS method has another special feature that distinguishes it from other MCDA sorting methods as well as from other linear programming classification approaches [162]. This involves the optimization framework used to develop the optimal sorting model (additive utility functions). In particular, during model development in the MHDIS method, three mathematical programming problems are solved. At each stage k of the hierarchical discrimination process ($k = 1, 2, \ldots, q-1$), two linear and one mixed-integer programming problems are solved to estimate the "optimal" pair of utility functions, where the term "optimal" refers both to the total number of misclassifications as well as to the clarity of the distinction between the groups. Initially, a linear programming problem (LP1) is solved to minimize the magnitude of the classification errors (in distance terms). Then, a mixed-integer programming problem (MIP) is solved to minimize the total number of misclassifications among the misclassifications that occur after the solution of LP1, while retaining the correct classifications. Finally, a second linear programming problem is solved to maximize the clarity of the classification obtained from the solutions of LP1 and MIP. A detailed description of the model optimization process in the MHDIS method can be found in Zopounidis and Doumpos [193].

4.3. Decision Rule Models: Rough Set Theory

Pawlak [129] introduced the rough set theory as a tool to describe dependencies between attributes, to evaluate the significance of attributes and to deal with inconsistent data. Generally, the rough set approach is a very useful tool in the study of sorting and classification problems, regarding the assignment of a set of alternatives into pre-specified groups. Recently, however, there have been several advances in this field to allow the application of the rough set theory to choice and ranking problems as well [60].

The rough set philosophy is founded on the assumption that with every alternative some information (data, knowledge) is associated. This information involves two types of attributes: condition and decision attributes. Condition attributes are those used to describe the characteristics of the alternatives (e.g., criteria), whereas the decision attributes define a partition of the alternatives into groups. Alternatives that have the same description in terms of the condition attributes are considered to be indiscernible. The indiscernibility relation constitutes the main mathematical basis of the rough set theory. Any set of all indiscernible alternatives is called an elementary set and forms a basic granule of knowledge about the universe. Any set of alternatives being a union of some elementary sets is referred to as crisp (precise) otherwise it is a rough set (imprecise, vague). A rough set can be approximated by a pair of crisp sets, called the lower and the upper approximation. The lower approximation includes the alternatives that certainly belong to the set and the upper approximation includes the alternatives that possibly belong to the set.

On the basis of these approximations, the first major capability that the rough set theory provides is to reduce the available information, so as to retain only what is absolutely necessary for the description and classification of the alternatives. This is achieved by discovering subsets of attributes, which provide the same quality of classification as the whole set of attributes. Such subsets of attributes are called reducts. Generally, the reducts are more than one. In such a case the intersection of all reducts is called the core. The core is the collection of the most relevant attributes, which cannot be excluded from the analysis without reducing the quality of the obtained description (classification).

The subsequent steps of the analysis involve the development of a set of "IF ... THEN ..." rules for the classification of the alternatives. The developed rules can be consistent if they include only one decision in their conclusion part, or approximate if their conclusion involves a disjunction of elementary decisions. Approximate rules are consequences of an approximate description of decision classes in terms of blocks of alternatives (granules) indiscernible by condition attributes. Such a situation indicates that using the available knowledge, one is unable to decide whether some alternatives belong to a given group (decision class) or not.

This traditional framework of the rough set theory, has been recently extended towards the development of a new preference modelling framework within the MCDA field [63; 62]. The main novelty of the recently developed rough set approach concerns the possibility of handling criteria, i.e., attributes with preference ordered domains, and preference ordered groups. Within this context the rough approximations of groups are defined according to the dominance relation, instead of the indiscernibility relation used in the basic rough sets approach. The decision rules derived from these approximations constitute a preference model.

4.4. Applications in Financial Decisions

MCDA discrete evaluation methods are well suited for the study of several financial decision-making problems. The diversified nature of the factors (evaluation criteria) that affect financial decisions, the complexity of the financial, business and economic environments, the subjective nature of many financial decisions, are only some of the features of financial decisions which are in accordance with the MCDA modelling framework. On the basis of these remarks this section reviews the up-to-date applications of MCDA discrete evaluation methods in several major financial decisions.

Bankruptcy and credit risk assessment. The assessment of bankruptcy and credit risk have been major research fields in finance for the last three decades. Bankruptcy risk is derived by the failure of

Table 1.1. Applications of MCDA approaches in bankruptcy and credit risk assessment

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[79; 160; 161]
Outranking relations	ELECTRE	[15; 38; 93]
	Other methods	[2; 180]
Preference disaggregation	UTA	[181; 184]
	UTADIS	[190; 191; 192]
	MHDIS	[41; 45]
	Other methods	[43; 64; 160]
Rough set theory		[37; 61; 153; 155]

a firm to meet its debt obligations to its creditors, thus leading the firm either to liquidation (discontinuity of the firm's operations) or to a reorganization program [188]. The concept of credit risk is similar to that of bankruptcy risk, in the sense that in both cases the likelihood that a debtor (firm, organization or individual) will not be able to meet its debt obligations to its creditors, is a key issue in the analysis. Credit risk assessment decisions, however, are not simply based on the estimation of this likelihood; furthermore, they take into account the opportunity cost that arises when a good client (firm or individual) is denied credit. In both cases, the most common approach used to address bankruptcy and credit risk assessment problems is to develop appropriate models that sort/classify the firms or the individuals into predefined groups (problematic β), e.g., classification of firms as bankrupt/non-bankrupt, or as high credit risk firms/low credit risk firms. Statistical and econometric techniques (discriminant analysis, logit and probit analysis, etc.) have dominated this field for several decades, but recently new methodologies have attracted the interest of researchers and practitioners including several MCDA methods [**?**; 188]. A representative list of the MCDA evaluation approaches applied in bankruptcy and credit risk assessment is presented in Table 1.1.

Portfolio selection and management. Portfolio selection and management involves the construction of a portfolio of securities (stocks, bonds, treasury bills, mutual funds, etc.) that maximizes the investor's utility. This problem can be realized as a two stage process [78; 77]: (1) the evaluation of the available securities to select the ones that best meet the investor's preferences, (2) specification of the amount of capital to be invested in each of the securities selected in the first stage.

Table 1.2. Applications of MCDA approaches in portfolio selection and management

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[143]
	MACBETH	[12; 11]
	Other methods	[29; 40; 53; 88; 134]
Outranking relations	ELECTRE	[76; 78; 77; 95; 115; 116; 167]
	PROMETHEE	[65; 94; 116]
Preference disaggregation	UTA	[77; 78; 183; 197]
	UTADIS	[194; 195]
	MHDIS	[48]
Rough set theory		[89]

The implementation of these two stages in the traditional portfolio theory is based on the mean-variance approach developed by Markowitz [111; 113]. Recently, however, the multi-dimensional nature of the problem has been emphasized by researchers in finance [84], as well as by MCDA researchers [**?**; 177; 178]. Within this multi-dimensional context, MCDA discrete evaluation methods provide significant support in evaluating securities according to the investor's policy. Studies conducted on this topic have focused on the modelling and representation of the investor's policy, goals and objectives in a mathematical model. The model aggregates all the pertinent factors describing the performance of the securities and provides their overall evaluation. The securities with the higher overall evaluation are selected for portfolio construction purposes in a latter stage of the analysis. Table 1.2 summarizes several studies involving the application of MCDA evaluations methods in portfolio selection and management.

Corporate performance evaluation. The evaluation of the performance of corporate entities and organizations is an important activity for their management and shareholders as well as for investors and policy makers. Such an evaluation provides the management and the shareholders with a tool to assess the strength and weakness of the firm as well as its competitive advantages over its competitors, thus providing guidance on the choice of the measures that need to be taken to overcome the existing problems. Investors (institutional and individual) are interested in the assessment of corporate performance for guidance to their investment decisions, while policy makers may use such an assessment to identify the existing problems in the business environment and take measures that will ensure a sustainable economic growth and

Table 1.3. Applications of MCDA approaches in the assessment of corporate performance

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[5; 102]
	Other methods	[36; 176]
Outranking relations	ELECTRE	[30]
	PROMETHEE	[5; 14; 30; 106; 107; 108; 109; 128; 179]
Preference disaggregation	UTA	[152; 187; 196]
	UTADIS	[121; 174]

social stability. The performance of a firm or an organization is clearly multi-dimensional, since it is affected by a variety of factors of different nature, such as: (1) financial factors indicating the financial position of the firm/organization, (3) strategic factors of qualitative nature that define the internal operation of the firm and its relation to the market (organization, management, market trend, etc. [181], (2) economic factors that define the economic and business environment. The aggregation of all these factors into a global evaluation index is a subjective process that depends on the decision maker's values and judgment policy. These findings are in accordance with the MCDA paradigm, thus leading several operational researchers to the investigation of the capabilities that MCDA methods provide in supporting decision maker's in making decisions regarding the evaluation of corporate performance. An indicative list of studies on this topic is given in Table 1.3.

Investment appraisal. In most cases the choice of investment projects is an important strategic decision for every firm, public or private, large or small. Therefore, the process of an investment decision should be conveniently modelled. In general, the investment decision process consists of four main stages: perception, formulation, evaluation and choice. The financial theory intervenes only in the stages of evaluation and choice based on traditional financial criteria such as the payback period, the accounting rate of return, the net present value, the internal rate of return, the index of profitability, the discounted payback method, etc. [28]. This approach, however, entails some shortcomings such as the difficulty in aggregating the conflicting results of each criterion and the elimination of important qualitative variables from the analysis [185]. MCDA, on the other hand, contributes in a very original way to the investment decision process, supporting all stages of the investment process. Concerning the stages of perception and formulation,

Table 1.4. Applications of MCDA approaches in investment appraisal

Approaches	Methods	Studies
Multiattribute utility theory	AHP	[96]
	Other methods	[53; 130]
Outranking relations	ELECTRE	[25; 33]
	PROMETHEE	[133; 175]
	ORESTE	[33]
Preference disaggregation	UTA	[16; 149]
	UTADIS	[80]

MCDA contributes to the identification of possible actions (investment opportunities) and to the definition of a set of potential actions (possible variants, each variant constituting an investment project in competition with others). Concerning the stages of evaluation and choice, MCDA supports the introduction in the analysis of both quantitative and qualitative criteria. Criteria such as the urgency of the project, the coherence of the objectives of the projects with those of the general policy of the firm, the social and environmental aspects should be taken into careful consideration. Therefore, MCDA contributes through the identification of the best investment projects according to the problematic chosen, the satisfactory resolution of the conflicts between the criteria, the determination of the relative importance of the criteria in the decision-making process, and the revealing of the investors' preferences and system of values. These attractive features have been the main motivation for the use of MCDA methods in investment appraisal in several real-world cases. A representative list of studies in presented in Table 1.4.

Other financial decision problems. Except for the above financial decision-making problems, discrete MCDA evaluation methods are also applicable in several other fields of finance. Table 1.5 list some additional applications of MCDA methods in other financial problems, including venture capital, country risk assessment and the prediction of corporate mergers and acquisitions. In venture capital investment decisions, MCDA methods are used both as tools to evaluate the firms that seek venture capital financing, as well as analysis tools to identify the factors that drive such financing decisions. In country risk assessment, MCDA methods are used to developed models that aggregate the appropriate economic, financial and socio-political factors, to support the evaluation of the creditworthiness and the future prospects of the coun-

Table 1.5. Applications of MCDA approaches in other financial decision-making problems

Topic	Methodology	Studies
Venture capital	Conjoint analysis	[123; 135]
	UTA	[150; 182]
Country risk	MAUT	[169]
	UTA	[1; 32]
	UTADIS	[1; 44; 189]
	MHDIS	[42; 44; 46]
	Other methods	[31; 125; 126]
Mergers and acquisitions	Rough sets	[154]

tries. Finally, in corporate mergers and acquisitions MCDA methods are used to assess the likelihood that a firm will be merged or acquired on the basis of financial information (financial ratios) and strategic factors.

5. Conclusions and Future Perspectives

This chapter discussed the contribution of MCDA in financial decision-making problems, focusing on the justification of the multi-dimensional character of financial decisions and the use of different MCDA methodologies to support them.

Overall, the main advantages that the MCDA paradigm provides in financial decision making, could be summarized in the following aspects [185]: (1) the possibility of structuring complex evaluation problems, (2) the introduction of both quantitative (i.e. financial ratios) and qualitative criteria in the evaluation process, (3) the transparency in the evaluation, allowing good argumentation in financial decisions, and (4) the introduction of sophisticated, flexible and realistic scientific methods in the financial decision-making process.

In conclusion, MCDA methods seem to have a promising future in the field of financial management, because they offer a highly methodological and realistic framework to decision problems. Nevertheless, their success in practice depends heavily on the development of computerized multicriteria decision support systems. Financial institutions as well as firms acknowledge the multi-dimensional nature of financial decision problems [17]. Nevertheless, they often use optimization or statistical approaches to address their financial problems, since optimization and statistical software packages are easily available in relatively low cost, even though many of these software packages are not specifically designed for financial decision-making problems. Consequently, the use of MCDA methods to support real time financial decision making, calls upon the development of integrated user-friendly multicriteria decision support systems that will be specifically designed to address financial problems. Examples of such systems are the CGX system [161], the BANKS system [108], the BANKADVISER system [106], the INVEX system [175], the FINEVA system [196], the FINCLAS system [190], the INVESTOR system [194], etc. The development and promotion of such systems is a key issue in the successful application of MCDA methods in finance.

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Notes

1. The underlying assumptions must be validated and the effectiveness and efficiency of the actions taken must be evaluated systematically. The latter calls for a sophisticated performance evaluation process that explicitly acknowledges the role of learning.
2. This section draws heavily on a part of Hallerbach & Spronk [66].

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